Cholesky Algorithm

□ **Theorem:** Every positive definitive matrix A can be decomposed as $A = LL^t$, where L is $lower - \Delta$.

□ **Algorithm:** $A \in R^{n\times n}$, $A = LL^t$, A is positive definite and L is $lower - \Delta$.

for $j = 0, 1, \cdots, n-1$

$L_{jj} \leftarrow \left[A_{jj} - \sum_{k=0}^{j-1} L_{jk}^2\right]^{1/2}$

for $i = j+1, j+2, \cdots, n-1$

$L_{ij} \leftarrow \left[A_{ij} - \sum_{k=0}^{j-1} L_{ik}L_{jk}\right]/L_{jj}$

endfor

endfor

$$C = \begin{bmatrix} 4 & -2 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix} = L_1L_1^t$$

$$A = \begin{bmatrix} 9 & 3 & -3 \\ 3 & 17 & 3 \\ -3 & 3 & 27 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 4 & 0 \\ -1 & 1 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 & -1 \\ 0 & 4 & 1 \\ 0 & 0 & 5 \end{bmatrix} = L_2L_2^t$$