Exam 1 for CS2334(01)

October 23, 2017

Name : ____________ *StudentNumber* : _____________ *Index* : ______

(15%)(1) Given

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 2 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & -1 \\ 0 & 2 \\ 3 & 1 \end{bmatrix}$$

(a) Find AB **(b)** Find $(AB)^{-1}$ **(c)** Find $B^t A^t$

(15%)(2) Let $C, D \in R^{3\times 3}$ with $det(C) = 4$ and $det(D) = 5$. Find the value of

(a) $det(CD)$ **(b)** $det(3C)$ **(c)** $det(2CD)$ **(d)** $det(C^{-1}D)$

(15%)(3) Let the matrices W and V be defined by the following Matlab commands, respectively.

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W=[1, 3, 9; 1, 4, 16; 1, 5, 25];
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V=[1, 12, 144; 1,13,169; 1, 14, 196];
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(a) Give the matrices W and V, respectively.

(b) Find $det(W)$

(c) Find $det(V)$

(15%)(4) Let $P, Q, R \in R^{3\times 3}$ be defined as

$$P = I - \mathbf{e}_2\mathbf{e}_1^t, \quad Q = I - 2\mathbf{e}_3\mathbf{e}_1^t, \quad R = I - 3\mathbf{e}_3\mathbf{e}_2^t$$

(a) Express P^{-1}, Q^{-1}, R^{-1} in a matrix form.

(b) Express $S = PQR$ in a matrix form.

(c) Express $T = R^{-1}Q^{-1}P^{-1}$ in a matrix form.

(25%)(5) A linear system of equations is given below.

$$\begin{array}{rcrcrcr} 2x & + & y & + & z & = & 5 \\ 4x & - & 6y & & & = & -2 \\ -2x & + & 7y & + & 2z & = & 9 \end{array}$$

(a) Express this system as $A\mathbf{x} = \mathbf{b}$, where $\mathbf{x} = [x, y, z]^t$. Show the augmented matrix for this system.

(b) Use *Gaussian elimination* and *back substitution* to solve this system of equations.

(c) Find $A = LU$, where L is unit lower-Δ and U is upper-Δ.

(d) Use *Gaussian elimination with partial pivoting* and *back substitution* to solve this system of equations.

(e) Give Matlab commands to solve $A\mathbf{x} = \mathbf{b}$ in (**a**).

(15%)(6) Prove the following statements.

(a) A matrix A is *idempotent* if $A^2 = A$. Show that if A is *idempotent*, so is $I - A$.

(b) Show that if A is *idempotent*, then $2A - I$ is invertible and is its own inverse.

(c) Let U_n be the $n \times n$ matrix, each of whose entries is 1. Show that for $n > 1$,

$$(I - U_n)^{-1} = I - \frac{1}{n-1} U_n$$