Exam 3 for CS2334(01)
January 8, 2018

$Name$: ____________ $StudentNumber$: _____________ $Index$: _____

(30%)1. Let

$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 4 \\ 3 & 5 \end{bmatrix}$$

(a) Compute eigenvalues and eigenvectors and give a *spectrum decomposition* for matrix A.

(b) Find the *Cholesky factorization* for A.

(c) Find the *singular value decomposition* for A.

(d) Find the $LU decomposition$ for matrix C.

(e) Find the *QR factorization* for C.

(f) Find the *singular value decomposition* for C.

(30%)2. Let $B \in R^{3\times 3}$ be a matrix defined as follows.

$$B = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

(a) Find the *characteristic polynomial* of B.

(b) Find the eigenvalues $\lambda_1,\ \lambda_2,\ \lambda_3$ and their corresponding *unit* eigenvectors $\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3$, that is, $\|\mathbf{v}_i\|_2 = 1,\ 1 \le i \le 3$.

(c) Find the eigenvalues of B^{-1} and their corresponding *unit eigenvectors.*

(d) Find an othogonal matrix U such that $U^t BU$ is a diagonal matrix.

(e) Find the eigenvalues $\mu_1,\ \mu_2,\ \mu_3$ of matrix $B + 2I$ and their corresponding *unit eigenvectors* $\mathbf{w}_1,\ \mathbf{w}_2,\ \mathbf{w}_3$.

(f) Find the eigenvalues $\tau_1,\ \tau_2,\ \tau_3$ of matrix $(B - I)^5$.

3.(20%) Mark ◯ if the statement is *true*, and mark × otherwise, or give your comments.

( ) **(a)** Cholesky decomposition is suitable for every real and symmetric matrix.

( ) **(b)** Every nonsingular matrix has an LU decomposition.

( ) **(c)** Every nonsingular matrix has a QR factorization.

( ) **(d)** Every real symmetric matrix must have nonnegative eigenvalues.

( ) **(e)** Every real diagonally dominant matrix must have positive eigenvalues.

( ) **(f)** Let $\mathbf{x}, \mathbf{y} \in R^n$ and $C \in R^{n\times n}$ is an orthogonal matrix, then $\langle C\mathbf{x}, C\mathbf{x}\rangle = \|\mathbf{x}\|_2^2$.

( ) **(g)** The product of all eigenvalues of a real matrix equals its determinant.

( ) **(h)** The product of two orthogonal and symmetric matrices is orthogonal and symmetric.

( ) **(i)** In a QR-factorization $A = QR$ of a nonsingular matrix A, $det(R) \neq 0$.

( ) **(j)** Let $H_i \in R^{n\times n}$, $1 \leq i \leq k$ be Householder matrices, then $\prod_{j=1}^{k} H_j = (-1)^n$.

( ) **(k)** Let $\{\mathbf{v}_1, \mathbf{v}_2, \cdots, \mathbf{v}_n\}$ be an orthonormal basis of R^n. If $C \in R^{n\times n}$ is nonsingular, then $\{C\mathbf{v}_1, C\mathbf{v}_2, \cdots, C\mathbf{v}_n\}$ is also an orthonormal basis of R^n.

4.(10%) Let $H \in R^{n\times n}$ be a Householder matrix.

(a) Show that H must have an eigenvalue -1.

(b) Show that if λ is an eigenvalue of H, then $|\lambda| = 1$.

6.(14%) Let $b = \begin{bmatrix} 1 \\ -5 \\ 4 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. *Writing* **a single Matlab command** *to solve each of the following questions* for **a** $\sim$ **h** and answer the questions for **i** $\sim$ **j**.

(a) Randomly generate a 3 by 3 matrix A whose elements are integers in [0, 10].

(b) Input vector b.

(c) Solve the linear system $Ax = b$ for x.

(d) Input matrix C given above.

(e) Find the characteristic polynomial for C.

(f) Find the eigenvalues and eigenvectors of C.

(g) Find the $QR - factorization$ of the matrix C.

(h) Find the *singular value decomposition* for the matrix C.

(3%)(i) Show the output results of **(f)**.

(3%)(j) Show the output results of **(h)**.