1. (20%) Mark ○ if the statement is true, otherwise mark × if the statement is false.

(a) The product of unit lower-$\Delta$ matrices is also unit lower-$\Delta$.

(b) If $A, B \in \mathbb{R}^{n \times n}$ are nonsingular, then $(AB)^{-1} = A^{-1}B^{-1}$.

(c) Let $A \in \mathbb{R}^{n \times n}$, then $\det(\alpha A) = \alpha^n \det(A)$, where $\alpha \in \mathbb{R}$.

(d) Let $H \in \mathbb{R}^{n \times n}$ such that $H^2 = I$, then $\det(H) = 1$.

(e) The product of two elementary matrices is also an elementary matrix.

2. (20%) Given

$$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 1 & -3 \\ -1 & 1 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 3 & 0 \\ 1 & -2 \end{bmatrix}$$

(a) Find $AB$

(b) Find $(AB)^{-1}$

(c) Find $(AB)^t$

(d) Find $BA$
3. (20%) Given

\[
A = \begin{bmatrix}
3 & 2 & 4 \\
1 & -2 & 3 \\
2 & 3 & 2
\end{bmatrix}
\]

(a) Find \( \text{det}(M_{21}) \), \( \text{det}(M_{22}) \), and \( \text{det}(M_{23}) \).

(b) Find the cofactors \( A_{21} \), \( A_{22} \), and \( A_{23} \).

(c) Compute \( A_{21} - 2A_{22} + 3A_{23} \).

(d) Compute \( \text{det}(A) \).

4. (15%) Let \( P, Q, R \in \mathbb{R}^{3 \times 3} \) be defined as

\[
P = I - 3e_{2}e_{1}^{t}, \quad Q = I + e_{3}e_{1}^{t}, \quad R = I - 2e_{3}e_{2}^{t}
\]

(a) Express \( P^{-1}, Q^{-1}, R^{-1} \) in matrix form.

(b) Write \( P^{-1}Q^{-1}R^{-1} \) in matrix form.

(c) Write \( R \ast Q \ast P \) in matrix form.
5. (25%) A linear system of equations is given below.

\[
\begin{align*}
3x + y - z &= 0 \\
-6x + 2z &= -4 \\
3x - 3y &= 9
\end{align*}
\]

(a) Express this system as \( Ax = b \), where \( x = [x, y, z]^t \). Show the augmented matrix for this system.

(b) Use Gaussian elimination and back substitution to solve this system of equations.

(c) Find \( A = LU \), where \( L \) is unit lower-\( \Delta \) and \( U \) is upper-\( \Delta \).