1. (20%) Given

\[ A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 3 & 0 \\ 1 & -3 \end{bmatrix} \]

(a) Find \( A \times B \)

(b) Find \((A \times B)^{-1}\)

(c) Find \((A \times B)^t\)

(d) Give Matlab commands to solve (a \sim c), respectively.

2. (20%) Given

\[ A = \begin{bmatrix} 3 & 2 & 4 \\ 1 & -2 & 3 \\ 2 & 3 & 2 \end{bmatrix} \]

(a) Find \( \det(M_{11}), \det(M_{12}), \) and \( \det(M_{13}). \)

(b) Find the cofactors \( A_{11}, A_{12}, \) and \( A_{13}. \)

(c) Compute \( 3A_{11} - 2A_{12} + 4A_{13}. \)

(d) Compute \( \det(A). \)
3. (20%) Let $P, Q, R \in \mathbb{R}^{3 \times 3}$ be defined as

$$P = I - 2e_2e_1^t, \quad Q = I + e_3e_1^t, \quad R = I + e_3e_2^t$$

(a) Express $P^{-1}, Q^{-1}, R^{-1}$ in matrix form.

(b) Write $(R \ast Q \ast P)^{-1}$ in matrix form.

(c) Write $P \ast Q \ast R$ in matrix form.

(d) Write a Matlab command to generate $P$ and $Q$, respectively.

4. (15%) A Hilbert matrix $H = [h_{ij}]$ of order 3 is defined as $h_{ij} = \frac{1}{i+j}$, where $1 \leq i, j \leq 3$.

(a) Write $H$ in a matrix form.

(b) Find $H^{-1}$. 
5. (25%) A linear system of equations is given below.

\begin{align*}
2x + y + z &= 5 \\
4x - 6y &= -2 \\
-2x + 7y + 2z &= 9
\end{align*}

(a) Express this system as $Ax = b$, where $\mathbf{x} = [x, y, z]^t$. Show the augmented matrix for this system.

(b) Use Gaussian elimination and back substitution to solve this system of equations.

(c) Find $A = LU$, where $L$ is unit lower-$\Delta$ and $U$ is upper-$\Delta$.

(d) Give Matlab commands to solve $Ax = b$ in this problem.