True-False Problems

November 6, 2017

StudentName : ______________ *StudentNumber* : ______________ *Index* : ___

Mark ◯ if the statement is true, otherwise mark ×.

(×) **(1)** $R^2 \leq R^3$.

(×) **(2)** It is possible to find a pair of 2-dimensional subspaes S and T of R^3 such that $S \cap T = \{\mathbf{0}\}$.

(×) **(3)** If S and T are subspaces of a vector space V, then $S \cup T$ is a subspace of V.

(◯) **(4)** If S and T are subspaces of a vector space V, then $S \cap T$ is a subspace of V.

(◯) **(5)** If S and T are subspaces of a vector space V, then $S + T$ is a subspace of V.

(◯) **(6)** If $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n$ are linearly independent, then they span R^n.

(×) **(7)** If $\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_n$ span a vector space V, then they are linearly independent.

(◯) **(8)** If $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k$ are vectors in a vector space V and

$$span(\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k) = span(\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_{k-1})$$

then $\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_k$ are linearly *dependent*.

(◯) **(9)** If $A \in R^{m \times n}$, then A and A^t have the same *rank*.

(×) **(10)** If $A \in R^{m \times n}$, then A and A^t have the same *nullity*.

(◯) **(11)** Let $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ be linearly independent vectors in R^n and let

$$\mathbf{y}_1 = \mathbf{x}_2 + \mathbf{x}_1, \quad \mathbf{y}_2 = \mathbf{x}_3 + \mathbf{x}_2, \quad \mathbf{y}_3 = \mathbf{x}_3 + \mathbf{x}_1$$

Then $\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3$ are linearly *independent*.

(×) **(12)** Let $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3$ be linearly independent vectors in R^n and let

$$\mathbf{y}_1 = \mathbf{x}_2 - \mathbf{x}_1, \quad \mathbf{y}_2 = \mathbf{x}_3 - \mathbf{x}_2, \quad \mathbf{y}_3 = \mathbf{x}_3 - \mathbf{x}_1$$

Then $\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3$ are linearly *independent*.

(◯) **(13)** $A \in R^{n \times n}$ and let $Null(A) = \{\mathbf{0}\}$. Then $rank(A) = n$ and A is nonsingular.