Quiz 3 for CS2334(01)

December 13, 2017

StudentName : ____________ *ID* : ____________ *Index* : _____

Let $A \in R^{3\times 3}$ be defined as follows.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 1 & -3 \end{bmatrix}$$

Answer the following questions.

(1) Find the three eigenvalues $\lambda_1,\ \lambda_2,\ \lambda_3$ and their corresponding *unit* eigenvectors $\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3$, that is, $\|\mathbf{v}_i\|_2 = 1,\ 1 \le i \le 3$.

(2) Find the *characteristic polynomial* of A.

(3) Evaluate the trace of A.

(4) Evaluate the determinant of A.

(5) What are the eigenvalues of A^{-1} and their corresponding eigenvectors?

(6) By **(1)**, let $V = \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3]$, then, we have $V^{-1}AV = \Lambda$, where $\Lambda = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -4 \end{bmatrix}$.

What are the three eigenvalues of A^3 and their corresponding eigenvectors?

(7) What is V^tAV?

(8) What is V^tA^3V?

(9) Can you use *Simple Matlab Commands* to solve the above problems?

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December 13, 2017

StudentName : <u>*Solutions*</u> *ID* : <u>20171213</u> *Index* : ____

Let $A \in R^{3\times 3}$ be defined as follows.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -3 & 1 \\ 0 & 1 & -3 \end{bmatrix}$$

Solutions:

(1) $\lambda_1 = 3,\ \lambda_2 = -2,\ \lambda_3 = -4.$ $\mathbf{v}_1 = [1,0,0]^t$, $\mathbf{v}_2 = [0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]^t$, $\mathbf{v}_3 = [0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}]^t$.

Matlab `[V lambda]=eig(A);`

(2) $poly(A) = P(\lambda) = \lambda^3 + 3\lambda^2 - 10\lambda - 24.$

Matlab `poly(A)`

(3) $trace(A) = \sum_{i=1}^{3} a_{ii} = \sum_{i=1}^{3} \lambda_i = -3.$

Matlab `trace(A)`

(4) $det(A) = \prod_{j=1}^{3} \lambda_j = 3 \times (-2) \times (-4) = 24.$

Matlab `det(A)`

(5) $\mu_1 = \frac{1}{3},\ \mu_2 = \frac{-1}{2},\ \mu_3 = \frac{-1}{4}$ and $\mathbf{v}_1 = [1,0,0]^t$, $\mathbf{v}_2 = [0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]^t$, $\mathbf{v}_3 = [0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}]^t$.

Matlab `[V, mu]=eig((inv(A))`

(6) $\lambda(A^3) = \{3^3, (-2)^3, (-4)^3\} = \{27, -8, -64\}.$ $\mathbf{v}_1 = [1,0,0]^t$, $\mathbf{v}_2 = [0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]^t$, $\mathbf{v}_3 = [0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}]^t$.

Matlab `[V, D]=eig(A^3)`

(7) $V^t A V = \Lambda.$

Matlab `Lambda=V'*A*V`

(8) $V^t A^3 V = \Lambda^3.$

Matlab `LambdaCube=V'*(A^3)*V`

(9) Can you use *Simple Matlab Commands* to solve the above problems?