

Linear Algebra

Let $A \in R^{n \times n}$ and $\mathbf{x}, \mathbf{b} \in R^n$ with $n = 3$ be defined as follows.

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix}$$

A linear system of equations $A\mathbf{x} = \mathbf{b}$ can be written as

$$\begin{array}{rcrcrcrcrl} 2y & + & z & = & 3 & & & \left[\begin{array}{ccc|c} & 2 & 1 & 3 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 3 \end{array} \right] \\ x & - & y & + & z & = & 1, & \\ x & + & y & + & z & = & 3 & \end{array}$$

Use Matlab commands to answer the following questions.

- (a) Input matrix A .
- (b) Input matrix b .
- (c) Apply Gaussian elimination with partial pivoting to solve $A\mathbf{x} = \mathbf{b}$.
- (d) In (c), report the matrices of P, L, U , respectively, where P is a permutation matrix, L is unit *lower* $-\Delta$ and U is *upper* $-\Delta$.
- (e) Find the determinant of A .
- (f) Find the rank of A .
- (g) Find the inverse matrix of A in a rational form.
- (h) Find the characteristic polynomial $p(x)$ of A .
- (i) Find the roots of $p(x) = 0$.
- (j) Find the eigenvalues of A .
- (k) Find the QR factorization of A .

- (l) Find the singular value decomposition of A .
- (m) Find $\|A\|_1$, $\|A\|_2$, and $\|A\|_\infty$, respectively.
- (n) Find LU -*decomposition* for matrix A , where L is unit *lower* - Δ and U is *upper* - Δ .