Crafting a Compiler

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Preface

Much has changed since *Crafting a Compiler*, by Fischer and LeBlanc, was published in 1988. While instructors may remember the $5\frac{1}{4}$-inch floppy disk of software that accompanied that text, most students today have neither seen nor held such a disk. Many changes have occurred in the programming languages that students experience in class and in the marketplace. In 1991 the book was available in two forms, with algorithms presented in either C or Ada. While C remains a popular language, Ada has become relatively obscure and did not achieve its predicted popularity. The C++ language evolved from C with the addition of object-oriented features. Java$^{\text{TM}}$ was developed as a simpler object-oriented language, gaining popularity because of its security and ability to be run within a Web browser. The College Board Advanced Placement curriculum moved from Pascal to C++ to Java.

While much has changed, students and faculty alike continue to study and teach the subject of compiler construction. Research in the area of compilers and programing language translation continues at a brisk pace, as compilers are tasked with accommodating an increasing diversity of architectures and programming languages. Software development environments depend on compilers interacting successfully with a variety of software toolchain components such as syntax-informed editors, performance profilers, and debuggers. All modern software efforts rely on their compilers to check vigorously for errors and to translate programs faithfully.

Some texts experience relatively minor changes over time, acquiring perhaps some new exercises or examples. This book reflects a substantive revision of the material from 1988 and 1991. While the focus of this text remains on teaching the fundamentals of compiler construction, the algorithms and approaches have been brought into modern practice:

- Coverage of topics that have faded from practical use (e.g., **attribute grammars**) has been minimized or removed altogether.

- Algorithms are presented in a **pseudocode** style that should be familiar to students who have studied the fundamental algorithms of our discipline.

Pseudocode enables a concise formulation of an algorithm and a rational discussion of the algorithm's purpose and construction.

The details of implementation in a particular language have been relegated to the *Crafting a Compiler Supplement* which is available online:

`http://www.pearsonhighered.com/fischer/`

- Parsing theory and practice are organized to facilitate a variety of pedagogical approaches.

  Some may study the material at a high level to gain a broad view of top-down and bottom-up parsing. Others may study a particular approach in greater detail.

- The front- and back-end phases of a compiler are connected by the **abstract syntax tree** (AST), which is created as the primary artifact of parsing. Most compilers build an AST, but relatively few texts articulate its construction and use.

  The **visitor pattern** is introduced for traversing the AST during semantic analysis and code generation.

- Laboratory and studio exercises are available to instructors.

  Instructors can assign some components as exercises for the students while other components are supplied from our course-support Web site.

Some texts undergo revision by the addition of more graduate-level material. While such information may be useful in an advanced course, the focus of *Crafting a Compiler* remains on the undergraduate-level study of compiler construction. A graduate course could be offered using Chapters 13 and 14, with the earlier portions of the text serving as reference material.

Text and Reference

As a classroom text, this book is oriented toward a curriculum that we have developed over the past 25 years. The book is very flexible and has been adopted for courses ranging from a three-credit upper-level course taught in a ten-week quarter to a six-credit semester-long graduate course. The text is accessible to any student who has a basic background in programming, algorithms, and data structures. The text is well suited to a single semester or quarter offering because its flexibility allows an instructor to craft a syllabus according to his or her interests. Author-sponsored solutions are available for those components that are not studied in detail. It is feasible to write portions of a compiler from parsing to code generation in a single semester.

This book is also a valuable professional reference because of its complete coverage of techniques that are of practical importance to compiler construction. Many of our students have reported, even some years after their graduation, of their successful application of these techniques to problems they encounter in their work.

Instructor Resources

The Web site for this book can be found at `http://www.pearsonhighered.com/fischer/`. The material posted for qualified instructors includes sample laboratory and project assignments, studio (active-learning) sessions, libraries of code that can be used as class-furnished solutions, and solutions to selected exercises.

For access to these materials, qualified instructors should contact their local Pearson Representative by visiting `http://www.pearsonhighered.com`, by sending email to `computing@aw.com`, or by visiting the Pearson Instructor Resource Center at `http://www.pearsonhighered.com/irc/`.

Student Resources

The book's Web site at `http://www.pearsonhighered.com/fischer/` contains working code for examples used throughout the book, including code for the toy language `ac` that is introduced in Chapter 2. The site also contains tutorial notes and a page with links to various compiler-construction tools.

Access to these materials may be guarded by a password that is distributed with the book or obtained from an instructor.

Project Approach

This book offers a comprehensive coverage of relevant theoretical topics in compiler construction. However, a cohesive implementation project is typically an important aspect of planning a curriculum in compiler construction. Thus, the book and the online materials are biased in favor of a sequence of exploratory exercises, culminating in a project, to support learning this material.

Lab exercises, studio sessions, and course projects appear in the *Crafting a Compiler Supplement*, and readers are invited to send us other materials or links for posting at our Web site. The exercises parallel the chapters and progression of material presented in the text. For example, Chapter 2 introduces the toy language ac to give an overview of the compilation process. The Web site contains full, working versions of the scanner, parser, semantics analyzer, and code generator for that language. These components will be available in a variety of source programming languages.

The Web site also offers material in support of developing a working compiler for a simple language modeled after Java. This allows instructors to assign some components as exercises while other components are provided to fill in any gaps. Some instructors may provide the entire compiler and ask students to implement extensions. Polishing and refining existing components can also be the basis of class projects.

Pseudocode and Guides

A significant change from the Fischer and LeBlanc text is that algorithms are no longer presented in any specific programming language such as C or Ada. Instead, algorithms are presented in **pseudocode** using a style that should be familiar to those who have studied even the most fundamental algorithms [CLRS01]. Pseudocode simplifies the exposition of an algorithm by omitting unnecessary detail. However, the pseudocode is suggestive of constructs used in real programming languages, so implementation should be straightforward. An index of all pseudocode methods is provided as a **guide** at the end of this book.

The text makes extensive use of abbreviations (including acronyms) to simplify exposition and to help readers acquire the terminology used in compiler construction. Each abbreviation is fully defined automatically at its first reference in each chapter. For example, AST has already been used in this preface, as an abbreviation of abstract syntax tree, but **context-free grammar** (CFG) has not. For further help, an index of all abbreviations appears as a **guide** at the end of the book. The full index contains abbreviations and indicates where they are referenced throughout the book. Terms such as **guide** are shown in boldface. Each reference to such terms is included in the full index.

Using this Book

An introductory course on compiler construction could begin with Chapters 1, 2, and 3. For parsing technique, either top-down (Chapter 5) or bottom-up (Chapter 6) could be chosen, but some instructors will choose to cover both. Material from Chapter 4 can be covered as necessary to support the parsing techniques that will be studied. Chapter 7 articulates the AST and presents the **visitor pattern** for its traversal. Some instructors may assign AST-management utilities as a lab exercise, while others may use the utilities provided by the

Web site. Various aspects of semantic analysis can then be covered at the instructor's discretion in Chapters 8 and 9. A quarter-based course could end here, with another quarter continuing with the study of code generation, as described next.

Chapter 10 provides an overview of the **Java Virtual Machine** (JVM), which should be covered if students will generate JVM code in their project. Code generation for such virtual machines is covered in Chapter 11. Instructors who prefer students to generate machine code could skip Chapters 10 and 11 and cover Chapters 12 and 13 instead. An introductory course could include material from the beginning of Chapter 14 on automatic program optimization.

Further study could include more detail of the parsing techniques covered in Chapters 4, 5, and 6. Semantic analysis and type checking could be studied in greater breadth and depth in Chapters 8 and 9. Advanced concepts such as **static single assignment** (SSA) Form could be introduced from Chapters 10 and 14. Advanced topics in program analysis and transformation, including data flow frameworks, could be drawn from Chapter 14. Chapters 13 and 14 could be the basis for a gradute compiler course, with earlier chapters providing useful reference material.

Chapter Descriptions

Chapter 1 Introduction

The text begins with an overview of the compilation process. The concepts of constructing a compiler from a collection of components are emphasized. An overview of the history of compilers is presented and the use of tools for generating compiler components is introduced.

Chapter 2 A Simple Compiler

The simple language ac is presented, and each of the compiler's components is discussed with respect to translating ac to another language, dc. These components are presented in pseudocode, and complete code can be found in the *Crafting a Compiler Supplement*.

Chapter 3 Scanning—Theory and Practice

The basic concepts and techniques for building the lexical analysis components of a compiler are presented. This discussion includes the development of hand-coded scanners as well as the use of scanner-generation tools for implementing table-driven lexical analyzers.

Chapter 4 Grammars and Parsing

This chapter covers the fundamentals of formal language concepts, including context-free grammars, grammar notation, derivations, and parse trees. Grammar-analysis algorithms are introduced that are used in Chapters 5 and 6.

Chapter 5 Top-Down Parsing

Top-down parsing is a popular technique for constructing relatively simple parsers. This chapter shows how such parsers can be written using explicit code or by constructing a table for use by a generic top-down parsing engine. Syntactic error diagnosis, recovery, and repair are discussed.

Chapter 6 Bottom-Up Parsing

Most compilers for modern programming languages use one of the bottom-up parsing techniques presented in this chapter. Tools for generating such parsers automatically from a context-free grammar are widely available. The chapter describes the theory on which such tools are built, including a sequence of increasingly sophisticated approaches to resolving **conflicts** that hamper parser construction for some grammars. Grammar and language **ambiguity** are thoroughly discussed, and heuristics are presented for understanding and resolving ambiguous grammars.

Chapter 7 Syntax-Directed Translation

This marks the mid-point of the book in terms of a compiler's components. Prior chapters have considered the lexical and syntactic analysis of programs. A goal of those chapters is the construction of an AST. In this chapter, the AST is introduced and an interface is articulated for constructing, managing, and traversing the AST. This chapter is pivotal in the sense that subsequent chapters depend on understanding both the AST and the **visitor pattern** that facilitates traversal and processing of the AST. The *Crafting a Compiler Supplement* contains a tutorial on the visitor pattern, including examples drawn from common experiences.

Chapter 8 Symbol Tables and Declaration Processing

This chapter emphasizes the use of a symbol table as an abstract component that can be utilized throughout the compilation process. A precise interface is defined for the symbol table, and various implementation issues and ideas are presented. This discussion includes a study of the implementation of nested scopes.

The semantic analysis necessary for processing symbol declarations is introduced, including types, variables, arrays, structures, and enumerations. An introduction to type checking is presented, including object-oriented classes, subclasses, and superclasses.

Chapter 9 Semantic Analysis

Additional semantic analysis is required for language specifications that are not easily checked while parsing. Various control structures are examined, including conditional branches and loops. The chapter includes a discussion of exceptions and the semantic analysis they require at compile-time.

Chapter 10 Intermediate Representations

This chapter considers two intermediate representations are are widely used by compilers. The first is the JVM instruction set and bytecode format, which has become the standard format for representing compiled Java programs. For readers who are interested in targeting the JVM in a compiler project, Chapters 10 and 11 provide the necessary background and techniques. The other representation is SSA Form, which is used by many optimizing compilers. This chapter defines SSA Form, but its construction is delayed until Chapter 14, where some requisite definitions and algorithms are presented.

Chapter 11 Code Generation for a Virtual Machine

This chapter considers code generation for a **virtual machine** (VM). The advantages of considering such a target is that many of the details of runtime support are subsumed by the VM. For example, most VMs offer an unlimited number of registers, so that the issue of **register allocation**, albeit interesting, can be postponed until the fundamentals of code generation are mastered. The VM's instruction set is typically at a higher level than machine code. For example, a method call is often supported by a single VM instruction, while the same call would require many more instructions in machine code.

While an eager reader interested in generating machine code may be tempted to skip Chapter 11, we recommend studying this chapter before attempting code generation at the machine-code level. The ideas from this chapter are easily applied to Chapters 12 and 13, but they are easier to understand from the perspective of a VM.

Chapter 12 Runtime Support

Much of the functionality embedded in a VM is its runtime support (e.g., its support for managing storage). This chapter discusses various concepts and implementation strategies for providing the runtime support needed for modern programming languages. Study of this material can provide an understanding of the construction of a VM. For those who write code generators for a target architecture (Chapter 13), runtime support must be provided, so the study of this material is essential to creating a working compiler.

The chapter includes discussion of storage that is statically allocated, stack allocated, and heap allocated. References to **nonlocal storage** are considered, along with implementation structures such as frames and displays to support such references.

Chapter 13 Target Code Generation

This chapter is similar to Chapter 11, except that the target of code generation is a relatively low-level instruction set when compared with a VM. The chapter includes a thorough discussion of topics that arise in such code generation, including register allocation, management of temporaries, code scheduling, instruction selection, and some basic peephole optimization.

Chapter 14 Program Optimization

Most compilers include some capability for improving the code they generate. This chapter considers some of the practical techniques commonly used by compilers for program optimization. Advanced control flow analysis structures and algorithms are presented. An introduction to **data flow analysis** is presented by considering some fundamental optimizations that are relatively easy to implement. The theoretical foundation of such optimizations is studied, and the chapter includes construction and use of SSA Form.

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Charles Fischer My fascination with compilers began in 1965 in Mr. Robert Eddy's computer lab. Our computer had all of 20 kilobytes of main memory, and our compiler used punched cards as its intermediate form, but the seed was planted.

My education really began at Cornell University, where I learned the depth and rigor of computing. David Gries' seminal compiler text taught me much and set me on my career path.

The faculty at Wisconsin, especially Larry Landweber and Tad Pinkerton, gave me free rein in developing a compiler curriculum and research program. Tad, Larry Travis and Manley Draper, at the Academic Computing Center, gave me the time and resources to learn the practice of compiling. The UW-Pascal compiler project introduced me to some outstanding students, including my co-author Richard LeBlanc. We learned by doing, and that became my teaching philosophy.

Over the years my colleagues, especially Tom Reps, Susan Horwitz, and Jim Larus, freely shared their wisdom and experience; I learned much. On the architectural side, Jim Goodman, Guri Sohi, Mark Hill, and David Wood taught me the subtleties of modern microprocessors. A compiler writer must thoroughly understand a processor to harness its full power.

My greatest debt is to my students who brought enormous energy and enthusiasm to my courses. They eagerly accepted the challenges I presented. A full compiler, from scanner to code generator, must have seemed impossible in one semester, but they did it, and did it well. Much of that experience has filtered its way into this text. I trust it will be helpful in teaching a new generation how to craft a compiler.

Ron K. Cytron My initial interest and subsequent research into programming languages and their compilers are due in large part to the outstanding mentors who have played pivotal roles in my career. Ken Kennedy, of blessed memory, taught my compilers classes at Rice University. The courses I now teach are patterned after his approach, especially the role that lab assignments play in helping students understand the material. Ken Kennedy was an outstanding educator, and I can only hope to connect with students as well as he could. He hosted me one summer at IBM T.J. Watson Research Labs, in Yorktown Heights, New York, where I worked on software for automatic parallelization. During that summer my investigations naturally led me to the research of Dave Kuck and his students at the University of Illinois.

I still consider myself so very fortunate that Dave took me on as his graduate student. Dave Kuck is a pioneer in parallel computer architecture and in the role compilers can play to make to make such advanced systems easier to program. I strive to follow his example of hard work, integrity, and perseverance and to pass those lessons on to my students. I also experienced the vibrancy and fun that stems from investigating ideas in a group, and I have tried to create similar communities among my students.

My experiences as an undergraduate and graduate student then led me to Fran Allen of IBM Research, to whom I shall always be grateful for allowing me to join her newly formed PTRAN group. Fran has inspired generations of research in data flow analysis, program optimization, and automatic parallelization. She has amazing intuition into the important problems and their likely solution. In talking with colleagues, some of our best ideas are due to Fran and the suggestions, advice, or critiques she has offered us.

Some of the best years of my professional life were spent learning from and working with Fran and my PTRAN colleagues: Michael Burke, Philippe Charles, Jong-Deok Choi, Jeanne Ferrante, Vivek Sarkar, and David Shields. At IBM I also had the privilege of learning from and working with Barry Rosen, Mark Wegman, and Kenny Zadeck. While the imprint of my friends and colleagues can be found throughout this text, any mistakes are mine.

If the reader notices that the number 431 appears frequently in this book, it is an homage to the students who have studied compilers with me at Washington University. I have learned as much from my students as I have taught them, and my contribution to this book stems largely from my experiences in the classroom and lab.

Finally, I thank my wife and children for putting up with the time I wanted to spend working on this book. They have shown patience and understanding throughout this effort. And thank you, Aunt Carole, for always asking how this book was coming along.

Richard LeBlanc After becoming more excited about computers than physics problem sets while getting my B.S. in physics, I moved to Madison and enrolled at the University of Wisconsin as a computer science Ph.D. student in 1972. Two years later, a young assistant professor, Charles Fischer, who had just received his Ph.D. from Cornell, joined the faculty of the Computer Science Department. The first course he taught was a graduate compiler course, CS 701. I was enrolled in that course and still remember it as a really remarkable learning experience, all the more impressive since it was his first time teaching the course. We obviously hit it off well, since this introduction has led to a rather lengthy series of collaborations.

Through the sponsorship of Larry Travis, I began working at the Academic Computing Center in the summer of 1974. I was thus already part of that organization when the UW-Pascal project began a year later. That project not only gave me the opportunity to apply what I had learned in the two courses I had just taken, but also some great lessons about the impact of good design and design reviews. I also benefited from working with two fellow graduate students, Steve Zeigler and Marty Honda, from whom I learned how much fun it can be to be part of an effective software development team. We all discovered the value of working in Pascal, a well-designed language that requires disciplined thought while programming, and of using a tool that you are developing, since we bootstrapped from the Pascal P-Compiler to our own compiler that generated native code for the Univac 1108 early in the project.

Upon completion of my graduate work, I took a faculty position at Georgia Tech, drawn by the warmer weather and an opportunity to be part of a distributed computing research project led by Phil Enslow, who provided invaluable guidance in the early years of my career. I immediately had the opportunity to teach a compiler course and attempted to emulate the CS 701 course at Wisconsin, since I strongly believed in the value of the project-based approach Charles used. I quickly realized that that having the students write a complete compiler in a 10-week quarter was too much of a challenge. I thus began using the approach of giving them a working compiler for a very tiny language and building the project around extending all of the components of that compiler to compile a more complex language. The base compiler that I used in my 10-week course became one of the support items distributed with the Fischer–LeBlanc text.

My career path has taken me to greater involvement with software engineering and educational activities than with compiler research. As I look back on my early compiler experiences at Wisconsin, I clearly see the seeds of my interests in both of these areas. The decision that Charles and I made to write the original *Crafting a Compiler* was based in our belief that we could help other instructors offer their students an outstanding educational experience through a project-based compiler course. With the invaluable help of our editor, Alan Apt, and a great set of reviewers, I believe we succeeded. Many colleagues have expressed to me their enthusiasm for our original book and *Crafting a Compiler with C*. Their support has been a great reward and it also served as encouragement toward finally completing this text. Particular thanks go to Joe Bergin, who went well beyond verbal support, translating some of our early software tools into new programming languages and allowing us to make his versions available to other instructors.

My years at Georgia Tech provided me with wonderful opportunities to develop my interests in computing education. I was fortunate to have been part of an organization led by Ray Miller and then Pete Jensen during the first part of my career. Beginning in 1990, I had the great pleasure of working with Peter Freeman as we created and developed the College of Computing. Beyond the many ways he mentored me during our work at Georgia Tech, Peter encouraged my broad involvement with educational issues through my work with the ACM Education Board, which has greatly enriched my professional life over the last 12 years.

Finally, I thank my family, including my new granddaughter, for sharing me with this book writing project, which at times must have seemed like it would never end.

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Dedication

CNF: *To Lisa, always*
In memory of Stanley J. Winiasz,
one of the greatest generation

RKC: *To Betsy, Jessica, Melanie, and Jacob*
In memory of Ken Kennedy

RJL: *To Lanie, Aidan, Maria and Evolette*

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1

Introduction

This chapter presents the history of compiler construction and an overview of compiler organization. Compilers have tracked and even precipitated the phenomenal gains in computing speed that have accrued in the relatively short history of computer science. Section 1.1 presents a historical review of the development and evolution of the programming languages, computer architectures, and compilers that are in widespread use today.

The general area we study is **language processing**, which is concerned with preparing a program to be run on a computer. Most programs are written in a relatively high-level language. Language processors ensure that a program conforms to its programming language's specification, and they often translate the program into a form that is easier to run on a computer. Some language processors perform more translation than others. At one extreme, an **interpreter** runs a program by examining its high-level constructs and simulating their actions. At the other extreme, a **compiler** translates the high-level constructs into low-level machine instructions that can be executed directly by a computer. The differences between compilers and interpreters are discussed in Section 1.3.

From there, we explain in Section 1.2 what a compiler does and how various compilers can be distinguished from each other: by the kind of machine code they generate and by the format of the target code they generate.

In Section 1.3, we discuss a kind of language processor called an *interpreter* and explain how an interpreter differs from a compiler. Section 1.4 discusses the *syntax* (structure) and *semantics* (meaning) of programs. Next,

Programming Language
Compiler
Machine Language

Figure 1.1: A user's view of a compiler.

in Section 1.5, we discuss the tasks that a compiler must perform, primarily *analysis* of the source program and *synthesis* of a target program. That section also covers the parts of a compiler, discussing each in some detail: scanner, parser, type checker, optimizer and code generator.

In Section 1.6, we discuss the mutual interaction of compiler design and programming language design. Similarly, in Section 1.7, the influence of computer architecture on compiler design is covered.

Section 1.8 introduces a number of important compiler variants, including *debugging* and *development compilers*, *optimizing compilers*, and *retargetable compilers*. Finally, in Section 1.9, we consider *program development environments* that integrate a compiler, editor, and debugger into a single tool.

1.1 History of Compilation

Compilers are fundamental to modern computing. They act as *translators*, transforming human-oriented programming languages into computer-oriented machine languages. For most users, a compiler can be viewed as a utility that performs the transformation illustrated in Figure 1.1. A compiler allows virtually all computer users to ignore the machine-dependent details of machine language. Therefore, compilers allow programs and programming expertise to be **portable** across a wide variety of computers. This is a particularly valuable capability in an age where the cost of software development is so high and the need for software exists at so many levels, from small embedded computers to extreme-scale supercomputers.

The term **compiler** was coined in the early 1950s by Grace Murray Hopper. Translation was then viewed as the *compilation* of a sequence of machine-language subprograms selected from a library. At that time, compilation was called **automatic programming** and there was almost universal skepticism that it would ever be successful. Today, the automatic translation of programming languages is an accomplished fact, but programming language translators are still called compilers.

Among the first real compilers in the modern sense were the Fortran compilers of the late 1950s. They presented the user with a problem-oriented, largely machine-independent source language. They also performed some rather ambitious optimizations to produce efficient machine code, since efficient code was deemed essential for Fortran to compete successfully against assembly language programming. Machine-independent languages such as Fortran proved the viability of **high-level** compiled languages. They paved the way for the flood of languages and compilers that was to follow.

In the early days, compilers were ad hoc structures; components and techniques were often devised as a compiler was built. This approach to constructing compilers lent an aura of mystery to them, and they were viewed as complex and costly. Today the compilation process is well understood and compiler construction is routine. Nonetheless, crafting an efficient and reliable compiler is still a complex task. This book's primary task is to teach a mastery of the fundamentals. A concomitant goal is to cover some advanced techniques and important innovations.

Compilers normally translate conventional programming languages like Java™, C, and C++ into executable machine-language instructions. Compiler technology, however, is far more broadly applicable and has been employed in rather unexpected areas. For example, text-formatting languages like TeX [Knu98] and LaTeX [Lam95] are really compilers. They translate text and formatting commands into detailed typesetting commands. PostScript® [Pos] on the other hand, which is generated by many programs, is really a programming language. It is translated and executed by printers and document previewers to produce a readable form of a document. This standardized document-representation language allows documents to be freely interchanged, independently of how they were created and how they will be viewed.

Mathematica [Wol99] is an interactive system that intermixes programming with mathematics, solving intricate problems in both symbolic and numeric forms. This system relies heavily on compiler techniques to handle the specification, internal representation, and solution of problems.

Languages like Verilog [TM08] and VHDL [VHD] address the creation of **very large scale integration** (VLSI) circuits. A **silicon compiler** specifies the layout and composition of a VLSI circuit mask using standard cell designs. Just as an ordinary compiler must understand and enforce the rules of a particular machine language, so must a silicon compiler understand and enforce the design rules that dictate the feasibility of a given circuit.

Compiler technology is of value in almost any program that presents a nontrivial text-oriented command set, including the command and scripting languages of operating systems and the query languages of database systems. Thus, while our discussion will focus on traditional compilation tasks, innovative readers will undoubtedly find new and unexpected applications for the techniques presented.

1.2 What Compilers Do

Figure 1.1 represents a compiler as a translator of the programming language being compiled (the *source*) to some machine language (the *target*). This description suggests that all compilers do about the same thing, the only difference being their choice of source and target languages. However, the situation is a bit more complicated. While the issue of the accepted source language is indeed simple, there are many alternatives in describing the output of a compiler. These go beyond simply naming a particular target computer. Compilers may be distinguished in two ways:

- By the kind of machine code they generate
- By the format of the target code they generate

These are discussed in the following sections.

1.2.1 Machine Code Generated by Compilers

Compilers may generate any of three types of code by which they can be differentiated:

- Pure Machine Code
- Augmented Machine Code
- Virtual Machine Code

Pure Machine Code

Compilers may generate code for a particular machine's instruction set without assuming the existence of any operating system or library routines. Such machine code is often called **pure code** because it includes nothing but instructions that are part of that instruction set. This approach is rare because most compilers rely on runtime libraries and operating system calls to interface with the generated code. Pure machine code is most commonly used in compilers for **system implementation languages**, which are intended for implementing operating systems or embedded applications. This form of target code can execute on bare hardware without dependence on any other software.

Augmented Machine Code

Far more often, compilers generate code for a machine architecture that is **augmented** with operating system routines and runtime language support routines. The execution of a program generated by such a compiler requires that a particular operating system be present on the target machine and a collection of language-specific runtime support routines (I/O, storage allocation, mathematical functions, etc.) be available to the program. Most Fortran compilers use such software support only for I/O and mathematical functions. Other compilers assume a much larger range of available functionality. These may include data transfer instructions (such as, to move bit fields), procedure call instructions (to pass parameters, save registers, allocate stack space, etc.), and dynamic storage instructions (to provide for heap allocation).

Virtual Machine Code

The third type of code generated is composed entirely of virtual instructions. This approach is particularly attractive as a technique for producing code that can be run easily on a variety of computers. This level of **portability** is achieved by writing an interpreter for the **virtual machine** (VM) on any target architecture of interest. Code generated by the compiler can then be run on any architecture for which a VM interpreter is available. Java is an example of a language for which a VM (the **Java Virtual Machine** (JVM) and its bytecode instructions) was defined to accompany the language. Java applications produce predictable results on any computer for which a JVM interpreter is available. Similarly, Java applets can be run in any web browser provisioned with a JVM interpreter.

The advantages of portability obtained by using a VM instruction set can also make the compiler itself easy to port. For the purposes of this discussion, assume that the compiler accepts some source language L. Any instance of this compiler can translate a program written in L into the VM instructions. If the compiler itself is written in L, then the compiler can compile *itself* into VM instructions, which can be executed on any architecture that hosts the VM interpreter. If the VM is kept simple and clean, the interpreter can be relatively easy to write. The process of porting such a compiler from one architecture to another is called **bootstrapping** and is illustrated in Figure 1.2. The very first instance of an L compiler cannot compile itself, since no such compiler exists yet. However, the first instance can be written in a language K for which a compiler or assembler already exists. As shown in Figure 1.2, the result of that compilation is the first executable instance of a compiler for L. That first instance is usually discarded after the reference compiler, written in L, is functioning correctly.

Examples of compilers that target a VM for portability include the early Pascal compilers and the Java compiler included in the **Java Development**

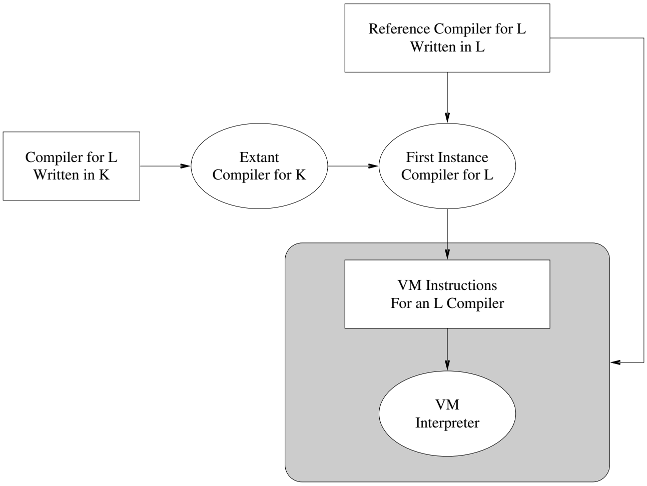

Figure 1.2: Bootstrapping a compiler that generates VM instructions. The shaded portion is a portable compiler for *L* that can run on any architecture supporting the VM.

Kit (JDK). Pascal uses P-code [Han85], while Java uses JVM **bytecodes** [Gos95] code. Both of these VMs are stack-based architectures. A rudimentary interpreter for P-code or JVM code can be written in a few weeks. Execution speed is roughly five to ten times slower than that of compiled code. Alternatively, the virtual machine code can be either translated into C code or expanded to machine code directly. This approach made Pascal and Java available for almost any platform. It was instrumental in Pascal's success in the 1970s and strongly influenced the acceptance of Java.

Virtual instructions serve a variety of purposes. They simplify the job of a compiler by providing primitives suitable for the particular language being translated (such as procedure calls and string manipulation). They also contribute to compiler transportability. Furthermore, they may allow for a significant *decrease* in the size of generated code because instructions can be designed to meet the needs of a particular programming language (such as JVM bytecodes for Java). Using this approach, one can realize as much as a two-thirds reduction in generated program size. This can be a crucial factor when a program is transmitted over a slow communications path (e.g., a Java applet sent from a slow server).

When an entirely virtual instruction set is used as the target language, the instruction set must be interpreted in software. In a **just-in-time** (JIT) approach, virtual instructions can be translated to target code just as they are about to be executed, or when they have been interpreted often enough to merit translation into target code.

If a virtual instruction set is used often enough, it is possible to develop special microprocessors that implement the virtual instruction set in hardware. For example, Jazelle$^{\text{TM}}$ [Jaz] offers hardware support to improve the performance and power usage of mobile phone applications that execute JVM instructions.

In summary, most compilers generate code that interfaces with runtime libraries, operating system utilities, and other software components. VMs can enhance compiler portability and increase consistency of program execution across diverse target architectures.

1.2.2 Target Code Formats

Another way that compilers differ from one another is in the format of the target code they generate. Target formats may be categorized as follows:

- Assembly or other source formats
- Relocatable binary
- Absolute binary

Assembly Language (Source) Format

The generation of assembly code simplifies and modularizes translation. A number of code-generation decisions (such as instruction and data addresses) can be left for the assembler. This approach is common for compilers developed as instructional projects or for prototyping programming language designs. One reason for this is that the assembly code is relatively easy to scrutinize, which makes the compilation process more transparent for students and prototyping activities.

Generating assembler code is also useful for **cross-compilation**, where the compiler executes on one computer but generates code that executes on another. The symbolic assembly code is easily transferred between different computers.

Sometimes another programming language, such as C, is generated by a compiler instead of a specific assembly language. C has in fact been called a **universal assembly language** because it is relatively low level yet it is far more platform independent than any particular assembly language. However, generation of C code leaves many decisions (such as the runtime representation of data structures) to a particular C compiler. Full control over such matters is retained if a compiler generates assembly language.

Relocatable Binary Format

Most production-quality compilers do not generate assembly language; direct generation of target code (in relocatable or absolute binary format) is more efficient and allows the compiler more control over the translation process. It is nonetheless beneficial for the compiler's output to be open to scrutiny. Compilers that produce binary format typically can also produce a **pseudoassembly language** listing of the generated code. Such a listing shows the instructions generated by the compiler with annotations to document storage references.

Relocatable binary format is essentially the form of code that most assemblers generate. This format can also be generated directly by a compiler. External references, local instruction addresses, and data addresses are not yet bound. Instead, addresses are assigned relative either to the beginning of the module or to some symbolically named locations. The latter alternative makes it easy to group together code sequences or data areas. A linkage step is required to incorporate any support libraries as well as other separately compiled routines referenced from within a compiled program. The result is an **absolute binary format** that is executable.

Both relocatable binary and assembly language formats allow **modular compilation**: the decomposition of a large program into separately compiled pieces. They also allow **cross-language support**: incorporation of assembler code and code written and compiled in other high-level languages. Such code can include I/O, storage allocation, and math libraries that supply functionality regarded as part of the language's definition.

Absolute Binary Format

Some compilers generate an **absolute binary format** that can be directly executed when the compiler is finished. This process is usually faster than the other approaches. However, the ability to interface with other code may be limited. In addition, the program must be recompiled for each execution unless some means is provided for archiving the memory image. Compilers that generate an absolute binary format are useful for student exercises and prototyping use, where frequent changes are the rule and compilation costs far exceed execution costs. It also can be useful to avoid saving compiled formats to save file space or to guarantee the use of only the most current library routines and class definitions.

Summary The code format alternatives and the target code alternatives discussed here show that compilers can differ quite substantially while still performing the same sort of translation task. Some compilers use a combination of the articulated alternatives. For example, most Java compilers emit **bytecodes**

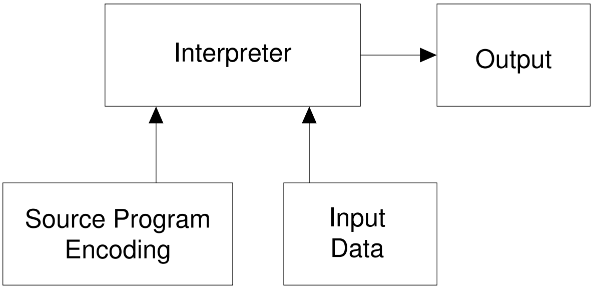

Figure 1.3: An interpreter.

that are subsequently subjected to interpretation or dynamic compilation to native machine code. The bytecodes are in a sense another source format, but their encoding is a standard and relatively compact binary format. Java has a **native interface** that is designed to allow Java code to interoperate with code written in other languages. Java also requires **dynamic linking** of classes used by an application, so that the origin of such classes can be controlled when an application is invoked. When a class is first referenced, a class definition may be remotely fetched, checked, and loaded during program execution.

1.3 Interpreters

Another kind of language processor is the **interpreter**. Interpreters share some of the functionality found in compilers, such as syntactic and semantic analyses. However, interpreters differ from compilers in that they execute programs without explicitly performing much translation. Figure 1.3 illustrates schematically how interpreters work. To an interpreter, a program is merely data that can be arbitrarily manipulated, just like any other data. The locus of control during execution resides in the interpreter, *not* in the user program (i.e., the user program is passive rather than active).

Interpreters provide a number of capabilities not usually found in compilers, as follows:

- Programs can be easily modified as execution proceeds. This provides a straightforward **interactive debugging** capability, since a program can be modified to pause at points of interest or to display the value of program variables. Depending on program structure, program modifications may require reparsing or repeating semantic analysis.

- Languages in which the type of an object is developed dynamically (e.g., Lisp and Scheme) are easily supported in an interpreter. Some languages (such as Smalltalk and Ruby) allow the type system itself to change dynamically. Since the user program is continuously reexamined as execution proceeds, symbols need not have a fixed meaning. For example, a symbol may denote an integer scalar at one point and a Boolean array at a later point. Such **fluid bindings** are more problematic for compilers, since dynamic changes in the meaning of a symbol make direct translation into machine code more difficult.

- Interpreters provide a significant degree of machine independence, since no machine code is generated. All operations are performed within the interpreter. Porting an interpreter can be as simple as recompiling the interpreter on a new machine, if the interpreter is written in a language already supported on that machine.

However, direct interpretation of source programs can involve significant overhead. As execution proceeds, program text must be continuously reexamined. Identifier bindings, types, and operations may have to be recomputed at each reference. For languages where such bindings can change arbitrarily, interpretation can be 100 times slower than compiled code. For more static languages such as C and Java, the cost difference is closer to 10.

Some languages (C, C++, and Java) have both interpreters (for debugging and program development) and compilers (for production work). JIT compilers offer a combination of interpretation and compilation/execution.

In summary, all language processing involves interpretation at some level. Interpreters directly interpret source programs or some syntactically transformed versions of them. They may exploit the availability of a source representation to allow program text to be changed as it is executed and debugged. While a compiler has distinct translation and execution phases, some form of "interpretation" is still involved. The translation phase may generate a virtual machine language that is interpreted by software or a real machine language that is interpreted by a particular computer, either in firmware or hardware.

1.4 Syntax and Semantics

A complete definition of a programming language must include the specification of its **syntax** (structure) and its **semantics** (meaning).

Syntax typically means context-free syntax because of the almost universal use of **context-free grammars** (CFGs) as a syntactic specification mechanism. Syntax defines the sequences of symbols that are legal; syntactic legality is independent of any notion of what the symbols mean. For example, a context-free syntax might specify that `a=b+c` is syntactically legal, while `b+c=a` is not.

However, not all aspects of well-formed programs can be described by context-free syntax. For example, CFGss cannot specify type compatibility and scoping rules. For example, a programming language may specify that a=b+c is illegal if any of the variables are undeclared or if b or c is of type Boolean.

Because of the limitations of CFGss, the semantics of a programming language are commonly divided into two classes:

- Static semantics
- Runtime semantics

1.4.1 Static Semantics

The **static semantics** of a language provide a set of rules that specify which syntactically legal programs are actually valid. Such rules typically require that all identifiers be declared, that operators and operands be type-compatible, and that procedures be called with the proper number of parameters. The common thread through all of these rules is that they cannot be expressed with a CFGs. Thus static semantics *augment* context-free specifications and complete the definition of valid programs.

Static semantics can be specified formally or informally. The prose descriptions found in most programming language specifications are informal. They tend to be relatively compact and easy to read, but often they are imprecise. Formal specifications can be expressed using any of a variety of notations. For example, **attribute grammars** [Knu68] can formalize many of the semantic checks found in compilers. The following rewriting rule, called a **production**, specifies that an expression, denoted by E, can be rewritten into an expression E plus a term T:

$$\mathsf{E} \rightarrow \mathsf{E} \ + \ \mathsf{T}$$

In an attribute grammar, this production might be augmented with a type attribute for E and T and a predicate testing for type compatibility, such as

```
E_result → E_v1 + T_v2
        if v1.type = numeric and v2.type = numeric
        then  result.type ← numeric
        else  call ERROR( )
```

Attribute grammars are a reasonable blend of formality and readability, but they can be rather verbose and tedious. Most compiler-writing systems do not use attribute grammars directly. Instead, they propagate semantic information through a program's **abstract syntax tree** (AST) in a manner similar to the evaluation of attribute grammar systems. The specifics of a portion of semantics checking are thus written in the compiler as a semantics-checking phase. Such is the approach taken in this book.

1.4.2 Runtime Semantics

Runtime, or **execution, semantics** are used to specify what a program computes. These semantics are often specified very informally in a language manual or report. Alternatively, a more formal *operational*, or *interpreter*, model can be used. In such a model, a program "state" is defined and program execution is described in terms of changes to that state. For example, the semantics of the statement a = 1 is that the state component corresponding to a is changed to 1.

A variety of formal approaches to defining the runtime semantics of programming languages have been developed. Three of them, natural semantics, axiomatic semantics and denotational semantics, are described below.

Natural Semantics

Natural semantics [NN92] (sometimes called **structured operational semantics**) formalizes the operational approach. Given assertions known to be true before the evaluations of a construct, we can infer assertions that will hold after the construct's evaluation. Natural semantics has been used to define the semantics of a variety of languages, including standard ML [MTHM97].

Axiomatic Semantics

Axiomatic definitions [Gri81] can be used to model execution at a more abstract level than operational models. They are based on formally specified *relations*, or *predicates*, that relate program variables. Statements are defined by how they modify these relations.

As an example of axiomatic definitions, the axiom defining $var \leftarrow exp$ states that a predicate involving *var* is **true** after statement execution if, and only if, the predicate obtained by replacing all occurrences of *var* by *exp* is **true** beforehand. Thus, for $y > 3$ to be **true** after execution of the statement $y \leftarrow x + 1$, the predicate $x + 1 > 3$ would have to be **true** before the statement is executed. Similarly, $y = 21$ is **true** after execution of $x \leftarrow 1$ if $y = 21$ is **true** before its execution (this is a roundabout way of saying that changing x doesn't affect y). However, if x is an **alias** (an alternative name) for y, the axiom is invalid. This is one reason why aliasing is discouraged (or forbidden) in some language designs.

The axiomatic approach is good for deriving proofs of program correctness because it avoids implementation details and concentrates on how relations among variables are changed by statement execution. Although axioms can formalize important properties of the semantics of a programming language, it is difficult to use them to define most programming languages completely. For example, they do not do a good job of modeling implementation considerations such as running out of memory.

Denotational Semantics

Denotational models [Sch86] are more mathematical in form than operational models, but they can accommodate memory stores and fetches that are central to procedural languages. They rely on notation and terminology drawn from mathematics, so they are often fairly compact, especially in comparison with operational definitions.

A denotational definition may be viewed as a syntax-directed definition that specifies the meaning of a construct in terms of the meaning of its immediate constituents. For example, to define addition, we might use the following rule:

$$E[T1 + T2]m = E[T1]m + E[T2]m$$

This definition says that the value obtained by adding two subexpressions, $T1$ and $T2$, in the context of a memory state m is defined to be the sum of the arithmetic values obtained by evaluating $T1$ in the context of m (denoted $E[T1]m$) and $T2$ in the context of m (denoted $E[T2]m$).

Denotational techniques are quite popular and form the basis for rigorous definitions of programming languages. Research has shown that it is possible to convert denotational representations *automatically* to equivalent representations that are directly executable [Set83, Wan82, App85].

Summary Regardless of how semantics are specified, our concern for precise semantics is motivated by the fact that writing a complete and accurate compiler for a programming language requires that the language itself be well defined. While this assertion may seem self-evident, many languages are defined by imprecise or informal language specifications. Attention is often given to formal specification of *syntax*, but the *semantics* of the language may be defined via informal prose. The resulting definition inevitably is ambiguous or incomplete on certain points.

For example, in Java all functions must return via a `return` *expr* statement, where `expr` is assignable to the function's return type. The following is therefore illegal:

```
public static int subr(int b) {
   if (b != 0)
      return b+100;
}
```

If b is equal to zero, subr fails to return a value. Now consider the following:

```
public static int subr(int b) {
   if (b != 0)
      return b+100;
   else if (10*b == 0)
           return 1;
}
```

In this case, a proper return is always executed, since the else part is reached only if b equals zero; this implies that 10*b is also equal to zero. Is the compiler expected to duplicate this rather involved chain of reasoning? Java compilers typically assume that a predicate could evaluate to **true** or **false**, even if a detailed program analysis refutes that assumption. Thus a compiler may reject subr as semantically illegal and in so doing trade simplicity for accuracy in its analysis. Indeed, the general problem of deciding whether a particular statement in a program is reachable is **undecidable**, proved by reduction from the famous **halting problem** [HU79]. We certainly cannot ask our Java compiler literally to do the impossible!

In practice, a trusted **reference compiler** can serve as a de facto language definition. That is, a programming language is, in effect, defined by what a compiler chooses to accept and how it chooses to translate language constructs. In fact, the operational and natural semantic approaches introduced previously take this view. A standard interpreter is defined for a language, and the meaning of a program is precisely whatever the interpreter says. An early (and very elegant) example of an operational definition is the seminal Lisp interpreter [McC60]. There, all of Lisp was defined in terms of the actions of a Lisp interpreter, assuming only seven primitive functions and the notions of argument binding and function call.

Of course, a reference compiler or interpreter is no substitute for a clear and precise semantic definition. Nonetheless, it is very useful to have a reference against which to test a compiler that is under development.

1.5 Organization of a Compiler

Compilers generally perform the following tasks:

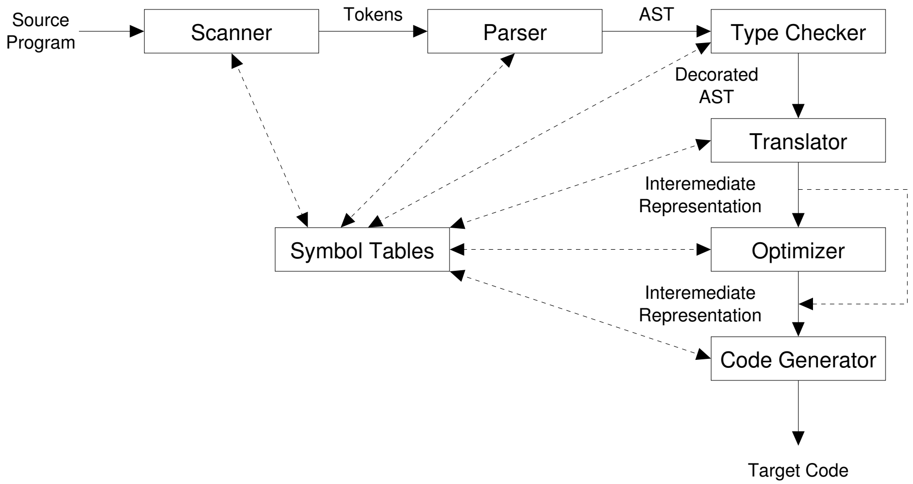

Figure 1.4: A syntax-directed compiler. AST denotes the Abstract Syntax Tree.

- *Analysis* of the source program being compiled
- *Synthesis* of a target program that, when executed, will correctly perform the computations described by the source program

Almost all modern compilers are **syntax-directed**. That is, the compilation process is driven by the syntactic structure of the source program, as recognized by the parser. Most compilers distill the source program's structure into an **abstract syntax tree** (AST) that omits unnecessary syntactic detail. The parser builds the AST out of **tokens**, the elementary symbols used to define a programming language syntax. Recognition of syntactic structure is a major part of the **syntax analysis** task.

Semantic analysis examines the meaning (semantics) of the program on the basis of its syntactic structure. It plays a dual role. It finishes the analysis task by performing a variety of correctness checks (for example, enforcing type and scope rules). It also begins the **synthesis phase**.

In the synthesis phase, source language constructs are translated into an **intermediate representation** (IR) of the program. Some compilers generate target code directly. If an IR is generated, it then serves as input to a *code generator* component that actually produces the desired machine-language program. The IR may optionally be transformed by an *optimizer* so that a more efficient program may be generated. A common organization of all of these compiler components is depicted schematically in Figure 1.4. Each of these components is described in more detail below. Chapter 2 presents a simple compiler to provide concrete examples of many of the concepts introduced in this overview.

1.5.1 The Scanner

The **scanner** begins the analysis of the source program by reading the input text (character by character) and grouping individual characters into **tokens** such as identifiers, integers, reserved words, and delimiters. This is the first of several steps that produce successively higher-level representations of the input. The tokens are encoded (often as integers) and fed to the parser for syntactic analysis. When necessary, the actual character string comprising the token is also passed along for use by the semantic phases. The scanner does the following:

- It puts the program into a compact and uniform format (a stream of tokens).
- It eliminates unneeded information (such as comments).
- It processes compiler control directives (for example, turn the listing on or off and include source text from a specified file).
- It sometimes enters preliminary information into symbol tables (for example, to register the presence of a particular label or identifier).
- It optionally formats and lists the source program.

The main action of building tokens is often driven by token descriptions. *Regular expression* notation (discussed in Chapter 3) is an effective approach to describing tokens. **Regular expressions** are a formal notation sufficiently powerful to describe the variety of tokens required by modern programming languages. In addition, they can be used as a specification for the automatic generation of finite automata (discussed in Chapter 3) that recognize **regular sets**, that is, the sets that regular expressions define. Recognition of regular sets is the basis of the **scanner generator**. A scanner generator is a program that actually produces a working scanner when given only a specification of the tokens it is to recognize. Scanner generators are a valuable compiler-building tool.

1.5.2 The Parser

The **parser** is based on a formal syntax specification such as a CFGs. It reads tokens and groups them into phrases according to the syntax specification. Grammars are discussed in Chapters 2 and 4, and parsing is discussed in

Chapters 5 and 6. Parsers are typically driven by tables created from a CFGs by a **parser generator**.

The parser verifies correct syntax. If a syntax error is found, it issues a suitable error message. Also, it may be able to repair the error (to form a syntactically valid program) or to recover from the error (to allow parsing to be resumed). In many cases, **syntactic error recovery** or **repair** can be done automatically by consulting structures created by a suitable parser generator.

As syntactic structure is recognized, the parser usually builds an AST as a concise representation of program structure. The AST then serves as a basis for semantic processing. ASTs are discussed in Chapters 2 and 7.

1.5.3 The Type Checker (Semantic Analysis)

The type checker checks the **static semantics** of each AST node. That is, it verifies that the construct the node represents is legal and meaningful (that all identifiers involved are declared, that types are correct, and so on). If the construct is semantically correct, the type checker **decorates** the AST node by adding type information to it. If a semantic error is discovered, a suitable error message is issued.

Type checking is purely dependent on the semantic rules of the source language. It is independent of the compiler's target.

1.5.4 Translator (Program Synthesis)

If an AST node is semantically correct, it can be *translated* into IR code that correctly implements the meaning of the AST node. For example, an AST for a while loop contains two subtrees, one representing the loop's expression and the other representing the loop's body. However, *nothing* in the AST explicitly captures the notion that a while loop loops! This meaning is captured when a while loop's AST is translated to IR form. In the IR, the notion of testing the value of the loop control expression and conditionally executing the loop body is made explicit.

The translator is largely dictated by the semantics of the source language. Little of the nature of the target machine needs to be made evident. As a convenience during translation, some general aspects of the target machine may be exploited (for example, that the machine is byte-addressable or that it has a runtime stack). However, detailed information on the nature of the target machine (operations available, addressing, register characteristics, etc.) is reserved for the code-generation phase.

In simple, nonoptimizing compilers, the translator may generate target code directly without using an explicit IR. This simplifies a compiler's design by removing an entire phase. However, it also makes retargeting the compiler to another machine much more difficult. Most compilers implemented as instructional projects generate target code directly from the AST, without using an IR.

More elaborate compilers such as the **GNU Compiler Collection** (GCC) may first generate a high-level IR (that is source-language oriented) and then subsequently translate it into a low-level IR (that is target-machine oriented). This approach allows a cleaner separation of source and target dependencies.

1.5.5 Symbol Tables

A **symbol table** is a mechanism that allows information to be associated with identifiers and shared among compiler phases. Each time an identifier is declared or used, a symbol table provides access to the information collected about it. Symbol tables are used extensively during type checking, but they can also be used by other compiler phases to enter, share, and later retrieve information about types, variables, procedures, and labels. Compilers may choose to use other structures to share information between compiler phases. For example, a program representation such as an AST may be expanded and refined to provide detailed information needed by optimizers, code generators, linkers, loaders, and debuggers.

1.5.6 The Optimizer

The IR code generated by the translator is analyzed and transformed into functionally equivalent but improved IR code by the **optimizer**. This phase can be complex, often involving numerous subphases, some of which may need to be applied more than once. Most compilers allow optimizations to be turned off so as to speed translation. Nonetheless, a carefully designed optimizer can significantly speed program execution by simplifying, moving, or eliminating unneeded computations.

If both a high-level and low-level IR are used, optimizations may be performed in stages. For example, a simple subroutine call may be expanded into the subroutine's body, with actual parameters substituted for formal parameters. This is a high-level optimization. Alternatively, a value already loaded from memory may be reused. This is a low-level optimization.

Optimization can also be done *after* code generation. An example is **peephole optimization**. Peephole optimization examines generated code a few instructions at a time (in effect, through a "peephole"). Common peephole optimizations include eliminating multiplications by 1 or additions of 0, eliminating a load of a value into a register when the value is already in another register, and replacing a sequence of instructions by a single instruction with the same effect. A peephole optimizer does not offer the payoff of a full-scale optimizer. However, it can significantly improve code and is often useful for "cleaning up" after earlier compiler phases.

1.5.7 The Code Generator

The IR code produced by the translator is mapped into target machine code by the **code generator**. This phase requires detailed information about the target machine and includes machine-specific optimization such as register allocation and code scheduling. Normally, code generators are hand-coded and can be quite complex, since generation of good target code requires consideration of many special cases.

The notion of **automatic construction** of code generators has been actively studied. The basic approach is to match a low-level IR to target-instruction templates, with the code generator automatically choosing instructions that best match IR instructions. This approach localizes the target-machine specifics of a compiler and, at least in principle, makes it easy to **retarget** a compiler to a new target machine. Automatic retargeting is an especially desirable goal, since a great deal of work is usually needed to move a compiler to a new machine. The ability to retarget by simply changing the set of target machine templates and generating (from the templates) a new code generator is compelling.

A well-known compiler using these techniques is the GCC [GNU]. GCC is a heavily optimizing compiler that can target over thirty computer architectures (including Intel®, Sparc™, and PowerPC®) and has at least six front ends (including C, C++, Fortran, Ada, and Java).

1.5.8 Compiler Writing Tools

Finally, note that in discussing compiler design and construction, we often talk of **compiler writing tools**. These are often packaged as **compiler generators** or **compiler compilers**. Such packages usually include scanner and parser generators. Some also include symbol table managers, attribute grammar evaluators, and code-generation tools. More advanced packages may aid in error repair generation.

These sorts of generators greatly assist the crafting of compilers, but much of the effort in crafting a compiler lies in writing and debugging the semantic phases. These routines can be numerous (a type checker and translator is apparently needed for each distinct AST node) and are usually hand coded. Judicious application of the **visitor pattern** can significantly reduce this effort and make the compiler easier to maintain. Chapters 2 and 7 introduce application of the visitor pattern to semantic analysis. This treatment continues beyond Chapter 7 as specific semantic issues are addressed.

1.6 Programming Language and Compiler Design

Our primary interest is the design and implementation of compilers for modern programming languages. An interesting aspect of this study is how programming language design and compiler design influence each other. Programming language design obviously influences, and indeed often dictates, how compilers are crafted. Many clever and sometimes subtle compiler techniques arise from the need to cope with some programming language construct. A good example of this is the **closure** mechanism that was invented to handle formal procedures. A closure is a special runtime representation for a function. It is usually implemented as a pointer to the function's body *and* to its execution environment. While the concept of a closure is attractive from a programming language design perspective, implementing closures efficiently has been challenging for compiler writers [App92, Ken07].

The state of the art in compiler design also strongly affects programming language design, if only because a programming language that cannot be compiled effectively has an uphill road to acceptance. Most successful programming language designers (such as the Java language development team) have extensive compiler design backgrounds.

A programming language that is easy to compile usually has the following advantages:

- It often is easier to learn, read, and understand. If a feature is hard to compile, it may well be difficult to understand.

- It will have quality compilers on a wide variety of machines. This fact is often crucial to a language's success. For example, C, C++, Java, and Fortran are widely available and very popular; Ada and Modula-3 have limited availability and are far less popular.

- Often, better code will be generated. Poor-quality code can be fatal in major applications.

- Fewer compiler bugs will occur. If a language cannot be easily understood, then discrepancies will arise in the difficult regions of the language's design. These will in turn lead to compilers that differ in their interpretation of a program's meaning.

- The compiler will be smaller, cheaper, faster, more reliable, and more widely used.

- Compiler diagnostic messages and program development tools will often be better.

Throughout our discussion of compiler design, we draw ideas, solutions, and shortcomings from many languages. Our primary focus is on Java and C, but we also consider Ada, C++, Smalltalk, ML, Pascal, and Fortran. We concentrate on Java and C because they are representative of the issues posed by modern language designs. We consider other languages so as to identify alternative design approaches for crafting a compiler.

1.7 Computer Architecture and Compiler Design

Advances in computer architecture and microprocessor fabrication have spearheaded the computer revolution. At one time, a computer offering one **megaflop** performance (1,000,000 floating-point operations per second) was considered advanced. Computers offering **teraflop** (one trillion flops) performance are available and **petaflop** computers (one thousand trillion flops) have become a matter of packaging (and cooling!) a sufficient number of individual computers. Meanwhile, each individual computer is often itself a multiprocessor, and each processor in the computer may have multiple **cores**, each offering an independent thread of control.

Compiler designers are responsible for making this vast computing capability available to programmers. Although compilers are rarely visibly to the end users of application programs, they are an essential enabling technology. The problems encountered in efficiently harnessing the capability of a modern computing platforms are numerous, as follows:

- Instruction sets for some popular architectures, particularly the Intel x86 series, are highly nonuniform. Some operations must be done in registers, while others can be done in memory. Often a number of distinct register classes exist, each suitable for only a particular class of operations.

- High-level programming language operations are not always easy to support. Virtual method dispatch, dynamic heap accesses, and **reflective programming** constructs can take hundreds or thousands of machine instructions to implement. Exceptions, threads, and concurrency management are typically more expensive and complex to implement than most users suspect.

- Essential architectural features such as hardware caches and distributed processors and memory are difficult to present to programmers in an architecturally independent manner. Yet misuse of these features can impose immense performance penalties.

- Effective use of a large number of processors has always posed challenges to application developers and compiler writers. Many developers have unrealistic expectations concerning how well a compiler can use large-scale systems without changing an application. While compilers continually improve [Wol95, AK01], languages are also evolving [CGS+05] to address these challenges.

For some programming languages, runtime checks for data and program integrity are dropped in favor of gains in execution speed. Programming errors can then go undetected because of that fear that extra checking will slow down execution unacceptably. The cost of software development and the consequences of program failure have reversed that trend for most programming efforts. A major complexity in implementing Java is efficiently enforcing the runtime integrity constraints it imposes.

1.8 Compiler Design Considerations

Compilers are often biased for a particular kind of deployment or user base. In this section we examine some common design criteria that affect how compilers are crafted.

1.8.1 Debugging (Development) Compilers

A **debugging compiler** such as `CodeCenter` [Cod] is specially designed to aid in the development and debugging of programs. It carefully scrutinizes programs and details programmer errors. Often it can tolerate or repair minor errors (for example, insert a missing comma or parenthesis). Some program errors can be detected only at runtime. Such errors include invalid subscripts, misuse of pointers, and illegal file manipulations.

These compilers may include the checking of code that can detect runtime errors and initiate a symbolic debugger. Although debugging compilers are particularly useful in instructional environments, diagnostic techniques are of value in all compilers. In the past, development compilers were used only in the initial stages of program development. When a program neared completion, compilation switched to a **production compiler**, which increased compilation and execution speed by ignoring diagnostic concerns. This strategy has been likened by Tony Hoare to wearing a life jacket in sailing classes held on dry land, but abandoning the jacket when at sea [Hoa89]! Indeed, it is becoming increasingly clear that for almost all applications, reliability is more important than speed. For example, Java mandates runtime checks that C and C++ do not.

For production systems where quality is a paramount concern, detecting possible or actual runtime errors is crucial. Tools such as `purify` [pur] can add initialization and array bounds checks to already compiled programs, thereby allowing illegal operations to be detected even when source files are not available. Other tools such as `Electric Fence` [Piz99] can detect dynamic storage problems such as buffer overruns and improperly deallocated storage.

1.8.2 Optimizing Compilers

An **optimizing compiler** is specially designed to produce efficient target code at the cost of increased compiler complexity and possibly increased compilation times. In practice, all production-quality compilers (those whose output will be used in everyday work) make some effort to generate reasonable target code. For example, no add instruction would normally be generated for the expression `i+0`.

The term *optimizing compiler* is actually a misnomer. This is because no compiler of any sophistication can produce *optimal* code for all programs. The reason for this is twofold. First, theoretical computer science has shown that even so simple a question as whether two programs are equivalent is **undecidable**: such questions cannot generally be answered by *any* computer program. Thus finding the simplest (and most efficient) translation of a program cannot always be done. Second, many program optimizations require time proportional to an exponential function of the size of the program being compiled. Thus, optimal code, even when theoretically possible, is often infeasible in practice.

Optimizing compilers actually use a wide variety of transformations that improve a program's performance. The complexity of an optimizing compiler arises from the need to employ a variety of transforms, some of which interfere with each other. For example, keeping frequently used variables in registers reduces their access time but makes procedure and function calls more expensive. This is because registers need to be saved across calls. Many optimizing compilers provide a number of levels of optimization, each providing increasingly greater code improvements at increasingly greater costs. The choice of which improvements are most effective (and least expensive) is a matter of judgment and experience. Chapter 13 discusses some optimizations that are specific to code generation, such as register allocation. Chapter 14 covers the theory of optimizing compilers in greater detail, including **data flow frameworks** and **static single-assignment form**. Further discussion of a comprehensive optimizing compiler is beyond the scope of this book. However, compilers that produce high-quality code at reasonable cost are an achievable goal.

1.8.3 Retargetable Compilers

Compilers are designed for a particular programming language (the source language) and a particular target computer (the computer for which it will generate code). Because of the wide variety of programming languages and computers that exist, apparently a large number of similar, but not identical, compilers must be written. While this situation has decided benefits for those of us in the compiler writing business, it does make for a lot of duplication of effort and for a wide variance in compiler quality. As a result, the **retargetable compiler** has become a concept of increasing importance for language designers, computer architects, and compiler writers.

A retargetable compiler is one whose target architecture can be changed without its machine-independent components having to be rewritten. A retargetable compiler is more difficult to write than an ordinary compiler because target-machine dependencies must be carefully localized. In addition, it is often difficult for a retargetable compiler to generate code that is as efficient as that of an ordinary compiler because special cases and machine idiosyncrasies are harder to exploit. Nonetheless, because a retargetable compiler allows development costs to be shared and provides for uniformity across computers, it is an important innovation. While discussing the fundamentals of compilation, we concentrate on compilers targeted to a single machine. Chapters 11 and 13 cover some of the techniques needed to provide retargetability.

1.9 Integrated Development Environments

In practice, a compiler is but one tool used in the program development cycle. Developers edit a program, compile it, and test its performance. This cycle is repeated many times as the application is developed, often in response to specification changes and bugs that are discovered. The **integrated development environment** (IDE) has become a popular tool to integrate this cycle within a single framework. An IDE allows programs to be built incrementally, with program checking and testing fully integrated. Of course, an important component within an IDE is its compiler. An IDE places special demands on its compiler as follows:

- Most IDEs provide immediate feedback concerning syntax and semantic problems in the code as the code is entered.

- The IDE focus is typically on the source of a program, with any derived files (such as object code) carefully managed beyond the user's view.

- Most IDEs provide key or mouse actions that provide information about the program as it is developed. For example, a program may have an object reference o and the developer may wish to see the methods that can be invoked on o. Such information depends on the declared type of o as well as the methods defined on objects of that type.

We focus on the traditional **batch compilation** approach in which an entire source file is translated. However, many of the techniques we develop can be reformulated into **incremental** form to support IDEs. For example, a parser can reparse only those portions of a program that have been changed [GM80, WG97], and a type checker can analyze only portions of an AST that are affected by program modification. An alternative is to write the compiler as a sequence of passes over the source code, with its first pass sufficiently fast to provide an IDE its requisite information. Subsequent passes can complete the compilation process and generate increasingly sophisticated code.

Summary In this book, we concentrate on the translation of C, C++, and Java. We use the JVM as a target in Chapter 11, and we address code generation for RISC processors such as the MIPS® and Sparc architectures in Chapter 13. At the code-generation stage, a variety of current techniques designed to exploit a processor's capabilities are explored. Like so much else in crafting a compiler, experience is the best guide. We begin with the translation of a very simple language in Chapter 2 and work our way up to ever more challenging translation tasks.

Exercises

1. The model of compilation we introduced is essentially batch-oriented. In particular, it assumes that an entire source program has been written and that the program will be fully compiled before the programmer can execute the program or make any changes. An interesting and important alternative is an **interactive compiler**. An interactive compiler, usually part of an integrated program development environment, allows a programmer to interactively create and modify a program, fixing errors as they are detected. It also allows a program to be tested before it is fully written, thereby providing for stepwise implementation and testing.

   Redesign the compiler structure of Figure 1.4 to allow incremental compilation. (The key idea is to allow individual phases of a compiler to be run or rerun without necessarily doing a full compilation.)

2. Most programming languages, such as C and C++, are compiled directly into the machine language of a "real" microprocessor (for example, an Intel x86 or Sparc). Java takes a different approach. It is commonly compiled into the machine language of the JVM. The JVM is not implemented in its own microprocessor, but is instead interpreted on some existing processor. This allows Java to be run on a wide variety of machines, thereby making it highly platform independent.

   Explain why building an interpreter for a virtual machine like the JVM is easier and faster than building a complete Java compiler. What are the disadvantages of this virtual machine approach?

3. C compilers are almost always written in C. This raises something of a "chicken and egg" problem—how was the *first* C compiler for a particular system created? If you need to create the first compiler for language X on system Y, one approach is to create a **cross-compiler**. A cross-compiler runs on system Z but generates code for system Y.

   Explain how, starting with a compiler for language X that runs on system Z, you might use cross-compilation to create a compiler for language X, written in X, that runs on system Y and generates code for system Y.

   What extra problems arise if system Y is "bare"—that is, has no operating system or compilers for any language? (Recall that Unix® is written in C and thus must be compiled before its facilities can be used.)

4. Cross-compilation assumes that a compiler for language X exists on some machine. When the first compiler for a new language is created, this assumption does not hold. In this situation, a **bootstrapping** approach can be taken. First, a subset of language X is chosen that is sufficient to implement a simple compiler. Next, a simple compiler for the X subset is written in any available language. This compiler must be correct, but it should not be any more elaborate than is necessary, since it will soon be discarded. Next, the subset compiler for X is rewritten in the X subset and then compiled using the subset compiler previously created. Finally, the X subset, and its compiler, can be enhanced until a complete compiler for X, written in X, is available.

   Assume you are bootstrapping C++ or Java (or some comparable language). Outline a suitable subset language. What language features must be in the language? What other features are desirable?

5. To allow the creation of camera-ready documents, languages like TeX and LaTeX have been created. These languages can be thought of as varieties of programming languages whose output controls a printer or display. Source language commands control details like spacing, font choice, point size, and special symbols. Using the syntax-directed compiler structure of Figure 1.4, suggest the kind of processing that might occur in each compiler phase if TeX or LaTeX input was being translated.

   An alternative to "programming" documents is to use a sophisticated editor such as that provided in Microsoft® `Word` or Adobe® FrameMaker® to interactively enter and edit the document. (Editing operations allow the choice of fonts, selection of point size, inclusion of special symbols, and so on.) This approach to document preparation is called **WYSIWYG—**what you see is what you get—because the exact form of the document is always visible.

   What are the relative advantages and disadvantages of the two approaches? Do analogues exist for ordinary programming languages?

6. Although compilers are designed to translate a particular language, they often allow calls to subprograms that are coded in some other language (typically, Fortran, C, or assembler). Why are such "foreign calls" allowed? In what ways do they complicate compilation?

7. Most C compilers (including the GCC compilers) allow a user to examine the machine instructions generated for a given source program. Run the following program through such a C compiler and examine the instructions generated for the `for` loop. Next, recompile the program, enabling optimization, and reexamine the instructions generated for the `for` loop. What improvements have been made? Assuming that the program spends all of its time in the `for` loop, estimate the speedup obtained. Write a suitable `main` C function that allocates and initializes a million-element array to pass to `proc`. Execute and time the unoptimized and optimized versions of the program and evaluate the accuracy of your estimate.

```
int proc(int a[]) {
    int sum = 0, i;
    for (i=0; i < 1000000; i++)
       sum += a[i];
    return sum;
}
```

8. C is sometimes called the **universal assembly language** in light of its ability to be very efficiently implemented on a wide variety of computer architectures. In light of this characterization, some compiler writers have chosen to generate C code as their output instead of a particular machine language. What are the advantages to this approach to compilation? Are there any disadvantages?

9. Many computer systems provide an interactive debugger (for example, `gdb` or `dbx`) to assist users in diagnosing and correcting runtime errors. Although a debugger is run long after a compiler has done its job, the two tools still must cooperate. What information (beyond the translation of a program) must a compiler supply to support effective runtime debugging?

10. Assume you have a source program *P*. It is possible to transform *P* into an equivalent program *P′* by reformatting *P* (by adding or deleting spaces, tabs, and line breaks), systematically renaming its variables (for example, changing all occurrences of `sum` to `total`), and reordering the definition of variables and subroutines.

    Although *P* and *P′* are equivalent, they may well look very different. How could a compiler be modified to compare two programs and determine if they are equivalent (or very similar)? In what circumstances would such a tool be useful?

11. The **Measure Of Software Similarity** (MOSS) [SWA03] tool can detect similarity of programs written in a variety of modern programming languages. Its main application has been in detecting similarity of programs submitted in computer science classes, where such similarity may indicate plagiarism (students, beware!). In theory, detecting equivalence of two programs is **undecidable**, but MOSS does a very good job of finding similarity in spite of that limitation.

    Investigate the techniques MOSS uses to find similarity. How does MOSS differ from other approaches for detecting possible plagiarism?

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2

A Simple Compiler

In this chapter we provide an overview of the compilation process by considering a simple translation task for a very small language. This language, called ac for *adding calculator*, accommodates two forms of numerical data types, allows computation and printing of numerical values, and offers a small set of variable names to hold the results of computations.

To simplify both the presentation and implementation of a compiler, we break the compilation process into a sequence of *phases*. Each phase is responsible for a particular aspect of the compilation process. The early phases analyze the syntax of the input program with the goal of generating an abstract representation of the program's essential information for translation. The subsequent phases analyze and transform the tree, eventually generating a translation of the input program in the target language.

The ac language and its compilation are sufficiently simple to facilitate a relatively quick overview of a compiler's phases and their associated data structures. The tools and techniques necessary for undertaking translation tasks of a more substantial nature are presented in subsequent chapters. Some code fragments are presented in this chapter to illustrate the basic concepts of a compiler's phases. A complete form of the code presented here can be found in the *Crafting a Compiler Supplement*.

2.1 An Informal Definition of the ac Language

Our language is called ac (for *adding calculator*). When compared with most programming languages, ac is relatively simple, yet it serves nicely as a study for examining the phases and data structures of a compiler. We first define ac informally:

Types Most programming languages offer a significant number of predefined data types, with the ability to extend existing types or specify new data types. In ac, there are only two data types: integer and float. An integer type is a sequence of decimal numerals, as found in most programming languages. A float type allows five fractional digits after the decimal point.

Keywords Most programming languages have a number of **reserved keywords**, such as `if` and `while`, which would otherwise serve as variable names. In ac, there are three reserved keywords, each limited for simplicity to a single letter: `f` (declares a float variable), `i` (declares an integer variable), and `p` (prints the value of a variable).

Variables Some programming languages insist that a variable be declared by specifying the variable's type prior to using the variable's name. The ac language offers only 23 possible variable names, drawn from the lowercase Roman alphabet and excluding the three reserved keywords `f`, `i`, and `p`. Variables must be declared prior to using them.

Most programming languages have rules that dictate circumstances under which a given type can be converted into another type. In some cases, such **type conversion** is handled automatically by the compiler, while other cases require explicit syntax (such as **casts**) to allow the type conversion. In ac, conversion from integer type to float type is accomplished automatically. Conversion in the other direction is not allowed under any circumstances.

For the target of translation, we use the widely available program dc (for *desk calculator*), which is a stack-based calculator that uses **reverse Polish notation** (RPN). When an ac program is translated into a dc program, the resulting instructions must be acceptable to the dc program and must faithfully represent the operations specified in an ac program. Stack-based languages commonly serve as targets of translation because they lend themselves to compact representation. Examples include the translation of Java™ into **Java Virtual Machine** (JVM), ActionScript® into AVM2 for Flash® media, and printable documents into PostScript®. Thus, compilation of ac to dc can be viewed as a study of such larger systems.

```
1  Prog  → Dcls Stmts $
2  Dcls  → Dcl Dcls
3        | λ
4  Dcl   → floatdcl id
5        | intdcl id
6  Stmts → Stmt Stmts
7        | λ
8  Stmt  → id assign Val Expr
9        | print id
10 Expr  → plus Val Expr
11       | minus Val Expr
12       | λ
13 Val   → id
14       | inum
15       | fnum
```

Figure 2.1: Context-free grammar for ac.

2.2 Formal Definition of ac

Before translating ac to dc we must first understand the syntax and semantics of the ac language. The informal definitions above may generally describe ac, but they are too vague to serve as a formal definition. We therefore follow the example of most programming languages and use a **context-free grammar** (CFG) to specify our language's syntax and **regular expressions** to specify the basic symbols of the language.

2.2.1 Syntax Specification

While CFGs are discussed in detail in Chapter 4, we presently view a CFG simply as a set of **productions** or **rewriting rules**. A CFG for the ac language is given in Figure 2.1. To improve readability, multiple productions for the same symbol can be specified using an arrow for the first production and bar symbols to separate the rest of the productions. For example, Stmt serves the same role in each of the productions:

```
Stmt → id assign Val Expr
     | print id
```

These productions indicate that a Stmt can be replaced by one of two strings of symbols. In the first rule, Stmt is rewritten by symbols that represent assignment to an identifier. In the second rule, Stmt is rewritten by symbols that print an identifier's value.

Productions reference two kinds of symbols: *terminals* and *nonterminals*. A **terminal** is a grammar symbol that cannot be rewritten. For example, the id, assign, and $ symbols have no productions in Figure 2.1 that specify how they can be rewritten. On the other hand, Figure 2.1 does contain productions for the **nonterminal** symbols Val and Expr. To ease readability in the grammar, we adopt the convention that nonterminals begin with an uppercase letter and terminals are all lowercase letters.

Consider a CFG for some programming language of interest. The CFG serves as a formal and relatively compact definition of *all* syntactically correct programs for that programming language. To generate such a program, we begin with a special nonterminal known as the CFG's **start symbol**, which is usually the symbol on the **left-hand side** (LHS) of the grammar's first rule. For example, the start symbol in Figure 2.1 is Prog. From the start symbol, we proceed by replacing it with the **right-hand side** (RHS) of some production for that symbol.

We continue by choosing some nonterminal symbol in our derived string of symbols, finding a production for that nonterminal, and replacing it with the string of symbols on the production's RHS. As a special case, the symbol λ denotes the **empty** or **null string** string, which indicates that there are *no* symbols on a production's RHS. The special symbol $ represents the end of the input stream or file.

We continue applying productions, rewriting nonterminals until none remain. Any string of terminals that can be produced in this manner is considered syntactically valid. Any other string has a **syntax error** and would not be a legal program.

To show how the grammar in Figure 2.1 defines legal ac programs, the derivation of one such program is given in Figure 2.2, beginning with the start symbol Prog. Each line represents one step in the derivation. In each line, the leftmost nonterminal (surrounded by angle brackets) is replaced by the boxed text shown on the next line. The right column shows the production number by which the derivation step is accomplished. For example, the production Stmt → id assign Val Expr is applied at step 8 to reach step 9.

Notice that some productions in a grammar serve to generate an unbounded list of symbols from a nonterminal using recursive rules. For example, Stmts → Stmt Stmts (Rule 6) allows an arbitrary number of Stmt symbols to be produced. Each use of the recursive rule—at steps 7, 11, and 17—generates another Stmt in Figure 2.2. The recursion is terminated by applying Stmts → λ (Rule 7) at step 19, thereby causing the remaining Stmts symbol to be erased. Rules 2 and 3 function similarly to generate an arbitrary number of Dcl symbols.

Step	Sentential Form	Production Number
1	⟨Prog⟩	
2	$\boxed{\langle\text{Dcls}\rangle\ \text{Stmts}\ \$}$	1
3	$\boxed{\langle\text{Dcl}\rangle\ \text{Dcls}}$ Stmts \$	2
4	$\boxed{\text{floatdcl id}}$ ⟨Dcls⟩ Stmts \$	4
5	floatdcl id $\boxed{\langle\text{Dcl}\rangle\ \text{Dcls}}$ Stmts \$	2
6	floatdcl id $\boxed{\text{intdcl id}}$ ⟨Dcls⟩ Stmts \$	5
7	floatdcl id intdcl id ⟨Stmts⟩ \$	3
8	floatdcl id intdcl id $\boxed{\langle\text{Stmt}\rangle\ \text{Stmts}}$ \$	6
9	floatdcl id intdcl id $\boxed{\text{id assign}\ \langle\text{Val}\rangle\ \text{Expr}}$ Stmts \$	8
10	floatdcl id intdcl id id assign $\boxed{\text{inum}}$ ⟨Expr⟩ Stmts \$	14
11	floatdcl id intdcl id id assign inum ⟨Stmts⟩ \$	12
12	floatdcl id intdcl id id assign inum $\boxed{\langle\text{Stmt}\rangle\ \text{Stmts}}$ \$	6
13	floatdcl id intdcl id id assign inum $\boxed{\text{id assign}\ \langle\text{Val}\rangle\ \text{Expr}}$ Stmts \$	8
14	floatdcl id intdcl id id assign inum id assign $\boxed{\text{id}}$ ⟨Expr⟩ Stmts \$	13
15	floatdcl id intdcl id id assign inum id assign id $\boxed{\text{plus}\ \langle\text{Val}\rangle\ \text{Expr}}$ Stmts \$	10
16	floatdcl id intdcl id id assign inum id assign id plus $\boxed{\text{fnum}}$ ⟨Expr⟩ Stmts \$	15
17	floatdcl id intdcl id id assign inum id assign id plus fnum ⟨Stmts⟩ \$	12
18	floatdcl id intdcl id id assign inum id assign id plus fnum $\boxed{\langle\text{Stmt}\rangle\ \text{Stmts}}$ \$	6
19	floatdcl id intdcl id id assign inum id assign id plus fnum $\boxed{\text{print id}}$ ⟨Stmts⟩ \$	9
20	floatdcl id intdcl id id assign inum id assign id plus fnum print id \$	7

Figure 2.2: Derivation of an ac program using the grammar in Figure 2.1.

Terminal	Regular Expression
floatdcl	"f"
intdcl	"i"
print	"p"
id	[a − e] | [g − h] | [j − o] | [q − z]
assign	"="
plus	"+"
minus	"-"
inum	[0 − 9]$^+$
fnum	[0 − 9]$^+$.[0 − 9]$^+$
blank	(" ")$^+$

Figure 2.3: Formal definition of ac tokens.

2.2.2 Token Specification

Thus far, a CFG formally defines the sequences of terminal symbols that comprise a language. The actual input characters that could correspond to each terminal symbol must also be specified. The ac grammar in Figure 2.1 uses the assign symbol as a terminal, but that symbol will appear in the input stream as the = character. The terminal id could be any alphabetic character except f, i, or p, which are **reserved** for special use in ac. In most programming languages, the strings that could correspond to an id are practically unlimited, and tokens such as if and while are often reserved keywords.

In addition to the grammar's terminal symbols, language definitions often include elements such as comments, blank space, and compilation directives that must be properly recognized as tokens in the input stream. The formal specification of a language's tokens is typically accomplished by associating a **regular expression** with each token, as shown in Figure 2.3. A full treatment of regular expressions can be found in Section 3.2 on page 60.

The specification in Figure 2.3 begins with rules for the language's reserved keywords: f, i, and p. The specification for id uses the | symbol to specify the union of four sets, each a range of characters, so that an id is any lower case alphabetic character not already reserved. The specification for inum allows one or more decimal digits. An fnum is like an inum except that it is followed by a decimal point and then one or more digits.

Figure 2.4 illustrates an application of the ac specification to the input stream shown at the bottom. The tokens corresponding to the input stream are shown just above the input stream. To save space, the blank tokens are not shown.

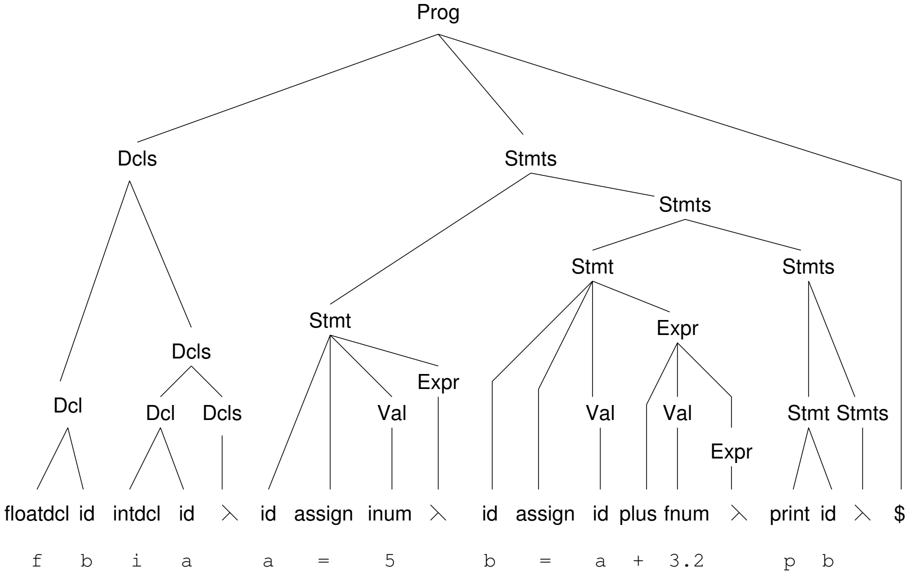
Prog
Dcls
Stmts
Stmts
Stmt
Stmts
Expr
Stmt
Dcls
Expr
Dcl
Dcl
Dcls
Val
Val
Val
Stmt
Stmts
Expr
floatdcl id intdcl id λ id assign inum λ id assign id plus fnum λ print id λ $
f b i a a = 5 b = a + 3.2 p b

Figure 2.4: An ac program and its parse tree.

We next consider the phases involved in compiling the ac program shown in Figure 2.4. The derivation shown textually in Figure 2.2 can be represented as a derivation (or parse) tree, also shown in Figure 2.4. An input stream can be automatically transformed into a stream of tokens using the techniques presented in Chapter 3.

In the following sections we examine each step of the compilation process for the ac language, assuming an input that would produce the derivation shown in Figure 2.2. While the treatment is somewhat simplified, the goal is to show the purpose and data structures of each phase.

2.3 Phases of a Simple Compiler

The rest of this chapter presents a simple compiler for ac, structured according to the illustration in Figure 1.4 on page 15. The phases in the translation process are as follows:

1. The *scanner* reads a source ac program as a text file and produces a stream of tokens. For example, strings such as `5` and `3.2` are recognized as `inum` and `fnum` tokens. Reserved keywords such as `f` and `p` are distinguished

from variable names such as a and b. For languages of greater complexity, the techniques presented in Chapter 3 automate much of this task.

2. The *parser* processes tokens produced by the scanner, determines the syntactic validity of the token stream, and creates an **abstract syntax tree** (AST) suitable for the compiler's subsequent activities. Given the simplicity of ac, we write its parser ad hoc using the *recursive-descent* style presented in Chapter 5. While such parsers work well in many cases, Chapter 6 presents a more popular technique for generating parsers automatically.

3. The AST created by the parsing task is next traversed to create a *symbol table*. This table associates type and other contextual information with variables used in an ac program. Most programming languages allow the use of an unbounded number of variable names. Techniques for processing symbols are discussed more generally in Chapter 8. This task can be greatly simplified for ac, which allows the use of at most 23 variable names.

4. The AST is next traversed to perform *semantic analysis*. For ac, such analysis is fairly minimal. For most programming languages, multiple passes over the AST may be required to enforce programming language rules that are difficult to check in the parsing task. Semantic analysis often decorates or transforms portions of an AST as the actual meaning of such portions becomes more clear. For example, an AST node for the + operator may be replaced with the actual meaning of +, which may mean floating point or integer addition.

5. Finally, the AST is traversed to generate a translation of the original program. Necessities such as register allocation and opportunities for program optimization may be implemented as phases that precede code generation. For ac, translation is sufficiently simple to be accommodated in a single code-generation pass.

2.4 Scanning

The scanner's job is to translate a stream of characters into a stream of **tokens**, where each token represents an instance of some terminal symbol. Rigorous methods for automatically constructing scanners based on regular expressions (such as those shown in Figure 2.3) are covered in Chapter 3. Here, the job at hand is sufficiently simple to undertake manually. Figure 2.5 shows pseudocode for a basic, ad hoc scanner that finds tokens for the ac language. Each token found by the scanner has the following two components:

- A token's *type* explains the token's membership in the terminal alphabet. All instances of a given terminal have the same token type.
- A token's **semantic value** provides additional information about the token.

For terminals such as plus, no semantic information is required, because only one token (+) can correspond to that terminal. Other terminals, such as id and num, require semantic information so that the compiler can record *which* identifier or number has been scanned.

The scanner in Figure 2.5 finds the beginning of a token by first skipping over any blanks. Scanners are often instructed to ignore comments and symbols that serve only to format the text, such as blanks and tabs. Next, using a single character of lookahead (the PEEK method), the scanner determines if the next token will be a num or some other terminal. Because the code for scanning a number is relatively complex, it is relegated to the ScanDigits procedure shown in Figure 2.6. Otherwise, the scanner is moved to the next input character (using ADVANCE), which suffices to determine the next token.

For most programming languages, the scanner's job is not so easy. Some tokens (+) can be prefixes of other tokens (++); other tokens such as comments and string constants have special symbols involved in their recognition. For example, a string constant is usually surrounded by quote symbols. If such symbols are meant to appear literally in the string constant, then they are usually **escaped** by a special character such as backslash (\). Variable-length tokens such as identifiers, constants, and comments must be matched character by character. If the next character is part of the current token, it is consumed. When a character that cannot be part of the current token is reached, scanning is complete. Some input files may contain character sequences that do not correspond to any token and should be flagged as errors.

The inum- and fnum-finding code in Figure 2.6 is written ad hoc, yet the logic of its construction is patterned after the tokens' regular expressions. A recurring theme in compiler construction is the use of such principled approaches and patterns to guide the crafting of a compiler's phases.

While the code in Figures 2.5 and 2.6 serves to illustrate the nature of a scanner, we emphasize that the most reliable and expedient methods for constructing scanners do so automatically from regular expressions, as covered in Chapter 3. Such scanners are reasonably efficient and correct by construction, given a correct set of regular-expression specifications for the tokens.

2.5 Parsing

The parser is responsible for determining if the stream of tokens provided by the scanner conforms to the language's grammar specification. In most

```
function SCANNER( ) returns Token
    while s.PEEK( ) = blank do call s.ADVANCE( )
    if s.EOF( )
    then ans.type ← $
    else
        if s.PEEK( ) ∈ { 0, 1, . . . , 9 }
        then ans ← SCANDIGITS( )
        else
            ch ← s.ADVANCE( )
            switch (ch)
                case { a, b, . . . , z } − { i, f, p }
                    ans.type ← id
                    ans.val ← ch
                case f
                    ans.type ← floatdcl
                case i
                    ans.type ← intdcl
                case p
                    ans.type ← print
                case =
                    ans.type ← assign
                case +
                    ans.type ← plus
                case -
                    ans.type ← minus
                case default
                    call LEXICALERROR( )
    return (ans)
end
```

Figure 2.5: Scanner for the ac language. The variable s is an input stream of characters.

```
function ScanDigits( ) returns token
    tok.val ← " "
    while $s.\text{peek}() \in \{0, 1, \ldots, 9\}$ do
        $tok.val \leftarrow tok.val + s.\text{advance}()$
    if $s.\text{peek}() \neq$ "."
    then  tok.type ← inum
    else
        tok.type ← fnum
        $tok.val \leftarrow tok.val + s.\text{advance}()$
        while $s.\text{peek}() \in \{0, 1, \ldots, 9\}$ do
            $tok.val \leftarrow tok.val + s.\text{advance}()$
    return (tok)
end
```

Figure 2.6: Finding inum or fnum tokens for the ac language.

compilers the grammar serves not only to define the syntax of a programming language, but also to guide the automatic construction of a parser, as described in Chapters 4, 5, and 6. In this section we build a parser for ac using a well-known parsing technique called *recursive descent*, which is described more fully in Chapter 5.

Recursive descent is one of the simplest parsing techniques used in practical compilers. The name is taken from the mutually recursive parsing routines that, in effect, descend through a derivation tree. In recursive-descent parsing, each nonterminal in the grammar has an associated *parsing procedure* that is responsible for determining if the token stream contains a sequence of tokens derivable from that nonterminal. For example, the nonterminal Stmt is associated with the parsing procedure shown in Figure 2.7.

We next illustrate how to write recursive descent parsing procedures for the nonterminals Stmt and Stmts from the grammar in Figure 2.1. Section 2.5.1 explains how such parsers predict which production to apply, and Section 2.5.2 explains the actions taken on behalf of a production.

2.5.1 Predicting a Parsing Procedure

Each procedure first examines the next input token to predict which production should be applied. For example, Stmt offers two productions:

Stmt → id assign Val Expr
Stmt → print id

In Figure 2.7, Markers ① and ⑥ pick which of those two productions should be followed by examining the next input token:

```
procedure Stmt( )
    if ts.peek( ) = id                                  ①
    then
        call match(ts, id )                             ②
        call match(ts, assign )                         ③
        call Val( )                                     ④
        call Expr( )                                    ⑤
    else
        if ts.peek( ) = print                           ⑥
        then
            call match(ts, print )
            call match(ts, id )
        else
            call error( )                               ⑦
end
```

Figure 2.7: Recursive-descent parsing procedure for Stmt. The variable *ts* is an input stream of tokens.

- If id is the next input token, then the parse must proceed with a rule that generates id as its first terminal. Because Stmt → id assign Val Expr is the only rule for Stmt that first generates an id, it must be uniquely predicted by the id token. Marker ① in Figure 2.7 performs this test.

  We say that the **predict set** for Stmt → id assign Val Expr is { id }.

- Similarly, if print is the next input token, the production Stmt → print id is predicted by the test at Marker ⑥. The predict set for Stmt → print id is { print }.

- Finally, if the next input token is neither id nor print, then neither rule can be predicted. Given that the Stmt procedure is called only where the nonterminal Stmt should be derived, the input must have a syntax error, as reported at Marker ⑦.

Computing the predict sets used in Stmt is relatively easy, because each production for Stmt begins with a distinct terminal symbol (id or print). However, consider the productions for Stmts:

Stmts → Stmt Stmts
Stmts → λ

The predict sets for Stmts in Figure 2.8 cannot be computed so easily by inspection because of the following:

```
procedure STMTS( )
    if ts.PEEK( ) = id or ts.PEEK( ) = print                 ⑧
    then
        call STMT( )                                          ⑨
        call STMTS( )                                         ⑩
    else
        if ts.PEEK( ) = $                                     ⑪
        then
            /*    do nothing for $\lambda$-production      */ ⑫
        else  call ERROR( )
end
```

Figure 2.8: Recursive-descent parsing procedure for Stmts.

- The production Stmts→Stmt Stmts begins with the nonterminal Stmt. To discover the terminals that predict this rule, we must find those symbols that predict *any* rule for Stmt. Fortunately, we have already done this in Figure 2.7. The predicate at Marker ⑧ in Figure 2.8 checks for id or print as the next token.

- The production Stmts→λ derives *no* symbols, so we must look instead for what symbols could occur *after* such a production. Grammar analysis (Chapter 4) can show that $ is the only such symbol, so it predicts Stmts→λ at Marker ⑪.

The analysis required to compute predict sets in general is covered in Chapters 4 and 5.

2.5.2 Implementing the Production

Once a given production has been predicted, the recursive descent procedure then executes code to trace across that production, one symbol at a time. For example, the production Stmt→id assign Val Expr in Figure 2.7 derives 4 symbols, and those will be considered in the order id, assign, Val, and Expr. The code for processing those symbols, shown at Markers ②, ③, ④, and ⑤, is written into the recursive descent procedure as follows:

- When a terminal such as id is encountered, a call to MATCH(*ts*, id) is placed into the code, as shown by Marker ② in Figure 2.7. The MATCH procedure (code shown in Figure 5.5 on page 149) simply consumes the expected token id if it is indeed the next token in the input stream. If some other token is found, then the input stream has a syntax error, and an appropriate message is issued. The call after Marker ② tries to match assign, which is the next symbol in the production.

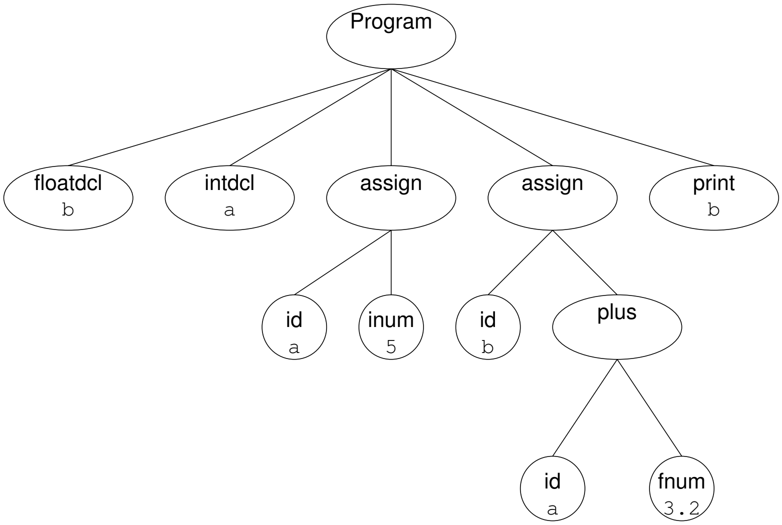

Figure 2.9: An abstract syntax tree for the ac program shown in Figure 2.4.

Throughout Figures 2.7 and 2.8, calls to match appear on behalf of terminal symbols within a production.

- The last two symbols in Stmt → id assign Val Expr are nonterminals. The recursive descent parser has a method responsible for the derivation of each nonterminal in a grammar. Thus, the code at Marker ④ calls the procedure Val associated with the nonterminal Val. Finally, the Expr method is called on behalf of the last symbol.

  In Figure 2.8, the code executed on behalf of Stmts → Stmt Stmts first calls Stmt at Marker ⑨ and then calls Stmts recursively at Marker ⑩. Recursive calls appear on behalf of grammar productions that reference each other. Recursive descent parsers are named after the manner in which the parser's methods call each other.

- The only other symbol that can be encountered is λ, as in Stmts → λ. For such productions, no symbols are derived from the nonterminal. Thus, no code is executed on behalf of such rules, as shown at Marker ⑫ in Figure 2.8.

The recursive-descent parser for Figure 2.1 is completed by writing a method for each nonterminal using the approach described above. The resulting parser can be found in the *Crafting a Compiler Supplement*.

2.6 Abstract Syntax Trees

The scanner and parser together accomplish the **syntax analysis** phase of a compiler. They ensure that the compiler's input conforms to a language's token and CFG specifications. While the process of compilation begins with scanning and parsing, following are some aspects of compilation that can be difficult or even impossible to perform during syntax analysis:

- Most programming language specifications include prose that describes aspects of the language that cannot be specified in a CFG. For example, strongly typed languages insist that symbols be used in ways consistent with their type declaration. For languages that allow new types to be declared, a CFG cannot presuppose the names of such types nor the manner in which they should properly be used. Even if the set of types is fixed by a language, enforcing proper usage usually requires some **context sensitivity** that is clearly not available in a CFG.

  Some languages use the same syntax to describe phrases whose meaning cannot be made clear in a CFG. For example, the phrase `x.y.z` in Java could mean a package `x`, a class `y`, and a static field `z`. That same phrase could also mean a local variable `x`, a field `y`, and another field `z`. In fact, many other meanings are possible: Java provides (6 pages of) rules to determine which of the possible interpretations holds for a given phrase, given the packages and classes that are present during a given compilation.

  Most languages allow operators to be **overloaded** to mean more than one actual operation. For example, the + operator might mean numerical addition or the appending of strings. Some languages allow the meaning of an operator to be defined in the program itself.

  In all of the above cases, a programming language's CFG alone provides insufficient information to understand the full meaning of a program.

- For relatively simple languages, **syntax-directed translation** can perform almost all aspects of program translation during syntax analysis. Compilers written in that fashion are arguably more efficient than compilers that perform a separate pass over a program for each phase. However, from a software engineering perspective, the separation of activities and concerns into phases (such as syntax analysis, semantic analysis, optimization, and code generation) makes the resulting compiler much easier to write and maintain.

In response to the above concerns, we might consider using the parse tree as the structure that survives syntax analysis and is used for the remaining phases. However, as Figure 2.4 shows, such trees can be rather large and unnecessarily detailed, even for very simple grammars and inputs.

It is therefore common practice to create an artifact of syntax analysis known as the **abstract syntax tree** (AST). This structure contains the essential information from a parse tree, but inessential punctuation and delimiters (braces, semicolons, parentheses, etc.) are not included. For example, Figure 2.9 shows an AST for the parse tree of Figure 2.4. In the parse tree, 8 nodes are devoted to generating the expression `a + 3.2`, but only 3 nodes are required to show the essence of that expression in Figure 2.9.

The AST serves as a common, intermediate representation for a program for all phases after syntax analysis. Such phases may make use of information in the AST, decorate the AST with more information, or transform the AST. Thus, the needs of the compiler's phases must be considered when designing an AST. For the ac language, such considerations are as follows:

- Declarations need not be retained in source form. However, a record of identifiers and their declared types must be retained to facilitate *symbol table construction* and *semantic type checking*, as described in Section 2.7. Each Dcl in the parse tree of Figure 2.4 is represented by a single node in the AST of Figure 2.9.
- The order of the executable statements is important and must be explicitly represented, so that *code generation* (Section 2.8) can issue instructions in the proper order.
- An assignment statement must retain the identifier that will hold the computed value and the expression that computes the value. Each assign node in Figure 2.9 has exactly two children.
- Nodes representing computation such as plus and minus can be represented in the AST as a node specifying the operation with two children for the operands.
- A print statement must retain the name of the identifier to be printed. In the AST, the identifier is stored directly in the print node.

It is common to revisit and modify the AST's design as the compiler is being written, in response to the needs of the various phases of the compiler. Object-oriented design patterns such **visitor** facilitate the design and implementation of the AST, as discussed in Chapter 7.

2.7 Semantic Analysis

The next phase to consider is **semantic analysis**, which is really a catchall term for any post-parsing processing that enforces aspects of a language's definition that are not easily accommodated by syntax analysis. Examples of such processing include the following:

```
/★  Visitor methods                                        ★/
procedure VISIT( SymDeclaring n )
    if n.GETTYPE( ) = floatdcl
    then  call ENTERSYMBOL(n.GETID( ), float)
    else  call ENTERSYMBOL(n.GETID( ), integer)
end

/★  Symbol table management                                ★/
procedure ENTERSYMBOL(name, type)
    if SymbolTable[name] = null
    then  SymbolTable[name] ← type
    else  call ERROR( "duplicate declaration" )
end

function LOOKUPSYMBOL(name) returns type
    return (SymbolTable[name])
end
```

Figure 2.10: Symbol table construction for ac.

- Declarations and name scopes are processed to construct a *symbol table*, so that declarations and uses of identifiers can be properly coordinated.
- Language- and user-defined types are examined for consistency.
- Operations and storage references are processed so that type-dependent behavior can become explicit in the program representation.

For the ac language, we focus on two aspects of semantic analysis: symbol-table construction and type checking.

2.7.1 Symbol Tables

In ac, identifiers must be declared prior to use, but this requirement is not easily enforced during syntax analysis. Symbol-table construction is a semantic-processing activity that traverses the AST to record all identifiers and their types in a **symbol table**. In most languages the set of potential identifiers is essentially infinite. In ac a program can mention at most 23 distinct identifiers. As a result, an ac symbol table has 23 entries indicating each identifier's type: integer, float, or unused (**null**). In most programming languages the type information associated with a symbol includes other attributes, such as the identifier's scope of visibility, storage class, and protection properties.

To create an ac symbol table, we traverse the AST, counting on the presence of a *symbol-declaring node* to trigger appropriate effects on the symbol

Symbol	Type	Symbol	Type	Symbol	Type
a	integer	k	**null**	t	**null**
b	float	l	**null**	u	**null**
c	**null**	m	**null**	v	**null**
d	**null**	n	**null**	w	**null**
e	**null**	o	**null**	x	**null**
g	**null**	q	**null**	y	**null**
h	**null**	r	**null**	z	**null**
j	**null**	s	**null**		

Figure 2.11: Symbol table for the ac program from Figure 2.4.

table. This can be arranged by having nodes such as floatdcl and intdcl implement an interface (or inherit from an empty class) called *SymDeclaring*, which implements a method to return the declared identifier's type. In Figure 2.10, visit(*SymDeclaring* n) shows the code to be applied at nodes that declare symbols. As declarations are discovered, EnterSymbol checks that the given identifier has not been previously declared. Figure 2.11 shows the symbol table constructed for our example ac program.

2.7.2 Type Checking

The ac language offers only two types, integer and float, and all identifiers must be type-declared in a program before they can be used. After the symbol table has been constructed, the declared type of each identifier is known, and the executable statements of the program can be checked for type consistency.

Most programming language specifications include a **type hierarchy** that compares the language's types in terms of their generality. Our ac language follows in the tradition of Java, C, and C++, in which a float type is considered **wider** (i.e., more general) than an integer. This is because every integer can be represented as a float. On the other hand, **narrowing** a float to an integer loses precision for some float values.

Most languages allow automatic widening of type, so an integer can be converted to a float without the programmer having to specify this conversion explicitly. On the other hand, a float cannot become an integer in most languages unless the programmer explicitly calls for this conversion.

Once symbol type information has been gathered, ac's executable statements can be examined for consistency of type usage. This process, known as **type checking**, walks the AST bottom-up, from its leaves toward its root. At each node, the appropriate visitor method (if any) in Figure 2.12 is applied:

```
/*  Visitor methods                                                  */
procedure VISIT( Computing $n$ )
    $n.type \leftarrow$ CONSISTENT( $n.child1, n.child2$ )
end
procedure VISIT( Assigning $n$ )
    $n.type \leftarrow$ CONVERT( $n.child2, n.child1.type$ )
end
procedure VISIT( SymReferencing $n$ )
    $n.type \leftarrow$ LOOKUPSYMBOL( $n.id$ )
end
procedure VISIT( IntConsting $n$ )
    $n.type \leftarrow$ integer
end
procedure VISIT( FloatConsting $n$ )
    $n.type \leftarrow$ float
end
/*  Type-checking utilities                                          */
function CONSISTENT( $c1, c2$ ) returns $type$
    $m \leftarrow$ GENERALIZE( $c1.type, c2.type$ )
    call CONVERT( $c1, m$ )
    call CONVERT( $c2, m$ )
    return ($m$)
end
function GENERALIZE( $t1, t2$ ) returns $type$
    if $t1$ = float or $t2$ = float
    then  $ans \leftarrow$ float
    else  $ans \leftarrow$ integer
    return ($ans$)
end
procedure CONVERT( $n, t$ )
    if $n.type$ = float and $t$ = integer
    then  call ERROR( "Illegal type conversion" )
    else
        if $n.type$ = integer and $t$ = float
        then
            /*  replace node $n$ by convert-to-float of node $n$         */ 13
        else  /* nothing needed */
end
```

Figure 2.12: Type analysis for ac.

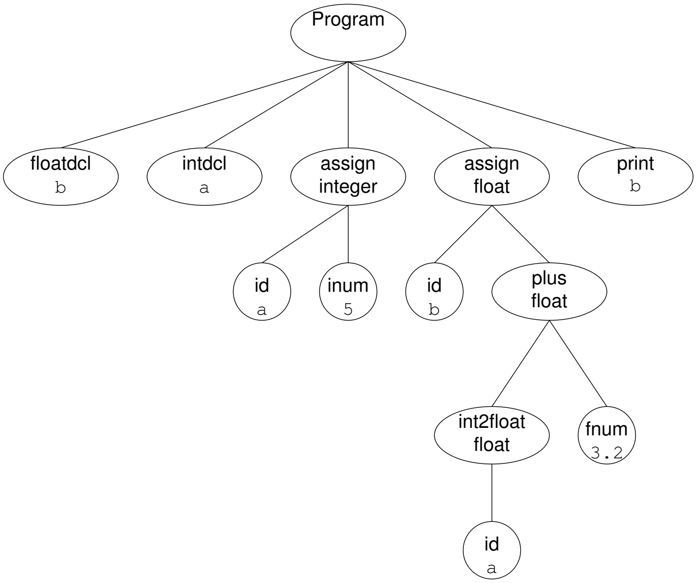

Figure 2.13: AST after semantic analysis.

- For constants and symbol references, the visitor methods simply set the supplied node's type based on the node's contents.
- For nodes that compute values, such as plus and minus, the appropriate type is computed by calling the utility methods in Figure 2.12. If both types are integer, the resulting computation is integer; otherwise, the resulting type is float.
- For an assignment operation, the visitor makes certain that the value computed by the second child is of the same type as the assigned identifier (the first child).

The Consistent method, shown in Figure 2.12, is responsible for reconciling the type of a pair of AST nodes using the following steps:

1. The Generalize function determines the least general (i.e., simplest) type that encompasses its supplied pair of types. For ac, if either type is float, then float is the appropriate type; otherwise, integer will do.
2. The Convert procedure checks whether conversion is necessary, possible, or impossible. An important consequence occurs at Marker ⑬ in Figure 2.12. If conversion is attempted from integer to float, then the

AST is transformed to represent this type conversion explicitly. Subsequent compiler passes (particularly code generation) can then assume a type-consistent AST in which all operations are explicit.

The results of applying semantic analysis to the AST of Figure 2.9 are shown in Figure 2.13.

2.8 Code Generation

The final task undertaken by a compiler is the formulation of target-machine instructions that faithfully represent the semantics (i.e., meaning) of the source program. This process is called **code generation**. Our translation exercise consists of generating source code that is suitable for the dc program, which is a simple calculator based on a *stack machine* model. In a **stack machine**, most instructions receive their input from the contents at or near the top of an operand stack. The result of most instructions is pushed on the stack. Programming languages such as C♯ and Java are frequently translated into a portable, stack machine representation.

Chapters 11 and 13 discuss code generation in detail. Modern compilers often generate code automatically, based on a description of the target machine's instruction set. Our translation task is sufficiently simple to allow an ad hoc approach.

The AST was transformed and decorated with type information during semantic analysis. Such information is required for selecting the proper instructions. For example, the instruction set on most computers distinguishes between float and integer data types.

Code generation proceeds by traversing the AST, starting at its root and working toward its leaves. As usual, we allow a visitor to apply methods based on the node's type, as shown in Figure 2.14.

- VISIT(*Computing* n) generates code for plus and minus. First, the code generator is called recursively to generate code for the left and right subtrees. The resulting values will be at top-of-stack, so the appropriate operator is then emitted (Marker ⑮) to perform the operation.

- VISIT(*Assigning* n) causes the expression to be evaluated. Code is then emitted to store the value in the appropriate dc register. The calculator's precision is then reset to integer by setting the fractional precision to zero (Marker ⑭)

- VISIT(*SymReferencing* n) causes a value to be retrieved from the appropriate dc register and pushed onto the stack.

```
procedure visit( Assigning n )
    call CodeGen( n.child2 )
    call Emit( "s" )
    call Emit( n.child1.id )
    call Emit( "0 k" )                                  ⑭
end
procedure visit( Computing n )
    call CodeGen( n.child1 )
    call CodeGen( n.child2 )
    call Emit( n.operation )                            ⑮
end
procedure visit( SymReferencing n )
    call Emit( "l" )
    call Emit( n.id )
end
procedure visit( Printing n )
    call Emit( "l" )
    call Emit( n.id )
    call Emit( "p" )
    call Emit( "si" )                                   ⑯
end
procedure visit( Converting n )
    call CodeGen( n.child )
    call Emit( "5 k" )                                  ⑰
end
procedure visit( Consting n )
    call Emit( n.val )
end
```

Figure 2.14: Code generation for ac

- visit(*Printing* n) is tricky because dc does not discard the value on top-of-stack after it is printed. The instruction sequence `si` is generated at Marker ⑯, thereby popping the stack and storing the value in dc's `i` register. Conveniently, the ac language precludes a program from using this register because the `i` token is reserved for spelling the terminal symbol integer.

- visit(*Converting* n) causes a change of type from integer to float at Marker ⑰. This is accomplished by setting dc's precision to five fractional decimal digits.

Figure 2.15 shows how code is generated for the AST shown in Figure 2.9. Each section shows the code generated for a particular subtree of Figure 2.9.

Code	Source	Comments
`5`	`a = 5`	Push `5` on stack
`sa`		Pop the stack, storing (s) the popped value in register a
`0 k`		Reset precision to integer
`la`	`b = a + 3.2`	Load (l) register a, pushing its value on stack
`5 k`		Set precision to float
`3.2`		Push `3.2` on stack
`+`		Add: `5` and `3.2` are popped from the stack and their sum is pushed
`sb`		Pop the stack, storing the result in register `b`
`0 k`		Reset precision to integer
`lb`	`p b`	Push the value of the `b` register
`p`		Print the top-of-stack value
`si`		Pop the stack by storing into the `i` register

Figure 2.15: Code generated for the AST shown in Figure 2.9.

Even in this ad hoc code generator, one can see a principled approach. The code sequences triggered by various AST nodes dovetail to carry out the instructions of the input program. Although the task of code generation for real programming languages and targets is more complex, the theme still holds that pieces of individual code generation contribute to a larger effect.

This finishes our tour of a compiler for the ac language. While each of the phases becomes more involved as we move toward working with real programming languages, the spirit of each phase remains the same. In the ensuing chapters, we discuss how to automate many of the tasks described in this chapter. We develop skills necessary to craft a compiler's phases to accommodate issues that arise when working with real programming languages.

Exercises

1. The CFG shown in Figure 2.1 defines the syntax of ac programs. Explain how this grammar enables you to answer the following questions.

   (a) Can an ac program contain only declarations (and no statements)?

   (b) Can a print statement precede all assignment statements?

2. Sometimes it is necessary to modify the syntax of a programming language. This is done by changing the CFG that the language uses. What changes would have to be made to ac's CFG (Figure 2.1) to implement the following changes?

   (a) All ac programs must contain at least one statement.

   (b) All integer declarations must precede all float declarations.

   (c) The first statement in any ac program must be an assignment statement.

3. Extend the ac scanner (Figure 2.5) in the following ways:

   (a) A floatdcl can be represented as either `f` or `float`, allowing a more Java-like syntax for declarations.

   (b) An intdcl can be represented as either `i` or `int`.

   (c) A num may be entered in exponential (scientific) form. That is, an ac num may be suffixed with an optionally signed exponent (`1.0e10`, `123e-22` or `0.31415926535e1`).

4. Write the recursive-descent parsing procedures for all nonterminals in Figure 2.1.

5. The recursive-descent code shown in Figure 2.7 contains redundant tests for the presence of some terminal symbols. How would you decided which ones are redundant?

6. Variables are considered **uninitialized** after they are declared in some programming languages. In ac a variable must be given a value in an assignment statement before it can be correctly used in an expression or print statement.

   Suggest how to extend ac's semantic analysis (Section 2.7) to detect variables that are used before they are properly initialized.

7. Implement semantic actions in the recursive-descent parser for ac to construct ASTs using the design guidelines in Section 2.6.

8. The grammar for ac shown in Figure 2.1 requires all declarations to precede all executable statements. In this exercise, the ac language is extended so that declarations and executable statements can be interspersed. However, an identifier cannot be mentioned in an executable statement until it has been declared.

   (a) Modify the CFG in Figure 2.1 to accommodate this language extension.

   (b) Discuss any revisions you would consider in the AST design for ac.

   (c) Discuss how semantic analysis is affected by the changes you envision for the CFG and the AST.

9. The abstract tree design for ac uses a single node to represent a print operation (see Figure 2.9). Consider an alternative design in which the print operation always has a single id child that represents the variable to be printed. What are the design and implementation issues associated with the two approaches?

10. The code in Figure 2.10 examines an AST node to determine its effect on the symbol table. Explain why the order in which nodes are visited does or does not matter with regard to symbol-table construction.

11. Figure 2.6 scans an input stream for an inum or fnum based on the regular expressions for those patterns shown in Figure 2.3. The code in Figure 2.6 does not check for errors.

    (a) Where could errors occur in Figure 2.6?

    (b) For each error, what action would you take should the error occur?

12. The last fragment of code generated in Figure 2.15 pops the dc stack and stores the resulting value in register `i`.

    (a) Why was register `i` chosen to receive the result?

    (b) Which other registers could have been chosen without causing any problems for code that might be generated subsequently?

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3

Scanning—Theory and Practice

In this chapter, we discuss the theoretical and practical issues involved in building a *scanner*. For the purposes of crafting a compiler, the scanner's job (as introduced in Section 2.4 on page 38) is to translate an input stream of characters into a stream of *tokens*, each corresponding to a *terminal* symbol of a programming language. More generally, scanners perform specified actions triggered by an associated pattern of input characters. Techniques related to scanning are found in most software components that are tasked with identifying structure in their input. For example, the processing of network packets, the display of Web pages, and the interpretation of digital video and audio media require some form scanning.

In Section 3.1, we give an overview of how a scanner operates. Section 3.2 revisits the declarative *regular expression* notation introduced in Section 2.2 on page 33, which is particularly well suited to the formal definition of tokens and the automatic generation of scanners. In Section 3.4, the correspondence between regular expressions and *finite automata* is studied. Section 3.5 considers a widely used scanner generator, *Lex*, as a case study. Lex uses regular expressions to produce a complete scanner component, ready to be compiled and deployed on its own or as part of a larger project. Section 3.6 briefly considers other scanner generators.

In Section 3.7, we discuss the practical considerations needed to build a scanner and integrate it with the rest of a compiler. These considerations include anticipating the tokens and contexts that may complicate scanning, avoiding performance bottlenecks, and recovering from lexical errors.

Section 3.8 describes the theory used by tools such as Lex to turn regular expressions into executable scanners. While this material is not strictly necessary to craft a compiler, the theoretical aspects of scanning are elegant, relatively straightforward, and helpful in understanding both the power and limitations of scanners.

3.1 Overview of a Scanner

The primary function of a scanner is to transform a character stream into a token stream. A scanner is sometimes called a **lexical analyzer**, or **lexer**. The names "scanner," "lexical analyzer," and "lexer" are used interchangeably. The ac scanner discussed in Chapter 2 was simple and could be coded by any competent programmer. In this chapter, we develop a thorough and systematic approach to scanning that will allow us to create scanners for complete programming languages.

We introduce formal notations for specifying the precise structure of tokens. At first glance, this may seem unnecessary because of the simple token structure found in most programming languages. However, token structure can be more detailed and subtle than one might expect. For example, consider **string constants** in C, C++, and Java™, which are surrounded by double quotes. The contents of the string can be any sequence of characters except the double quote, as that would terminate the string. A double quote can appear literally in the string only if it is preceded (**escaped**) by a backslash. Is this simple definition really correct? Can a newline character appear in a string? In C it cannot, unless it is escaped with a backslash. This notation avoids a "runaway string" that, lacking a closing quote, matches characters intended to be part of other tokens. While C, C++, and Java allow escaped newlines in strings, Pascal forbids them. Ada goes further still and forbids all unprintable characters (precisely because they are normally unreadable). Similarly, are null (zero-length) strings allowed? C, C++, Java, and Ada allow them, but Pascal forbids them. In Pascal, a string is a packed array of characters and zero-length arrays are disallowed.

A precise definition of tokens is necessary to ensure that lexical rules are clearly stated and properly enforced. Formal definitions also allow a language designer to anticipate design flaws. For example, virtually all languages provide syntax for specifying certain kinds of **rational constants**. Such constants are often specified using decimal numerals such as `0.1` and `10.01`. Should the notation `.1` or `10.` also be allowed? In C, C++, and Java, such notation is permitted, but in Pascal and Ada, it is not—and for an interesting reason. Scanners normally seek to match as many characters as possible. Thus `ABC` is scanned as one identifier rather than three. But now consider the character sequence `1..10`. In Pascal and Ada, this should be interpreted as a range specifier (1 to 10). However, if we were careless in our token definitions, we might well scan `1..10` as two constants, `1.` and `.10`, which would lead to an immediate (and unexpected) syntax error. The fact that two constants *cannot* be adjacent is reflected in the **context-free grammar** (CFG), which is enforced by the parser, not the scanner.

When a formal specification of token and program structure is given, it is possible to examine a language for design flaws. For example, we could analyze all pairs of tokens that can be adjacent to each other and determine whether the two, if catenated, might be incorrectly scanned. If so, a separator may be required. In the case of adjacent identifiers and reserved words, a blank space (whitespace) suffices to distinguish the two tokens. Sometimes, though, the lexical or program syntax might need to be redesigned. The point is that language design is far more involved than one might expect, and formal specifications allow flaws to be discovered before the design is completed.

All scanners, independent of the tokens to be recognized, perform much the same function. Thus, writing a scanner from scratch means reimplementing components that are common to all scanners; this leads to a significant duplication of effort. The goal of a **scanner generator** is to limit the effort of building a scanner to that of specifying which tokens the scanner is to recognize. Using a formal notation, we tell the scanner generator what tokens we want recognized. It is then the generator's responsibility to produce a scanner that meets our specification. Some generators do not produce an entire scanner. Rather, they produce tables that can be used with a standard driver program, and this combination of generated tables and standard driver yields the desired custom scanner.

Programming a scanner generator is an example of **declarative programming**. That is, unlike in ordinary, or **procedural programming**, we do not tell a scanner generator *how* to scan but simply *what* to scan. This is a higher-level approach and in many ways a more natural one. Much recent research in computer science is directed toward declarative programming styles; examples are database query languages and Prolog, a "logic" programming language. Declarative programming is most successful in limited domains, such as scanning, where the range of implementation decisions that must be made automatically is limited. Nonetheless, a long-standing (and as yet unrealized) goal of computer scientists is to generate an entire production-quality compiler automatically from a specification of the properties of the source language and target computer.

Although our primary focus in this book is on producing correct compilers, performance is sometimes a real concern, especially in widely used "production compilers." Surprisingly, even though scanners perform a simple task, they can be significant performance bottlenecks if poorly implemented. This because scanners must wade through the text of a program character by character.

Suppose we want to implement a very fast compiler that can compile a program in a few seconds. We will use 30,000 lines per minute (500 lines per second) as our goal. (Compilers such as Turbo C++ achieve such speeds.) If an average line contains 20 characters, the compiler must scan 10,000 characters per second. On a processor that executes 10,000,000 instructions per second, even if we did nothing but scanning, we would have only 1,000 instructions per input character to spend. But because scanning is not the only thing a compiler does, 250 instructions per character is more realistic. This is a rather tight budget, considering that even a simple assignment takes several instructions on a typical processor. Although faster processors are common these days and 30,000 lines per minute is an ambitious speed, clearly a poorly coded scanner can dramatically impact a compiler's performance.

3.2 Regular Expressions

Regular expressions are a convenient way to specify various simple (although possibly infinite) sets of strings. They are of practical interest because they can specify the structure of the tokens used in a programming language. In particular, you can use regular expressions to program a scanner generator.

Regular expressions are widely used in computer applications other than compilers. The Unix® utility grep uses them to define search patterns in files. Unix shells allow a restricted form of regular expressions when specifying file lists for a command. Most editors provide a "context search" command that enables you to specify desired matches using regular expressions.

A set of strings defined by regular expressions is called a **regular set**. For purposes of scanning, a token class is a regular set whose structure is defined by a regular expression. A particular instance of a token class is sometimes called a **lexeme**; however, we simply call a string in a token class an *instance* of that token. For example, we call the string abc an identifier if it matches the regular expression that defines the set of valid identifier tokens.

Our definition of regular expressions starts with a finite character set, or **vocabulary** (denoted Σ). This vocabulary is normally the character set used by a computer. Today, the *ASCII* character set, which contains 128 characters, is very widely used. Java, however, uses the *Unicode* character set. This set includes all of the ASCII characters as well as a wide variety of other characters.

An empty, or null, string is allowed (denoted λ). This symbol represents an empty buffer in which no characters have yet been matched. It also represents an optional part of a token. Thus, an integer literal may begin with a plus or minus, or, if it is unsigned, it may begin with λ.

Strings are built from characters in the character set Σ via *catenation* (that is, by joining individual characters to form a string). As characters are catenated to a string, it grows in length. For example, the string do is built by first catenating d to λ and then catenating o to the string d. The null string, when catenated with any string s, yields s. That is, $s\ \lambda \equiv \lambda\ s \equiv s$. Catenating λ to a string is like adding 0 to an integer—nothing changes.

Catenation is extended to sets of strings as follows. Let P and Q be sets of strings. The symbol $\in$ represents set membership. If $s_1 \in P$ and $s_2 \in Q$, then string $s_1 s_2 \in (P\ Q)$. Small finite sets are conveniently represented by listing their elements, which can be individual characters or strings of characters. Parentheses are used to delimit expressions, and |, the alternation operator, is used to separate alternatives. For example, D, the set of the ten single digits, is defined as $D =$ (0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9). In this text, we often use abbreviations such as (0 | . . . | 9) rather than enumerate a complete list of alternatives. The . . . symbol is *not* part of our regular expression notation.

A **meta-character** is any punctuation character or regular expression operator. A meta-character must be quoted when used as an ordinary character in order to avoid ambiguity. (Any character or string may be quoted, but unnecessary quotation is avoided to enhance readability.) The following six symbols are meta-characters: ( ) ' * + |. The expression ('(' | ')' | ; | ,) defines four single character tokens (left parenthesis, right parenthesis, semicolon, and comma) that we might use in a programming language. The parentheses are quoted to show they are meant to be individual tokens and not delimiters in a larger regular expression.

Alternation can be extended to sets of strings. Let P and Q be sets of strings. Then string $s \in (P \mid Q)$ if, and only if, $s \in P$ or $s \in Q$. For example, if LC is the set of lowercase letters and UC is the set of uppercase letters, then $(LC \mid UC)$ denotes the set of all letters (in either case).

Large (or infinite) sets are conveniently represented by operations on finite sets of characters and strings. Catenation and alternation may be used. A third operation, **Kleene closure**, as defined below, is also allowed. The operator * is the postfix *Kleene closure operator*. For example, let P be a set of strings. Then $P^\star$ represents all strings formed by the catenation of zero or more selections (possibly repeated) from P. (Zero selections are represented by λ.) For example, $LC^\star$ is the set of all words composed only of lowercase letters and of any length (including the zero-length word, λ).

Precisely stated, a string $s \in P^\star$ if, and only if, s can be broken into zero or more pieces: $s = s_1 s_2 ... s_n$ such that each $s_i \in P (n \geq 0, 1 \leq i \leq n)$. We explicitly allow $n = 0$ so that λ is always in $P^\star$.

Now that we have introduced the operators used in regular expressions, we can define regular expressions as follows:

- $\emptyset$ is a regular expression denoting the empty set (the set containing no strings). $\emptyset$ is rarely used but is included for completeness.

- λ is a regular expression denoting the set that contains only the empty string. This set is not the same as the empty set because it does contain one element.
- The symbol s is a regular expression denoting $\{s\}$: a set containing the single symbol $s \in \Sigma$.
- If A and B are regular expressions, then $A \mid B$, AB, and $A^\star$ are also regular expressions. They denote, respectively, the alternation, catenation, and Kleene closure of the corresponding regular sets.

Each regular expression denotes a regular set. Any finite set of strings can be represented by a regular expression of the form $(s_1 \mid s_2 \mid \ldots \mid s_k)$. Thus, the reserved words of ANSI C can be defined as (`auto` | `break` | `case` | ...).

The following additional operations are also useful. They are not strictly necessary because their effect can be obtained (perhaps somewhat clumsily) using the three standard regular operators (alternation, catenation, and Kleene closure):

- P^+, sometimes called **positive closure**, denotes all strings consisting of one or more strings in P catenated together: $P^\star = (P^+ \mid \lambda)$ and $P^+ = P\,P^\star$. For example, the expression $(0 \mid 1)^+$ is the set of all strings containing one or more bits.
- If A is a set of characters, Not(A) denotes (Σ - A), that is, all *characters* in Σ not included in A. Since Not(A) can never be larger than Σ and Σ is finite, Not(A) must also be finite. Therefore it is regular. Not(A) does not contain λ because λ is not a character (it is a zero-length string). As an example, Not(Eol) is the set of all characters excluding Eol (the end-of-line character; in Java or C, `\n`).

  It is possible to extend Not() to strings, rather than just Σ. If S is a set of strings, we can define Not(S) to be ($\Sigma^\star$ - S), that is, the set of all strings except those in S. Although Not(S) is usually infinite, it also is regular if S is regular (Exercise 18).
- If k is a constant, then the set A^k represents all strings formed by catenating k (possibly different) strings from A. That is, $A^k = (AAA \ldots)$ (k copies). Thus, $(0 \mid 1)^{32}$ is the set of all bit strings exactly 32 bits long.

3.3 Examples

We next provide some examples that use regular expressions to specify some common programming language tokens. In these definitions, D is the set of the ten single digits and L is the set of all upper- and lower-case letters.

- A Java or C++ single-line comment that begins with // and ends with Eol can be defined as

  $$\mathit{Comment} = /\!/\, (\mathsf{Not}(\mathsf{Eol}))^{\star}\, \mathsf{Eol}$$

  This regular expression says that a comment begins with two slashes and ends at the *first* end-of-line. Within the comment, any sequence of characters is allowed that does not contain an end-of-line. (This guarantees that the first end-of-line we see ends the comment.)

- A fixed-decimal literal (for example, 12.345) can be defined as

  $$\mathit{Lit} = D^{+}.D^{+}$$

  One or more digits must be on both sides of the decimal point, so `.12` and `35.` are excluded.

- An optionally signed integer literal can be defined as

  $$\mathit{IntLiteral} = (\text{'+'} \mid - \mid \lambda\ )\ D^{+}$$

  An integer literal is one or more digits preceded by a plus, minus, or no sign at all (λ). So that the plus sign is not confused with the positive closure operator, it is quoted.

- A more complicated example is a comment delimited by ## markers, which allows single #'s within the comment body:

  $$\mathit{Comment2} = \#\#\ ((\# \mid \lambda)\ \ \mathsf{Not}(\#))^{\star}\ \#\#$$

  Any # that appears within this comment's body must be followed by a non-# so that a premature end-of-comment marker, ##, is not found.

All *finite* sets are regular. However, some (but not all) *infinite* sets are regular. For example, consider the set of balanced brackets of the form [[[. . .]]]. This set is defined formally as $\{[^{m}]^{m} \mid m \geq 1\}$, and it can be proven that this set is not regular (Exercise 14). The problem is that any regular expression that tries to define it either does not get all balanced nestings or includes extra, unwanted strings.

On the other hand, it is straightforward to write a CFG that defines balanced brackets precisely. Moreover, all regular sets can be defined by CFGs. Thus, the bracket example shows that CFGs are a more powerful descriptive mechanism than regular expressions. Regular expressions are, however, quite adequate for specifying token-level syntax. Moreover, for every regular expression we can create an efficient device, called a *finite automaton*, that recognizes exactly those strings that match the regular expression's pattern.

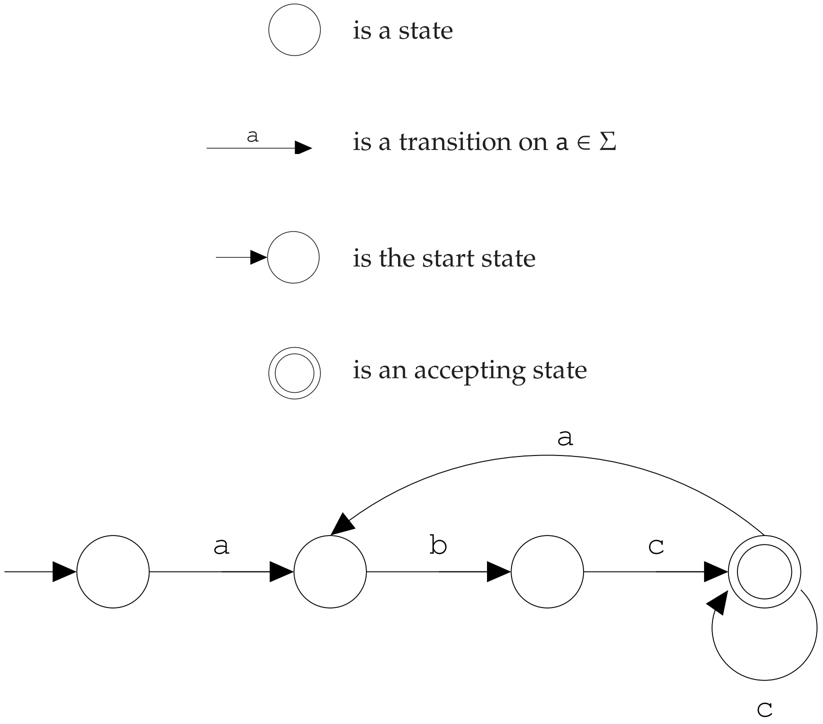


Figure 3.1: Components of a finite automaton drawing and their use to construct an automaton that recognizes $(\mathtt{a\,b\,c}^{+})^{+}$.

3.4 Finite Automata and Scanners

A **finite automaton** (FA) can be used to recognize the tokens specified by a regular expression. An FA (plural: finite automata) is a simple, idealized computer that recognizes strings as belonging to regular sets. An FA consists of the following:

- A finite set of *states*

- A finite *vocabulary*, denoted Σ

- A set of *transitions* (or *moves*) from one state to another, labeled with characters in Σ

- A special state called the *start* state

- A subset of the states called the *accepting*, or *final*, states

These components of an FA can be represented graphically as shown in Figure 3.1.

An FA also can be represented graphically using a **transition diagram**, composed of the components shown in Figure 3.1. Given a transition diagram, we begin at the start state. If the next input character matches the label on a transition from the current state, then we go to the state to which it points. If no move is possible, then we stop. If we finish in an accepting state, the sequence of characters read forms a valid token; otherwise, a valid token has not been seen. In the transition diagram shown in Figure 3.1, the valid tokens are the strings described by the regular expression $(\mathsf{a\,b\,c}^+)^+$.

As an abbreviation, a transition may be labeled with more than one character (for example, Not(c)). The transition may be taken if the current input character matches any of the characters labeling the transition.

3.4.1 Deterministic Finite Automata

An FA that always has a *unique* transition (for a given state and character) is a **deterministic finite automaton** (DFA). DFAs are simple to program and are often used to drive a scanner. A DFA is conveniently represented in a computer by a **transition table**. A transition table, T, is a two-dimensional array indexed by a DFA state and a vocabulary symbol. Table entries are either a DFA state or an error flag (often represented as a blank table entry). If we are in state s and read character c, then $T[s,c]$ will be the next state we visit, or $T[s,c]$ will contain an error flag indicating that c cannot extend the current token. For example, the regular expression

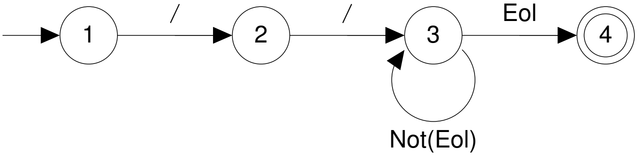
$$\mathsf{/\,/\,(Not(Eol))^{\star}\,Eol}$$

which defines a Java or C++ single-line comment, might be recognized by the DFA shown in Figure 3.2(a). The corresponding transition table is shown in Figure 3.2(b).

A full transition table will contain one column for each character. To save space, *table compression* is sometimes utilized. In that case, only nonerror entries are explicitly represented in the table. This is done by using hashing or linked structures [CLRS01].

Any regular expression can be translated into a DFA that accepts (as valid tokens) the set of strings denoted by the regular expression. This translation can be done manually by a programmer or automatically by a scanner generator.

(a)



(b)

State	Character				
	/	Eol	a	b	...
1	2				
2	3				
3	3	4	3	3	3
4					

Figure 3.2: DFA for recognizing a single-line comment. (a) transition diagram; (b) corresponding transition table.

Coding the DFA

A DFA can be coded in one of two forms:

1. Table-driven

2. Explicit control

In the *table-driven* form, the transition table that defines a DFA's actions is explicitly represented in a runtime table that is "interpreted" by a driver program. In the *explicit control* form, the transition table that defines a DFA's actions appears implicitly as the control logic of the program. Typically, individual program statements correspond to distinct DFA states. For example, suppose *CurrentChar* is the current input character. End-of-file is represented by a special character value, Eof. Using the DFA for the Java comments illustrated previously, the two approaches would produce the programs illustrated in Figures 3.3 and 3.4.

The table-driven form is commonly produced by a scanner generator; it is token independent. It uses a simple driver that can scan any token, provided the transition table is properly stored in *T*. The explicit control form may be produced automatically or by hand. The token being scanned is "hardwired" into the code. This form of a scanner is usually easy to read and often is more efficient, but it is specific to a single token definition.

The following are two more examples of regular expressions and their corresponding DFAs:

```
/⋆ Assume $CurrentChar$ contains the first character to be scanned ⋆/
$State \leftarrow StartState$
while true do
    $NextState \leftarrow \mathrm{T}[State, CurrentChar]$
    if $NextState$ = error
    then break
    $State \leftarrow NextState$
    $CurrentChar \leftarrow$ READ( )
if $State \in AcceptingStates$
then /⋆ Return or process the valid token ⋆/
else /⋆ Signal a lexical error ⋆/
```

Figure 3.3: Scanner driver interpreting a transition table.

1. A Fortran-like real literal (which requires either digits on one or both sides of a decimal point or just a string of digits) can be defined as

   $$RealLit = (D^+ \ (\lambda \mid .\ )) \mid (D^\star \ .\ D^+)$$

   which corresponds to the DFA shown in Figure 3.5(a).

2. Another form of identifier consists of letters, digits, and underscores. It begins with a letter and allows no adjacent or trailing underscores. It may be defined as

   $$ID = L\,(L \mid D)^\star(\_\,(L \mid D)^+)^\star$$

   This definition includes identifiers such as sum or `unit_cost` but excludes `_one`, `two_`, and `grand__total`. The corresponding DFA is shown in Figure 3.5(b).

Transducers

The scanners shown in Figures 3.3 and 3.4 begin processing characters at some point in the input stream. They finish either by accepting the token for which they are programmed or by signaling a lexical error. It is often useful for a scanner to process the input stream not only to recognize tokens but also to associate a **semantic value** with the discovered tokens. For example, a scanner can find that the input `431` is an integer constant, but it is useful to associate the value of 431 with that token.

An FA that analyzes or transforms its input beyond simply accepting tokens is called a **transducer**. The FAs shown in Figure 3.5 recognize a particular kind of constant and identifier. A transducer that recognizes constants might

```
/⋆  Assume CurrentChar contains the first character to be scanned  ⋆/
if CurrentChar = '/'
then
    CurrentChar ← READ( )
    if CurrentChar = '/'
    then
        repeat
            CurrentChar ← READ( )
        until CurrentChar ∈ { Eol, Eof }
    else  /⋆ Signal a lexical error ⋆/
else  /⋆ Signal a lexical error ⋆/
if CurrentChar = Eol
then  /⋆ Finished recognizing a comment ⋆/
else  /⋆ Signal a lexical error ⋆/
```

Figure 3.4: Explicit control scanner.

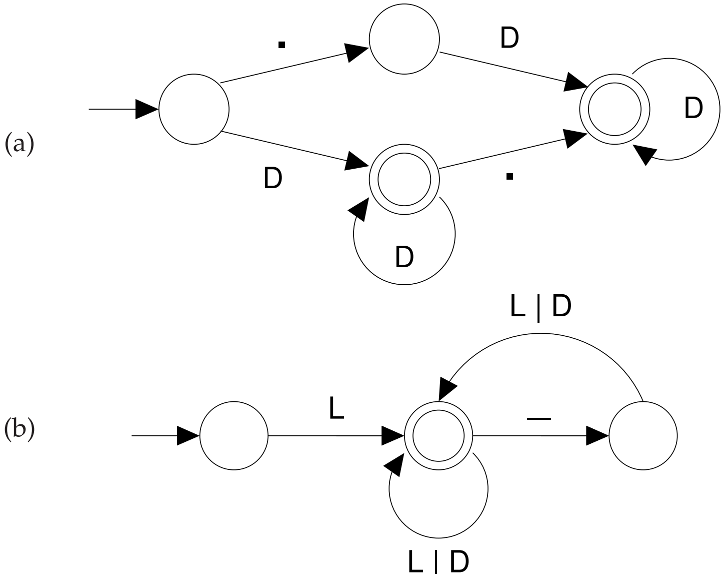


Figure 3.5: DFAs: (a) floating-point constant; (b) identifier with embedded underscore.

be responsible for developing the appropriate bit pattern to represent the constant. A transducer that processes identifiers may only have to retain the name of the identifier. For some languages, the scanner may be further required to classify the type of the identifier by referring to a **symbol table**.

A scanner can be turned into a transducer by the appropriate insertion of actions based on state transitions. Consider the table-driven scanner shown in Figure 3.3. The transition table shown in Figure 3.2(b) expresses the next state in terms of the current state and input symbol. An **action table** can be formulated that parallels the transition table. Based on the current state and input symbol, the action table encodes the action that should be performed as the FA makes the corresponding transition. The encoding could be formulated as an integer that is then demultiplexed by a `switch` statement to choose an appropriate sequence of actions. A more object-oriented approach would encode the action as an object instance containing a method that performs the action.

3.5 The Lex Scanner Generator

As a case study in the design of scanner generation tools, we first discuss a very popular scanner generator, Lex. We then briefly discuss several other scanner generators.

Lex was developed by M. E. Lesk and E. Schmidt of AT&T Bell Laboratories. It is used primarily with programs written in C or C++ running under the Unix operating system. Lex produces an entire scanner module, coded in C, that can be compiled and linked with other compiler modules. A complete description of Lex and its usage can be found in [LS83] and [Joh83]. Flex [Pax] is a widely used, freely distributed reimplementation of Lex that produces faster and more reliable scanners. JFlex is a similar tool for use with Java [KD]. Valid Lex scanner specifications may, in general, be used with Flex without modification.

The operation of Lex is illustrated in Figure 3.6. The steps are as follows:

1. A scanner specification that defines the tokens to be scanned and how they are to be processed is presented to Lex.

2. Lex generates a complete scanner coded in C.

3. This scanner is compiled and linked with other compiler components to create a complete compiler.

Using Lex saves a great deal of effort when programming a scanner. Many low-level details of the scanner (reading characters efficiently, buffering them, matching characters against token definitions, and so on) need not be explicitly

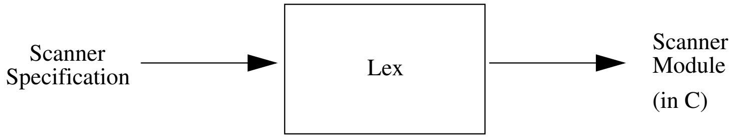


Figure 3.6: The operation of the Lex scanner generator.

programmed. Rather, we can focus on the character structure of tokens and how they are to be processed.

The primary purpose of this section is to show how regular expressions and related information are presented to scanner generators. A helpful way to learn Lex is to start with the simple examples presented here and then gradually generalize them to solve the problem at hand. To inexperienced readers, Lex's rules may seem unnecessarily complex. It is best to keep in mind that the key is always the specification of tokens as regular expressions. The rest is there simply to increase efficiency and handle various details.

3.5.1 Defining Tokens in Lex

Lex's approach to scanning is simple. It allows the user to associate regular expressions with commands coded in C (or C++). When input characters that match the regular expression are read, the associated commands are executed. Users of Lex do not specify *how* to match tokens, except by providing the regular expressions. The associated commands specify *what* should be done when a particular token is matched.

Lex creates a file `lex.yy.c` that contains an integer function `yylex()`. This function is normally called from the parser whenever another token is needed. The value that `yylex()` returns is the token code of the token scanned by Lex. Tokens such as whitespace are deleted simply by having their associated command not return anything. Scanning continues until a command with a return in it is executed.

Figure 3.7 illustrates a simple Lex definition for the three reserved words—`f`, `i`, and `p`—of the ac language introduced in Chapter 2. When a string matching any of these three reserved keywords is found, then the appropriate token code is returned. It is vital that the token codes that are returned when a token is matched are identical to those expected by the parser. If they are not, then the parser will not *see* the same token sequence produced by the scanner. This will cause the parser to generate false syntax errors based on the incorrect token stream it sees.

It is standard for the scanner and parser to share the definition of token codes to guarantee that consistent values are seen by both. The file `y.tab.h`,

```
%%
f          { return(FLOATDCL); }
i          { return(INTDCL); }
p          { return(PRINT); }
%%
```

Figure 3.7: A Lex definiton for ac's reserved words.

```
declarations
%%
regular expression rules
%%
subroutine definitions
```

Figure 3.8: The structure of Lex definiton files.

produced by the yacc parser generator (see Chapter 7), is often used to define shared token codes. A Lex token specification consists of three sections delimited by the pair %%. The general form of a Lex specification is shown in Figure 3.8.

In the simple example shown in Figure 3.7, we use only the second section, in which regular expressions and corresponding C code are specified. The regular expressions are simple single-character strings that match only themselves. The code executed returns a constant value representing the appropriate ac token.

We could quote the strings representing the reserved keywords (f, i, or p), but since these strings contain no delimiters or operators, quoting them is unnecessary. If you want to quote such strings to avoid any chance of misinterpretation, that is allowed in Lex.

3.5.2 The Character Class

Our specification so far is incomplete. None of the other tokens in ac have been correctly handled, particularly identifiers and numbers. To do this, we introduce a useful concept: the **character class**. A character class is a set of characters treated identically in a token definition. Thus, in the definition of an ac identifier, all letters (except f, i, and p) form a class since any of them can be used to form an identifier. Similarly, in a number, any of the ten digits characters can be used.

Character Class	Set of Characters Denoted
[abc]	Three characters: a, b, and c
[cba]	Three characters: a, b, and c
[a-c]	Three characters: a, b, and c
[aabbcc]	Three characters: a, b, and c
[^abc]	All characters except a, b, and c
[\^\-\]]	Three characters: ^, -, and]
[^]	All characters
"[abc]"	Not a character class. This is an example of one five-character *string*: [abc].

Figure 3.9: Lex character class definitions.

A character class is delimited by [and]; individual characters are catenated without any quotation or separators. However \, ^,], and - must be escaped because of their reserved meanings (see below) in character classes. Thus [xyz] represents the class that can match a single x, y, or z. The expression [\])] represents the class that can match a single] or). The] is escaped so that it is not misinterpreted as the end-of-character-class symbol.

Ranges of characters are separated by a -; for example, [x-z] is the same as [xyz]. [0-9] is the set of all digits, and [a-zA-Z] is the set of all letters, both uppercase and lowercase. \ is the escape character; it is used to represent unprintables and to escape special symbols. Following C conventions, \n is the newline (that is, end-of-line), \t is the tab character, \\ is the backslash symbol itself, and \010 is the character corresponding to 10 in octal (base 8) form.

The ^ symbol complements a character class; it is Lex's representation of the Not() operation. For example, [^xy] is the character class that matches any single character except x and y. The ^ symbol applies to all characters that follow it in the character class definition, so [^0-9] is the set of all characters that are not digits. [^] can be used to match all characters. (Avoid the use of \0 in character classes because it can be confused with the null character's special use as the end-of-string terminator in C.) Figure 3.9 illustrates various character classes and the character sets they define.

Using character classes, we can easily define ac identifiers, as shown in Figure 3.10. The character class includes the characters a through e, g and h, j through o, and finally q through z. We can concisely represent the 23 characters that may form an ac identifier without having to enumerate them all.

```
%%
[a-eghj-oq-z]      { return(ID); }
%%
```

Figure 3.10: A Lex definition for ac's identifiers.

3.5.3 Using Regular Expressions to Define Tokens

Tokens are defined using regular expressions. Lex provides the standard regular expression operators, as well as others. Catenation is specified by the juxtaposition of two expressions; no explicit operator is used. Thus [ab][cd] will match any of ad, ac, bc, or bd. Individual letters and numbers match themselves when outside of character class brackets. Other characters should be quoted (to avoid misinterpretation as regular expression operators). For example, while (as used in C, C++, and Java) can be matched by the expressions while, "while", or [w][h][i][l][e].

Case *is* significant. The alternation operator is |. As usual, parentheses can be used to control grouping of subexpressions. Therefore, to match the reserved word while and allow any mixture of uppercase and lowercase (as required in Pascal and Ada), we can use (w|W)(h|H)(i|I)(l|L)(e|E).

Postfix operators * (Kleene closure) and $^+$ (positive closure) are also provided, as is ? (optional inclusion). For example, expr? matches expr zero times or once. It is equivalent to (expr) | λ and obviates the need for an explicit λ symbol. The character . matches any single character (other than a newline). The character ˆ (when used outside a character class) matches the beginning of a line. Similarly, the character $ matches the end of a line. Thus ˆA.⋆ e $ could be used to match an entire line that begins with A and ends with e. We now define all of ac's tokens using Lex's regular expression facilities. This is shown in Figure 3.11.

Recall that a Lex specification of a scanner consists of three sections. The first, not used so far, contains symbolic names associated with character classes and regular expressions. Symbolic definitions can often make Lex specifications easier to read, as illustrated in Figure 3.12. There is one definition per line. Each definition line contains an identifier and a definition string, separated by a blank or tab. The { and } symbols signal the macro-expansion of a symbol. For example, the expression {Blank}+ in Figure 3.12 expands to any positive number of occurrences of Blank, which is in turn defined as a single space.

The first section can also include source code, delimited by %{ and %}, that is placed before the commands and regular expressions of section two. This source code may include statements, as well as variable, procedure, and type

```
%%
(" ")+                              { /* delete blanks */}
f                                   { return(FLOATDCL); }
i                                   { return(INTDCL); }
p                                   { return(PRINT); }
[a-eghj-oq-z]                       { return(ID); }
([0-9]+)|([0-9]+"."[0-9]+)          { return(NUM);  }
"="                                 { return(ASSIGN); }
"+"                                 { return(PLUS); }
"-"                                 { return(MINUS); }
%%
```

Figure 3.11: A Lex definition for ac’s tokens.

```
%%
Blank                               " "
Digits                              [0-9]+
Non_f_i_p                           [a-eghj-oq-z]
%%
{Blank}+                            { /* delete blanks */}
f                                   { return(FLOATDCL); }
i                                   { return(INTDCL); }
p                                   { return(PRINT); }
{Non_f_i_p}                         { return(ID); }
{Digits}|({Digits}"."{Digits})      { return(NUM);  }
"="                                 { return(ASSIGN); }
"+"                                 { return(PLUS); }
"-"                                 { return(MINUS); }
%%
```

Figure 3.12: An alternative definition for ac’s tokens.

declarations that are needed to allow the commands of section two to be compiled. For example,

```
%{
#include "tokens.h"
%}
```

can include the definitions of token values returned when tokens are matched.

Lex's second section defines a table of regular expressions and corresponding commands in C. The first blank or tab not escaped or not part of a quoted string or character class is taken as the end of the regular expression. Thus, one should avoid embedded blanks that are within regular expressions.

When an expression is matched, its associated command is executed. If an input sequence matches no expression, then the sequence is simply copied verbatim to the standard output file. Input that is matched is stored in a global string variable `yytext` (whose length is `yyleng`). Commands may alter `yytext` in any way. The default size of `yytext` is determined by `YYLMAX`, which is initially defined to be 200. *All* tokens, even those that will be ignored (such as comments), are stored in `yytext`. Hence, you may need to redefine `YYLMAX` to avoid overflow. An alternative approach to scanning comments that is not prone to the danger of overflowing `yytext` involves the use of start conditions [LS83, Joh83]. Flex, an improved version of Lex discussed in the next section, automatically extends the size of `yytext` when necessary. This removes the danger that a very long token may overflow the text buffer.

The content of `yytext` is overwritten as each new token is scanned. Therefore, care must be taken to avoid returning the text of a token using a reference into `yytext`. It is safer to copy the contents of `yytext` (e.g., using `strcpy()`) *before* the next call to `yylex()`.

Lex allows regular expressions to overlap (that is, to match the same input sequences). In the case of overlap, two rules are used to determine which regular expression is matched:

1. The longest possible match is performed. Lex automatically buffers characters while deciding how many characters can be matched.

2. If two expressions match exactly the same string, the earlier expression (in order of definition in the Lex specification) is preferred.

Reserved words are often special cases of the pattern used for identifiers, so their definitions are placed before the expression that defines an identifier token. Often a "catchall" pattern is placed at the very end of section two. It is used to catch characters that do not match any of the earlier patterns and hence are probably erroneous. Recall that `.` matches any single character (other than a newline). It is useful in a catchall pattern. However, avoid a pattern such as `.*` because it will consume all characters up to the next newline.

3.5.4 Character Processing Using Lex

Although Lex is often used to produce scanners, it is really a general-purpose character-processing tool, programmed using regular expressions. Lex provides no character-tossing mechanism because this would be too special purpose. We may need to process the token text (stored in yytext) before returning a token code. This is normally done by calling a subroutine in the command associated with a regular expression. The definitions of such subroutines may be placed in the final section of the Lex specification. For example, we might want to call a subroutine to insert an identifier into a symbol table before it is returned to the parser. For ac, the line

```
{Non_f_i_p}         {insert(yytext); return(ID);}
```

could do this, with insert defined in the final section. Alternatively, the definition of insert could be placed in a separate file containing symbol table routines. This would allow insert to be changed and recompiled without Lex's having to be rerun. (Some implementations of Lex generate scanners rather slowly.)

In Lex, end-of-file is not handled by regular expressions. A predefined EndFile token, with a token code of zero, is automatically returned when end-of-file is reached at the beginning of a call to yylex(). It is up to the parser to recognize the zero return value as signifying the EndFile token.

If more than one source file must be scanned, this fact is hidden inside the scanner mechanism. yylex() uses three user-defined functions to handle character-level I/O:

input()	Reads a single character; zero is returned on end-of-file.
output(c)	Writes a single character to output.
unput(c)	Puts a single character back into the input to be reread.

When yylex() encounters end-of-file, it calls a user-supplied integer function named yywrap(). The purpose of this routine is to "wrap up" input processing. It returns the value 1 if there is no more input. Otherwise, it returns zero and arranges for input() to provide more characters.

The definitions for the input(), output(), unput(), and yywrap() functions may be supplied by the compiler writer (usually as C macros). Lex supplies default versions that read characters from the standard input and write them to the standard output. The default version of yywrap() simply returns 1, thereby signifying that there is no more input. (The use of output() allows Lex to be used as a tool for producing stand-alone data "filters" for transforming a stream of data.)

Lex-generated scanners normally select the longest possible input sequence that matches some token definition. Occasionally this can be a problem.

For example, if we allow Fortran-like fixed-decimal literals such as `1.` and `.10` and the Pascal subrange operator "..", then `1..10` will most likely be misscanned as two fixed-decimal literals rather than two integer literals separated by the subrange operator. Lex allows us to define a regular expression that applies only if some other expression immediately follows it. For example, `r/s` tells Lex to match regular expression `r`, but only if regular expression `s` immediately follows it. The expression `s` is *right-context*. That is, it is not part of the token that is matched, but it must be present for `r` to be matched. Thus `[0-9]+/".."` would match an integer literal, but only if `..` immediately follows it. Since this pattern covers more characters than the one defining a fixed-decimal literal, it takes precedence. The longest match is still chosen, but the right-context characters are returned to the input so that they can be matched as part of a later token.

The operators and special symbols most commonly used in Lex are summarized in Figure 3.13. Note that a symbol sometimes has one meaning in a regular expression and an entirely different meaning in a character class (that is, within a pair of brackets). If you find Lex behaving unexpectedly, it is a good idea to check this table to be sure how the operators and symbols you have used behave. Ordinary letters and digits, as well as symbols not mentioned (such as `@`), represent themselves. If you are not sure whether a character is special, you can always escape it or make it part of a quoted string.

In summary, Lex is a very flexible generator that can produce a complete scanner from a succinct definition. The difficult part of working with Lex is learning its notation and rules. Once you have done this, Lex will relieve you of the many chores of writing a scanner (for example, reading characters, buffering them, and deciding which token pattern matches). Moreover, Lex's notation for representing regular expressions is used in other Unix programs, most notably the grep pattern matching utility.

Lex can also transform input as a preprocessor, as well as scan it. It provides a number of advanced features beyond those discussed here. It does require that code segments be written in C, and hence it is not language-independent.

3.6 Other Scanner Generators

Lex is certainly the most widely known and widely available scanner generator because it is distributed as part of the Unix system. Even after years of use, it still has bugs, however, and produces scanners too slow to be used in production compilers. This section briefly discussed some of the alternatives to Lex, including Flex, `JLex`, `Alex`, `Lexgen`, `GLA`, and `re2c`.

It has been shown that Lex can be improved so that it is always faster than a handwritten scanner [Jac87]. This is done using *Flex*, a widely used, freely

Symbol	Meaning in Regular Expressions	Meaning in Character Classes
(	matches with) to group subexpressions.	Represents itself.
)	matches with (to group subexpressions.	Represents itself.
[	Begins a character class.	Represents itself.
]	Represents itself.	Ends a character class.
{	Matches with } to signal macro-expansion.	Represents itself.
}	Matches with { to signal macro-expansion.	Represents itself.
"	Matches with " to delimit strings.	Represents itself.
\	Escapes individual characters. Also used to specify a character by its octal code.	Escapes individual characters. Also used to specify a character by its octal code.
.	Matches any one character except \n.	Represents itself.
|	Alternation (or) operator.	Represents itself.
*	Kleene closure operator (zero or more matches).	Represents itself.
+	Positive closure operator (one or more matches).	Represents itself.
?	Optional choice operator (one or more matches)	Represents itself.
/	Context-sensitive matching operator.	Represents itself.
^	Matches only at the beginning of a line.	Complements the remaining characters in the class.
$	Matches only at the end of a line.	Represents itself.
-	Represents itself.	The range of characters operator.

Figure 3.13: Meaning of operators and special symbols in Lex.

distributed Lex clone. It produces scanners that are considerably faster than the ones produced by Lex. It also provides options that allow the tuning of the scanner size versus its speed, as well as some features that Lex does not have (such as support for 8-bit characters). If Flex is available on your system, you should use it instead of Lex.

Lex also has been implemented in languages other than C. JFlex [KD] is a Lex-like scanner generator written in Java that generates Java scanner classes. It is of particular interest to people writing compilers in Java. Versions of Lex are also available for Ada and ML.

An interesting alternative to Lex is `GLA` (Generator for Lexical Analyzers) [Gra88]. `GLA` takes a description of a scanner based on regular expressions and a library of common lexical idioms (such as "Pascal comment") and produces a *directly executable* (that is, not transition table-driven) scanner written in C. `GLA` was designed with both ease of use and efficiency of the generated scanner in mind. Experiments show it to be typically twice as fast as Flex and only slightly slower than a trivial program that reads and "touches" each character in an input file. The scanners it produces are more than competitive with the best hand-coded scanners.

Another tool that produces directly executable scanners is `re2c` [BC93]. The scanners it produces are easily adaptable to a variety of environments and yet scanning speed is excellent.

Scanner generators are usually included as parts of complete suites of compiler development tools. Other than those already mentioned, some of the most widely used and highly recommended scanner generators are `DLG` (part of the `PCCTS` tools suite, [Par97]), `CoCo/R` [Moe90], an integrated scanner/parser generator, and `Rex` [GE91], part of the Karlsruhe/CoCoLab `Cocktail Toolbox`.

3.7 Practical Considerations of Building Scanners

In this section, we discuss the practical considerations involved in building real scanners for real programming languages. As one might expect, the finite automaton model developed earlier in the chapter sometimes falls short and must be supplemented. Efficiency concerns must be addressed. In addition, some provision for error handling must be incorporated.

We discuss a number of potential problem areas. In each case, solutions are weighed, particularly in conjunction with the Lex scanner generator discussed in Section 3.5.

3.7.1 Processing Identifiers and Literals

In simple languages that have only global variables and declarations, the scanner commonly will immediately enter an identifier into the symbol table, if it is not already there. Whether the identifier is entered or is already in the table, a pointer to the symbol table entry is then returned from the scanner.

In block-structured languages, the scanner generally is not expected to enter or look up identifiers in the symbol table because an identifier can be used in many contexts (for example, as a variable, member of a class, or label). The scanner usually cannot know when an identifier should be entered into the symbol table for the current scope or when it should return a pointer to an instance from an earlier scope. Some scanners just copy the identifier into a private string variable (that cannot be overwritten) and return a pointer to it. A later compiler phase, the type checker, then resolves the identifier′s intended usage.

Sometimes a **string space** is used to store identifiers in conjunction with a symbol table (see Chapter 8). A string space is an extendable block of memory used to store the text of identifiers. A string space eliminates frequent calls to memory allocators such as `new` or `malloc`. It also avoids the space overhead of storing multiple copies of the same string. The scanner can enter an identifier into the string space and return a pointer into the string space rather than the actual text.

An alternative to a string space is a **hash table** that stores identifiers and assigns to each a unique **serial number**. A serial number is a small integer that can be used instead of a string space pointer. All identifiers that have the same text get the same serial number; identifiers with different texts get different serial numbers. Serial numbers are ideal indices into symbol tables (which need not be hashed) because they are small, contiguously assigned integers. A scanner can hash an identifier when it is scanned and return its serial number as part of the identifier token.

In some languages, such as C, C++, and Java, case is significant; in others, such as Ada and Pascal, it is not. When case is significant, identifier text must be stored or returned exactly as it was scanned. Reserved word lookup must distinguish between identifiers and reserved words that differ only in case. However, when case is insignificant, case differences in the spelling of an identifier or reserved word must be guaranteed to not cause errors. This can be done by putting all tokens scanned as identifiers into a uniform case before they are returned or looked up in a reserved word table.

Other tokens, such as literals, require processing before they are returned. Integer and real (floating) literals are converted to numeric form and returned as part of the token. Numeric conversion can be tricky because of the danger of overflow or roundoff errors. It is wise to use standard library routines such as `atoi` and `atof` (in C) (`Integer.intValue` and `Float.floatValue` in Java). For string literals, a pointer to the text of the string (with escaped characters expanded) should be returned.

The design of C contains a flaw that requires a C scanner to do a bit of special processing. The character sequence `a (* b);` can be a call to procedure a, with *b as the parameter. If a has been declared in a typedef to be a type name, then this character sequence can also be the declaration of an identifier b that is a pointer variable (the parentheses are not needed, but they are legal).

C contains no special marker that separates declarations from statements, so the parser will need some help in deciding whether it is seeing a procedure call or a variable declaration. One way to do this is for the scanner to create, while scanning and parsing, a table of currently visible identifiers that have been defined in typedef declarations. When an identifier in this table is scanned, a special typeid token is returned (rather than an ordinary identifier token). This allows the parser to distinguish the two constructs easily, since they now begin with different tokens.

Why does this complication exist in C? The typedef statement was not in the original definition of C in which the lexical and syntactic rules were established. When the typedef construct was added, the ambiguity was not immediately recognized (parentheses, after all, are rarely used in variable declarations). When the problem was finally recognized, it was too late, and the "trick" described previously had to be devised to resolve the correct usage.

Processing Reserved Words

Virtually all programming languages have symbols (such as if and while) that match the lexical syntax of ordinary identifiers. These symbols are called *keywords*. If the language has a rule that keywords may not be used as programmer-defined identifiers, then they are *reserved words*, that is, they are reserved for special use.

Most programming languages choose to make keywords reserved. This simplifies parsing, which drives the compilation process. It also makes programs more readable. For example, in Pascal and Ada, subprograms without parameters are called as name; (no parentheses required). But what if, for example, begin and end are not reserved and some devious programmer has declared procedures named begin and end? The result is a program whose meaning is not well defined, as shown in the following example, which can be parsed in many ways:

```
begin
  begin;
  end;
  end;
  begin;
end
```

With careful design, you can avoid outright ambiguities. For example, in PL/I keywords are not reserved; procedures are called using an explicit `call` keyword. Nonetheless, opportunities for convoluted usage abound. Keywords may be used as variable names, allowing the following:

```
if if then else = then;
```

The problem with reserved words is that if they are too numerous, they may confuse inexperienced programmers, who may unknowingly choose an identifier name that clashes with a reserved word. This usually causes a syntax error in a program that "looks right" and in fact *would* be right if the symbol in question was not reserved. COBOL is infamous for this problem because it has several hundred reserved words. For example, in COBOL, `zero` is a reserved word. So is `zeros`. So is `zeroes`!

In Section 3.5.1, we showed how to recognize reserved words by creating distinct regular expressions for each. This approach was feasible because Lex (and Flex) allows more than one regular expression to match a character sequence, with the earliest expression that matches taking precedence. Creating regular expressions for each reserved word increases the number of states in the transition table that a scanner generator creates. In as simple a language as Pascal (which has only 35 reserved words), the number of states increases from 37 to 165 [Gra88]. With the transition table in uncompressed form and having 127 columns for ASCII characters (excluding null), the number of transition table entries increases from 4,699 to 20,955. This may not be a problem with modern multimegabyte memories. Still, some scanner generators, such as Flex, allow you to choose to optimize scanner size or scanner speed.

Exercise 18 establishes that any regular expression may be complemented to obtain all strings not in the original regular expression. That is, $\overline{A}$, the complement of A, is regular if A is. Using complementation of regular expressions we can write a regular expression for nonreserved identifiers:

$$\overline{(\overline{ident} \mid if \mid while \mid \ldots)}$$

That is, if we take the complement of the set containing reserved words and all nonidentifier strings, then we get all strings that *are* identifiers, *excluding* the reserved words. Unfortunately, neither Lex nor Flex provides a complement operator for regular expressions (ˆ works only on character sets).

We could just write down a regular expression directly, but this is too complex to consider seriously. Suppose `END` is the only reserved word and identifiers contain only letters. Then

$$L \mid (LL) \mid ((LLL)L^{+}) \mid ((L -' E')L^{\star}) \mid (L(L -' N')L^{\star}) \mid (LL(L -' D')L^{\star})$$

defines identifiers that are shorter or longer than three letters, that do not start with `E`, that are without `N` in position two, and so on.

Many hand-coded scanners treat reserved words as ordinary identifiers (as far as matching tokens is concerned) and then use a separate table lookup to detect them. Automatically generated scanners can also use this approach, especially if transition table size is an issue. After an apparent identifier is scanned, an exception table is consulted to see if a reserved word has been matched. When case is significant in reserved words, the exception lookup requires an exact match. Otherwise, the token should be translated to a standard form (all uppercase or lowercase) before the lookup.

There are several ways of organizing an exception table. One obvious mechanism is a sorted list of exceptions suitable for a binary search. A hash table also may be used. For example, the length of a token may be used as an index into a list of exceptions of the same length. If exception lengths are well distributed, then few comparisons will be needed to determine whether a token is an identifier or a reserved word. Perfect hash functions are also possible [Spr77, Cic80]. That is, each reserved word is mapped to a unique position in the exception table and no position in the table is unused. A token is either the reserved word selected by the hash function or an ordinary identifier.

If identifiers are entered into a string space or given a unique serial number by the scanner, then reserved words can be entered in advance. Then, when a string that looks like an identifier is found to have a serial number or string space position *smaller* than the initial position assigned to identifiers, we know that a reserved word rather than an identifier has been scanned. In fact, with a little care we can assign initial serial numbers so that they exactly match the token codes used for reserved words. That is, if an identifier is found to have a serial number s, where s is less than the number of reserved words, then s must be the correct token code for the reserved word just scanned.

3.7.2 Using Compiler Directives and Listing Source Lines

Compiler directives and pragmas control compiler options (for example, listings, source file inclusion, conditional compilation, optimizations, and profiling). They may be processed either by the scanner or by subsequent compiler phases. If the directive is a simple flag, then it can be extracted from a token. The command is then executed, and finally the token is deleted. More elaborate directives, such as Ada pragmas, have nontrivial structure and need to be parsed and translated like any other statement.

A scanner may have to handle source inclusion directives. These directives cause the scanner to suspend the reading of the current file and begin the reading and scanning of the contents of the specified file. Since an included file may itself contain an include directive, the scanner maintains a stack of open files. When the file at the top of the stack is completely scanned, it is popped and scanning resumes with the file now at the top of the stack. When the entire stack is empty, end-of-file is recognized and scanning is completed.

Because C has a rather elaborate macro definition and expansion facility, macro processing and included files are typically handled by a preprocessing phase prior to scanning and parsing. The preprocessor, `cpp`, may in fact be used with languages other than C to obtain the effects of source file inclusion, macro processing, and so on.

Some languages (such as C and PL/I) include conditional compilation directives that control whether statements are compiled or ignored. Such directives are useful in creating multiple versions of a program from a common source. Usually, these directives have the general form of an `if` statement; hence, a conditional expression will be evaluated. Characters following the expression will either be scanned and passed to the parser, or ignored until an `end if` delimiter is reached. If conditional compilation structures can be nested, a skeletal parser for the directives may be needed.

Another function of the scanner is to list source lines and to prepare for the possible generation of error messages. While straightforward, this requires a bit of care. The most obvious way to produce a source listing is to echo characters as they are read, using end-of-line characters to terminate a line, increment line counters, and so on. However, this approach has a number of shortcomings:

- Error messages may need to be printed. These should appear merged with source lines, with pointers to the offending symbol.
- A source line may need to be edited before it is written. This may involve inserting or deleting symbols (for example, for error repair), replacing symbols (because of macro preprocessing), and reformatting symbols (to prettyprint a program, that is, to print a program with text properly indented, `if-else` pairs aligned, and so on).
- Source lines that are read are not always in a one-to-one correspondence with source listing lines that are written. For example, in Unix a source program can legally be condensed into a single line (Unix places no limit on line lengths). A scanner that attempts to buffer entire source lines may well overflow buffer lengths.

In light of these considerations, it is best to build output lines (which normally are bounded by device limits) *incrementally* as tokens are scanned. The token image placed in the output buffer may not be an exact image of the token that was scanned, depending on error repair, prettyprinting, case conversion, or whatever else is required. If a token cannot fit on an output line, then the line is written and the buffer is cleared. (To simplify editing, you should place source line numbers in the program's listing.) In rare cases, a token may need to be broken; for example, if a string is so long that its text exceeds the output line length.

Even if a source listing is not requested, each token should contain the line number in which it appeared. The token's position in the source line may also be useful. If an error involving the token is noted, the line number and position marker can be used to improve the quality of error messages by specifying where in the source file the error occurred. It is straightforward to open the source file and then list the source line containing the error, with the error message immediately below it. Sometimes, an error may not be detected until long after the line containing the error has been processed. An example of this is a `goto` to an undefined label. If such delayed errors are rare (as they usually are), then a message citing a line number can be produced, for example, "Undefined label in statement 101." In languages that freely allow forward references, delayed errors may be numerous. For example, Java allows declarations of methods after they are called. In this case, a file of error messages keyed with line numbers can be written and later merged with the processed source lines to produce a complete source listing. Source line numbers are also required for reporting post-scanning errors in multipass compilers. For example, a type conversion error may arise during semantic analysis; associating a line number with the error message greatly helps a programmer understand and correct the error.

A common view is that compilers should just concentrate on translation and code generation and leave the listing and prettyprinting (but not error messages) to other tools. This considerably simplifies the scanner.

3.7.3 Terminating the Scanner

A scanner is designed to read input characters and partition them into tokens. When the end of the input file is reached, it is convenient to create an end-of-file pseudocharacter.

In Java, for example, `InputStream.read()`, which reads a single byte, returns `-1` when end-of-file is reached. A constant, `Eof`, defined as `-1`, can be treated as an "extended" ASCII character. This character then allows the definition of an EndFile token that can be passed back to the parser. The EndFile token is useful in a CFG because it allows the parser to verify that the logical end of a program corresponds to its physical end. In fact, LL(1) parsers (discussed in Chapter 5) and LALR(1) parsers (discussed in Chapter 6) require an EndFile token.

What will happen if a scanner is called after end-of-file is reached? Obviously, a fatal error could be registered, but this would destroy our simple model in which the scanner always returns a token. A better approach is to continue to return the EndFile token to the parser. This allows the parser to handle termination cleanly, especially since the EndFile token is normally syntactically valid only after a complete program is parsed. If the EndFile token

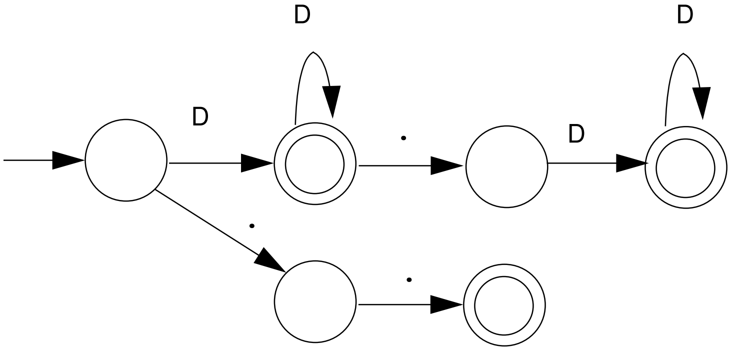


Figure 3.14: An FA that scans integer and real literals and the subrange operator.

appears too soon or too late, the parser can perform error repair or issue a suitable error message.

3.7.4 Multicharacter Lookahead

We can generalize FAs to look ahead beyond the next input character. This feature is important for implementing a scanner for Fortran. In Fortran, the statement `DO 10 J = 1,100` specifies a loop, with index `J` ranging from `1` to `100`. In contrast, the statement `DO 10 J = 1.100` is an assignment to the variable `DO10J`. In Fortran, blanks are not significant except in strings. A Fortran scanner can determine whether the `O` is the last character of a `DO` token only after reading as far as the comma (or period). (In fact, the erroneous substitution of a `.` for a `,` in a Fortran `DO` loop once caused a 1960s-era space launch to fail! Because the substitution resulted in a valid statement, the error was not detected until runtime, which in this case was *after* the rocket had been launched. The rocket deviated from course and had to be destroyed.)

We have already shown you a milder form of the extended lookahead problem that occurs in Pascal and Ada. Scanning, for example, `10..100` requires two-character lookahead after the `10`. Using the FA of Figure 3.14 and given `10..100`, we would scan three characters and stop in a nonaccepting state. Whenever we stop reading in a nonaccepting state, we can back up over accepted characters until an accepting state is found. Characters over which we back up are rescanned to form later tokens. If no accepting state is reached during backup, then we have a lexical error and invoke lexical error recovery.

In Pascal or Ada, more than two-character lookahead is not needed; this simplifies the buffering of characters to be rescanned. Alternatively, we can add a new accepting state to the previous FA that corresponds to a pseudotoken of the form (D^+.). If this token is recognized, then we strip the trailing `.` from the token text and buffer it for later reuse. We then return the token code of an integer literal. In fact, we are simulating the effect of a context-sensitive match as provided by Lex's / operator.

Multiple character lookahead may also be a consideration in scanning *invalid* programs. For example, in C (and many other programming languages) `12.3e+q` is an invalid token. Many C compilers simply flag the entire character sequence as invalid (a floating-point value with an illegal exponent). If we follow our general scanning philosophy of matching the longest *valid* character sequence, the scanner could be backed up to produce four tokens. Since this token sequence (`12.3`, `e`, `+`, `q`) is invalid, the parser will detect a syntax error when it processes the sequence. Whether we decide to consider this a lexical error or a syntax error (or both) is unimportant. Some phase of the compiler must detect the error.

We can build a scanner that can perform general backup. This allows the scanner to operate correctly no matter how token definitions overlap. As each character is scanned, it is buffered and a flag is set indicating whether the character sequence scanned so far is a valid token (the flag might be the appropriate token code). If we are not in an accepting state and cannot scan any more characters, then backup is invoked. We extract characters from the right end of the buffer and queue them for rescanning. This process continues until we reach a prefix of the scanned characters flagged as a valid token. This token is returned by the scanner. If no prefix is flagged as valid, then we have a lexical error. (Lexical errors are discussed in Section 3.7.6.)

Buffering and backup are essential in general-purpose scanners such as those generated by Lex. It is impossible to know in advance which regular expression pattern will be matched. Instead, the generated scanner (using its internal DFA) follows all patterns that are possible matches. If a particular pattern is found to be unmatchable, then an alternative pattern that matches a shorter input sequence may be chosen. The scanner will back up to the longest input prefix that can be matched, saving buffered characters that will be matched in a later call to the scanner.

As an example of scanning with backup, consider the previous example of `12.3e+q`. Figure 3.15 shows how the buffer is built and flags are set. When the `q` is scanned, backup is invoked. The longest character sequence that is a valid token is `12.3`, so a floating-point literal is returned. The remaining input `e+` is requeued so that it can be rescanned later.

3.7.5 Performance Considerations

Our main concern in this chapter is showing how to write correct and robust scanners. Because scanners do so much character-level processing, they can

Buffered Token	Token Flag
`1`	Integer literal.
`12`	Integer literal.
`12.`	Floating-point literal.
`12.3`	Floating-point literal.
`12.3e`	Invalid (but valid prefix).
`12.3e+`	Invalid (but valid prefix).

Figure 3.15: Building the token buffer and setting token flags when scanning with a backup.

be a real performance bottleneck in production compilers. Hence, it is a good idea to consider how to increase scanning speed.

One approach to increasing scanner speed is to use a scanner generator such as Flex or `GLA` that is designed to generate fast scanners. These generators will incorporate many "tricks" that increase speed in clever ways.

If you hand-code a scanner, a few general principles can increase scanner performance dramatically. Try to block character-level operations whenever possible. It is usually better to do one operation on *n* characters rather than *n* operations on single characters. This is most apparent in reading characters. In the examples herein, characters are input one at a time, perhaps using Java's `InputStream.read` (or a C or C++ equivalent). Using single-character processing can be quite inefficient. A subroutine call can cost hundreds or thousands of instructions to execute—far too many for a single character. Routines such as `InputStream.read(buffer)` perform block reads, putting an entire block of characters directly into `buffer`. Usually, the number of characters read is set to the size of a disk block (512 or perhaps 1024 bytes) so that an entire disk block can be read in one operation. If fewer than the requested number of characters are returned, then we know we have reached end-of-file. An **end-of-file** (EOF) character can be set to indicate this.

One problem with reading blocks of characters is that the end of a block won't usually correspond to the end of a token. For example, the beginning of a quoted string may be found near the end of a block, but not the string's end. Another read operation to get the rest of the string may overwrite the first part.

Double-buffering can avoid this problem, as shown in Figure 3.16. Input is first read into the left buffer, then into the right buffer, and then the left buffer is overwritten. Unless a token whose text we want to save is longer than the buffer length, tokens can cross a buffer boundary without difficulty. If the buffer size is made large enough (say 512 or 1,024 characters), then the chance of losing part of a token is very low. If a token's length is near the buffer's

`System.out.println("Four score`	`and seven years ago,");`

Figure 3.16: An example of double buffering.

length, then we can extend the buffer size, perhaps by using Java-style `Vector` objects rather than arrays to implement buffers.

We can speed up a scanner not only by doing block reads, but also by avoiding unnecessary copying of characters. Because so many characters are scanned, moving them from one place to another can be costly. A block read enables direct reading into the scanning buffer rather than into an intermediate input buffer. As characters are scanned, we need not copy characters from the input buffer unless we recognize a token whose text must be saved or processed (an identifier or a literal). With care, we can process the token's text directly from the input buffer.

At some point, using a profiling tool such as `qpt`, `prof`, `gprof`, or `pixie` may allow you to find unexpected performance bottlenecks in a scanner.

3.7.6 Lexical Error Recovery

A character sequence that cannot be scanned into any valid token results in a **lexical error**. Although uncommon, such errors must be handled by a scanner. It is unreasonable to stop compilation because of what is often a minor error, so usually we try some sort of *lexical error recovery*. Two approaches come to mind:

1. Delete the characters read so far and restart scanning at the next unread character.

2. Delete the first character read by the scanner and resume scanning at the character following it.

Both approaches are reasonable. The former can be done by resetting the scanner and beginning scanning anew. The latter is a bit harder to do but also is a bit safer (because fewer characters are immediately deleted). Non-deleted characters can be rescanned using the buffering mechanism described previously for scanner backup.

In most cases, a lexical error is caused by the appearance of some illegal character, which usually appears as the beginning of a token. In this case, the two approaches work equally well. The effects of lexical error recovery might well create a syntax error, which will be detected and handled by the parser. Consider . . . `for$tnight`. . . . The `$` would terminate scanning of `for`. Since no valid token begins with `$`, it would be deleted. Then `tnight` would be scanned as an identifier. The result would be `...for tnight...`, which will cause a syntax error. Such occurrences are unavoidable.

However, a good syntactic error-repair algorithm will often make some reasonable repair. In this case, returning a special warning token when a lexical error occurs can be useful. The semantic value of the warning token is the character string that is deleted to restart scanning. The warning token warns the parser that the next token is unreliable and that error repair may be required. The text that was deleted may be helpful in choosing the most appropriate repair.

Certain lexical errors require special care. In particular, runaway strings and comments should receive special error messages.

Handling Runaway Strings and Comments Using Error Tokens

In Java, strings are not allowed to cross line boundaries, so a runaway string is detected when an end-of-line character is reached within the string body. Ordinary recovery heuristics are often inappropriate for this error. In particular, deleting the first character (the double quote character) and restarting scanning will almost certainly lead to a cascade of further "false" errors because the string text is inappropriately scanned as ordinary input.

One way to catch runaway strings is to introduce an **error token**. An error token is not a valid token; it is never returned to the parser. Rather, it is a pattern for an error condition that needs special handling. We use an error token to represent a string terminated by an Eol rather than a double quote. For a valid string, in which internal double quotes and backslashes are escaped (and no other escaped characters are allowed), we can use

$$\text{″}\ (\text{Not}(\ \text{″} \mid \text{Eol} \mid \backslash) \mid \backslash\text{″} \mid \backslash\backslash\ )^{\star}\ \text{″}$$

For a runaway string, we can use

$$\text{″}\ (\text{Not}(\ \text{″} \mid \text{Eol} \mid \backslash) \mid \backslash\text{″} \mid \backslash\backslash\ )^{\star}\ \text{Eol}$$

When a runaway string token is recognized, a special error message should be issued. Further, the string may be repaired and made into a correct string by returning an ordinary string token with the opening double quote and closing Eol stripped (just as ordinary opening and closing double quotes are stripped). Note, however, that this repair may or may not be "correct." If the closing double quote is truly missing, the repair will be good. If it is present on a succeeding line, however, a cascade of inappropriate lexical and syntactic errors will follow until the closing double quote is finally reached.

Some PL/I compilers issue special warnings if comment delimiters appear within a string. Although such strings are legal, they almost always result from errors that cause a string to extend farther than was intended. A special string token can be used to implement such warnings. A valid string token is returned *and* an appropriate warning message is issued.

In languages such as C, C++, Java, and Pascal, which allow multiline comments, improperly terminated (that is, runaway) comments present a similar problem. A runaway comment is not detected until the scanner finds a close comment symbol (possibly belonging to some other comment) or until end-of-file is reached. Clearly, a special error message is required.

Consider the Pascal-style comments that begin with a { and end with a }. (Comments that begin and end with a pair of characters, such as `/*` and `*/` in Java, C, and C++, are a bit trickier to get right; see Exercise 6.)

Correct Pascal comments are defined quite simply: $\{\ \mathsf{Not}(\})^{\star}\ \}$

To handle comments terminated by Eof, the error token approach can be used: $\{\ \mathsf{Not}(\})^{\star}\ \mathsf{Eof}$

To handle comments closed by a close comment belonging to another comment (for example, {. . . `missing close comment`. . . { `normal comment` }), we issue a warning (but not an error message; this form of comment is lexically legal). In particular, a comment containing an open comment symbol in its body is most probably a symptom of the kind of omission depicted previously. We therefore split the legal comment definition into two tokens. The one that accepts an open comment in its body causes a warning message to be printed (`"Possible unclosed comment"`). This results in the following token definitions:

$\{\ \mathsf{Not}(\{ \mid \})^{\star}\ \}$	matches correct comments that do not contain an open comment in their bodies
$\{\ (\mathsf{Not}(\{ \mid \})^{\star}\ \{\ \mathsf{Not}(\{ \mid \})^{\star})^{+}\ \}$	matches correct, but suspect, comments that contain at least one open comment in their bodies
$\{\ \mathsf{Not}(\})^{\star}\ \mathsf{Eof}$	matches a **runaway comment** terminated by end-of-file

Single-line comments, found in Java and C++, are always terminated by an end-of-line character and so do not fall prey to the runaway comment problem. They do, however, require that each line of a multiline comment contain an open comment marker. Note, too, that we mentioned previously that balanced brackets cannot be correctly scanned using regular expressions and finite automata. A consequence of this limitation is that nested comments cannot be properly scanned using conventional techniques. This limitation causes problems when we want comments to nest, particularly when we "comment-out" a piece of code (which itself may well contain comments). Conditional compilation constructs, such as `#if` and `#endif` in C and C++, are designed to safely disable the compilation of selected parts of a program.

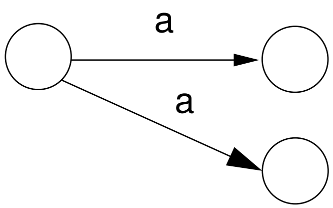


Figure 3.17: An NFA with two a transitions.

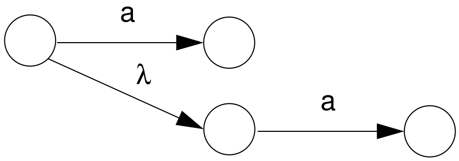


Figure 3.18: An NFA with a λ transition.

3.8 Regular Expressions and Finite Automata

Regular expressions are equivalent to FAs. In fact, the main job of a scanner generator program such as Lex is to transform a regular expression definition into an equivalent FA. It does this by first transforming the regular expression into a **nondeterministic finite automaton (NFA)**. An NFA is a generalization of a DFA that allows transitions labeled with λ as well as multiple transitions from a state that have the same label.

A scanner generator first creates an NFA from a set of regular-expression specifications. The NFA is then transformed into a DFA. Both of these steps are discussed in greater detail in this section.

An NFA, upon reading a particular input, need not make a unique (deterministic) choice of which state to visit. For example, as shown in Figure 3.17, an NFA is allowed to have a state that has two transitions (shown by the arrows) coming out of it, labeled by the same symbol. As shown in Figure 3.18, an NFA may also have transitions labeled with λ.

Transitions are normally labeled with individual characters in Σ, and although λ is a string (the string with no characters in it), it is definitely *not* a character. In the last example, when the FA is in the state at the left and the next input character is a, it may choose either to use the transition labeled a or to first follow the λ transition (you can always find λ wherever you look for it) and *then* follow an a transition. FAs that contain no λ transitions and that always have unique successor states for any symbol are *deterministic*.

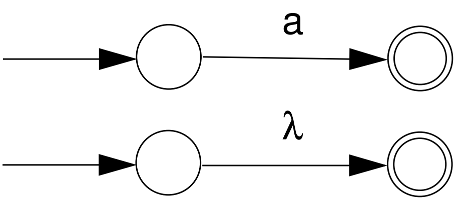


Figure 3.19: NFAs for a and λ.

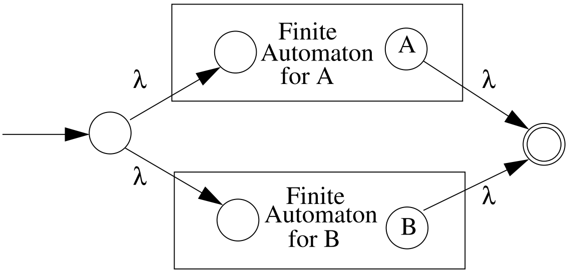


Figure 3.20: An NFA for $A \mid B$.

The algorithm to make an FA from a regular expression proceeds in two steps. First, it transforms the regular expression into an NFA. Then it transforms the NFA into a DFA.

3.8.1 Transforming a Regular Expression into an NFA

Transforming a regular expression into an NFA is easy. A regular expression is built of the *atomic* regular expressions a (where a is a character in Σ) and λ by using the three operations AB, $A \mid B$, and $A^{\star}$. Other operations (such as A^{+}) are just abbreviations for combinations of these. As shown in Figure 3.19, NFAs for a and λ are trivial.

Now suppose we have NFAs for A and B and want one for $A \mid B$. We construct the NFA shown in Figure 3.20. The states labeled A and B were the accepting states of the automata for A and B; we create a new accepting state for the combined FA.

As shown in Figure 3.21, the construction of AB is straightforward. The accepting state of the combined FA is the same as the accepting state of B.

Finally, the NFA for $A^{\star}$ is shown in Figure 3.22. The start state is an accepting state, so λ is accepted. Alternatively, we can follow a path through

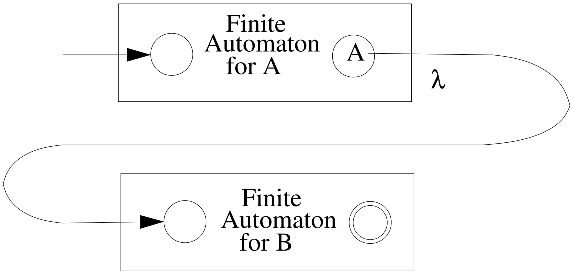


Figure 3.21: An NFA for AB.

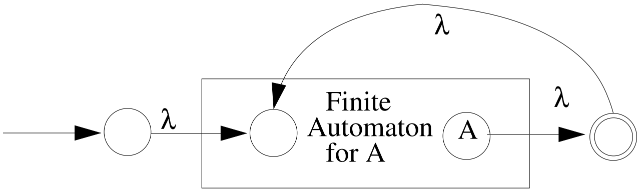


Figure 3.22: An NFA for $A^\star$.

the FA for A one or more times so that zero or more strings that belong to A are matched.

3.8.2 Creating the DFA

The transformation from an NFA N to an equivalent DFA D works by what is sometimes called the **subset construction**. The subset construction algorithm is shown in Figure 3.23. The algorithm associates each state of D with a *set* of states of N. The idea is that D will be in state $\{x, y, z\}$ after reading a given input string if, and only if, N could be in *any* of the states x, y, or z, depending on the transitions it chooses. Thus, D keeps track of all of the possible routes N might take and runs them simultaneously. Because N is a *finite* automaton, it has only a finite number of states. The number of subsets of N's states is also finite. This makes tracking various sets of states feasible.

The start state of D is the set of all states to which N can transition without reading any input characters—that is, the set of states reachable from the start state of N following only λ transitions. In Figure 3.23, Algorithm Close, called from RecordState, computes those states that can be reached after only λ transitions. Once the start state of D is built, we begin to create successor states.

```
function MakeDeterministic( $N$ ) returns $DFA$
    $D.StartState \leftarrow$ RecordState( $\{ N.StartState \}$ )
    foreach $S \in WorkList$ do
        $WorkList \leftarrow WorkList - \{ S \}$
        foreach $c \in \Sigma$ do $D.T(S,c) \leftarrow$ RecordState( $\bigcup_{s \in S} N.T(s,c)$ )
    $D.AcceptStates \leftarrow \{ S \in D.States \mid S \cap N.AcceptStates \neq \emptyset \}$
end

function Close( $S, T$ ) returns $Set$
    $ans \leftarrow S$
    repeat
        $changed \leftarrow$ false
        foreach $s \in ans$ do
            foreach $t \in T(s, \lambda)$ do
                if $t \notin ans$
                then
                    $ans \leftarrow ans \cup \{ t \}$
                    $changed \leftarrow$ true
    until not $changed$
    return ($ans$)
end

function RecordState( $s$ ) returns $Set$
    $s \leftarrow$ Close( $s, N.T$ )
    if $s \notin D.States$
    then
        $D.States \leftarrow D.States \cup \{ s \}$
        $WorkList \leftarrow WorkList \cup \{ s \}$
    return ($s$)
end
```

Figure 3.23: Construction of a DFA D from an NFA N.

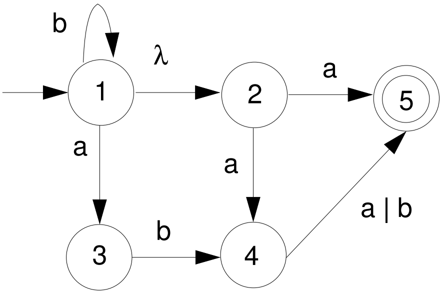


Figure 3.24: An NFA showing how subset construction operates.

To do this, we place each state S of D on a work list when it is created. For each state S on the work list and each character c in the vocabulary, we compute S's successor under c. S is identified with some set of N's states $\{n1, n2, \ldots\}$. We find all of the possible successor states to $\{n1, n2, \ldots\}$ under c and obtain a set $\{m1, m2, \ldots\}$. Finally, we include the λ-successors of $\{m1, m2, \ldots\}$. The resulting set of NFA states is included as a state T in D, and a transition from S to T, labeled with c, is added to D. We continue adding states and transitions to D until all possible successors to existing states are added. Because each state corresponds to a finite subset of N's states, the process of adding new states to D must eventually terminate.

An accepting state of D is any set that contains an accepting state of N. This reflects the convention that N accepts if there is *any* way it could get to its accepting state by choosing the "right" transitions.

To see how the subset construction operates, consider the NFA shown in Figure 3.24. In the NFA, we start with state 1, the start state of N, and add state 2, its λ-successor. Hence, D's start state is $\{1, 2\}$. Under a, $\{1, 2\}$'s successor is $\{3, 4, 5\}$. State 1 has itself as a successor under b. When state 1's λ-successor, 2, is included, $\{1, 2\}$'s successor is $\{1, 2\}$. $\{3, 4, 5\}$'s successors under a and b are $\{5\}$ and $\{4, 5\}$. $\{4, 5\}$'s successor under b is $\{5\}$. Accepting states of D are those state sets that contain N's accepting state (5). The resulting DFA is shown in Figure 3.25.

It can be established that the DFA constructed by MakeDeterministic is equivalent to the original NFA (see Exercise 20). What is not obvious is the fact that the DFA that is built can sometimes be *much* larger than the original NFA. States of the DFA are identified with sets of NFA states. If the NFA has n states, there are 2^n distinct sets of NFA states and hence the DFA may have as many as 2^n states. Exercise 16 discusses an NFA that actually exhibits this exponential blowup in size when it is made deterministic. Fortunately, the NFAs built from the kind of regular expressions used to specify programming language

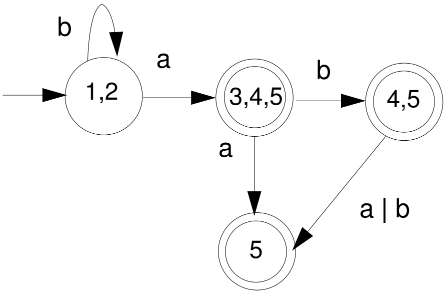


Figure 3.25: DFA created for NFA of Figure 3.24.

tokens do not exhibit this problem when they are made deterministic. As a rule, DFAs used for scanning are simple and compact.

When creating a DFA is impractical (either because of speed-of-generation or size concerns), an alternative is to scan using an NFA (see Exercise 17). Each possible path through an NFA can be tracked, and reachable accepting states can be identified. Scanning is slower using this approach, so it is usually used only when the construction of a DFA is not cost-effective.

3.8.3 Optimizing Finite Automata

We can improve the DFA created by MakeDeterministic. Sometimes this DFA will have more states than necessary. For every DFA, there is a *unique* smallest (in terms of number of states) equivalent DFA. Suppose a DFA D has 75 states and there is a DFA D' with 50 states that accepts exactly the same set of strings. Suppose further that no DFA with fewer than 50 states is equivalent to D. Then D' is the only DFA with 50 states equivalent to D. Using the techniques discussed in this section, we can optimize D by replacing it with D'.

Some DFAs contain **unreachable states**, states that cannot be reached from the start state. Other DFAs may contain **dead states**, states that cannot reach any accepting state. It is clear that neither unreachable states nor dead states can participate in scanning any valid token. So we eliminate all such states as part of our optimization process.

We optimize the resulting DFA by merging states we know to be equivalent. For example, two accepting states that have no transitions out of them are equivalent. Why? Because they behave exactly the same way—they accept the string read so far but will accept no additional characters. If two states, s_1 and s_2, are equivalent, then all transitions to s_2 can be replaced with transitions to s_1. In effect, the two states are merged into one common state.

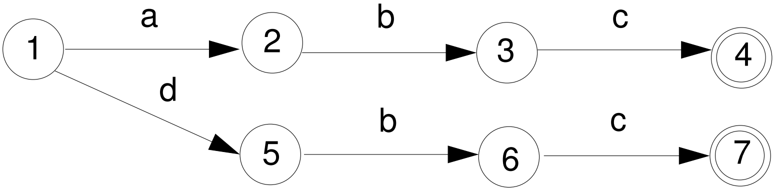


Figure 3.26: Example FA before merging.

How do we decide what states to merge? We take a *greedy* approach and try the most optimistic merger. By definition, accepting and nonaccepting states are distinct, so we initially try to create only two states: one representing the merger of all accepting states and the other representing the merger of all nonaccepting states. Having only two states is almost certainly too optimistic. In particular, all of the constituents of a merged state must agree on the same transition for each possible character. That is, for character *c* all of the merged states either must have no successor under *c* or must go to a single (possibly merged) state. If all constituents of a merged state do not agree on the transition to follow for some character, then the merged state is split into two or more smaller states that *do* agree.

As an example, assume we start with the FA shown in Figure 3.26. Initially, we have a merged nonaccepting state $\{1,2,3,5,6\}$ and a merged accepting state $\{4,7\}$. A merger is legal if, and only if, all constituent states agree on the same successor state for all characters. For example, states 3 and 6 would go to an accepting state when given character *c*; states 1, 2, and 5 would not, so a split must occur. We add an error state s_E to the original DFA that will be the successor state under any illegal character. (Thus, reaching s_E becomes equivalent to detecting an illegal token.) s_E is not a real state. Rather, it allows us to assume that every state has a successor under every character. s_E is never merged with any real state.

Algorithm SPLIT, shown in Figure 3.27, splits merged states whose constituents do not agree on a single successor state for a particular character. When SPLIT terminates, we know that the states that remain merged are equivalent in that they always agree on common successors.

Returning to the example, we initially have states $\{1,2,3,5,6\}$ and $\{4,7\}$. Invoking SPLIT, we first observe that states 3 and 6 have a common successor under *c* and states 1, 2, and 5 have no successor under *c* (or, equivalently, they have the error state s_E). This forces a split that yields $\{1,2,5\}$, $\{3,6\}$, and $\{4,7\}$. Now, for character *b*, states 2 and 5 go to the merged state $\{3,6\}$, but state 1 does not, so another split occurs. We now have $\{1\}$, $\{2,5\}$, $\{3,6\}$, and $\{4,7\}$. At this point, all constituents of merged states agree on the same successor for each input symbol, so we are done.

```
procedure Split($MergedStates$)
    repeat
        $changed \leftarrow$ false
        foreach $S \in MergedStates, c \in \Sigma$ do
            $targets \leftarrow \bigcup_{s \in S}$ TargetBlock($s, c, MergedStates$)
            if $|targets| > 1$
            then
                $changed \leftarrow$ true
                foreach $t \in targets$ do
                    $newblock \leftarrow \{ s \in S \mid$ TargetBlock$(s, c, MergedStates) = t \}$
                    $MergedStates \leftarrow MergedStates \cup \{ newblock \}$
                $MergedStates \leftarrow MergedStates - \{ S \}$
    until not $changed$
end

function TargetBlock($s, c, MergedStates$) returns $MergedState$
    return $(B \in MergedStates \mid T(s, c) \in B)$
end
```

Figure 3.27: An algorithm to split FA states.

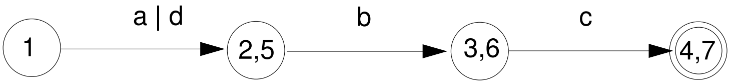
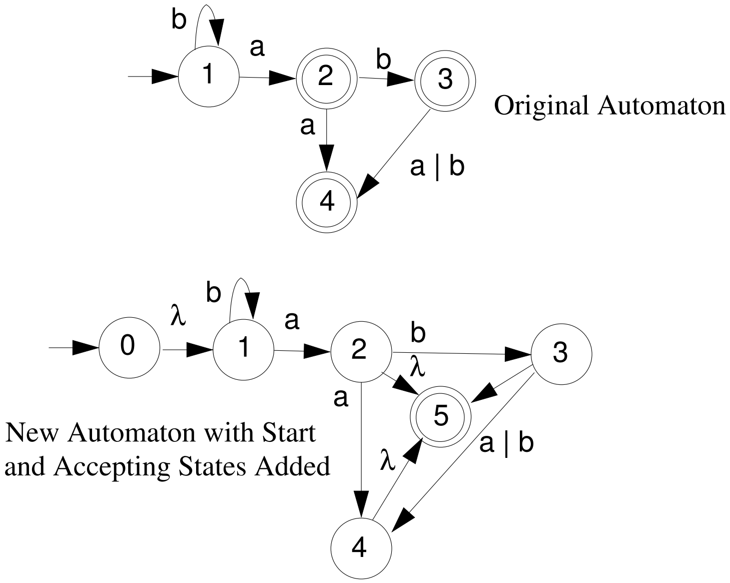


Figure 3.28: The minimum state automaton for Figure 3.26.

Once Split is executed, we are essentially done. Transitions between merged states are the same as the transitions between states in the original DFA. That is, if there was a transition between states s_i and s_j under character c, then there is now a transition under c from the merged state containing s_i to the merged state containing s_j. The start state is that merged state that contains the original start state. An accepting state is a merged state that contains accepting states (recall that accepting and nonaccepting states are never merged).

Returning to the example, the minimum state automaton we obtain is shown in Figure 3.28.

A proof of the correctness and optimality of this minimization algorithm can be found in most texts on automata theory, such as [HU79].



Original Automaton

New Automaton with Start and Accepting States Added

Figure 3.29: An FA with new start and accepting states added.

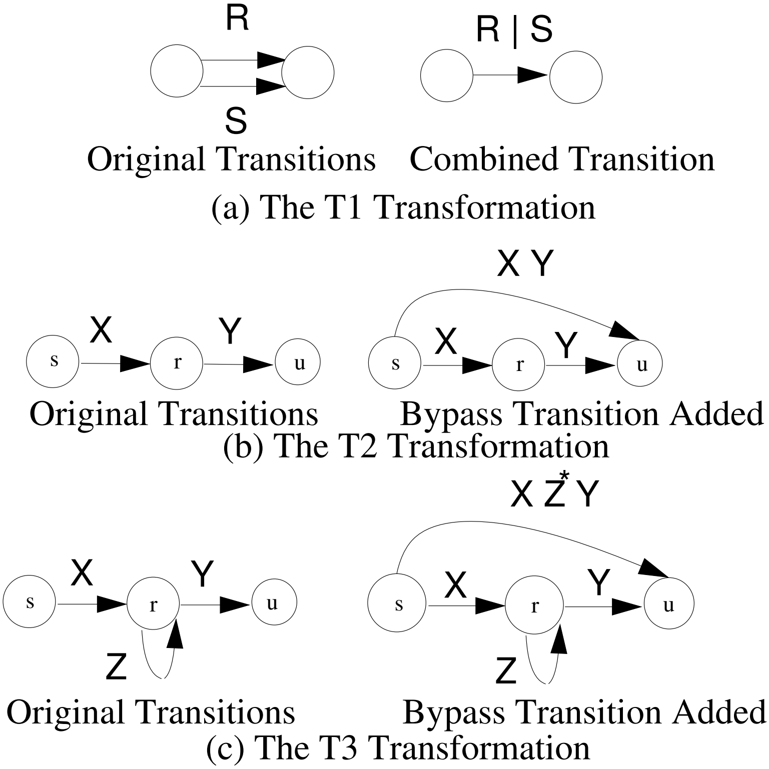
3.8.4 Translating Finite Automata into Regular Expressions

So far, we have concentrated on the process of converting a given regular expression into an equivalent FA. This is the key step in Lex's construction of a scanner from a set of regular expression token patterns.

Since regular expressions, DFAs, and NFAs are interconvertible, it is also possible to derive for any FA a regular expression that describes the strings that the FA matches. In this section, we briefly discuss an algorithm that does this derivation. This algorithm is sometimes useful when you already have an FA you want to use but you need a regular expression to program Lex or to describe the FA's effect. This algorithm also helps you to see that regular expressions and FAs really are equivalent.

The algorithm we use is simple and elegant. We start with an FA and simplify it by removing states, one by one. Simplified FAs are equivalent to the original, except that transitions are now labeled with regular expressions rather than individual characters. We continue removing states until we have an FA with a single transition from the start state to a single accepting state. The regular expression that labels that single transition correctly describes the effect of the original FA.

To start, we assume our FA has a start state with no transitions into it and a single accepting state with no transitions out of it. If it fails to meet these



(a) The T1 Transformation

(b) The T2 Transformation

(c) The T3 Transformation

Figure 3.30: The $T1$, $T2$, and $T3$ transformations.

requirements, then we can easily transform it by adding a new start state and a new accepting state linked to the original automaton with λ transitions. This is illustrated in Figure 3.29 using the FA we created with MakeDeterministic in Section 3.8.2. We define three simple transformations, $T1$, $T2$, and $T3$, that will allow us to progressively simplify FAs. The first, illustrated in Figure 3.30(a), notes that if there are two different transitions between the same pair of states, with one transition labeled R and the other labeled S, then we can replace the two transitions with a new transition labeled $R \mid S$. $T1$ simply reflects that we can choose to use either the first transition *or* the second.

Transformation $T2$, illustrated in Figure 3.30(b) allows us to *bypass* a state. That is, if state s has a transition to state r labeled X and state r has a transition to state u labeled Y, then we can go directly from state s to state u with a transition labeled XY.

Transformation $T3$, illustrated in Figure 3.30(c), is similar to transformation

$T2$. It, too, allows us to bypass a state. Suppose state s has a transition to state r labeled X and state r has a transition to itself labeled Z, as well as a transition to state u labeled Y. We can go directly from state s to state u with a transition labeled $XZ^{\star}Y$. The $Z^{\star}$ term reflects that once we reach state r, we can cycle back into r zero or more times before finally proceeding to u.

We use transformations $T2$ and $T3$ as follows. We consider, in turn, each pair of predecessors and successors a state s has and use $T2$ or $T3$ to link a predecessor state directly to a successor state. In this case, s is no longer needed—all paths through the FA can bypass it. Since s is not needed, we remove it. The FA is now simpler because it has one fewer states. By removing all states other than the start state and the accepting state (using transformation $T1$ when necessary), we will reach our goal. We will have an FA with only one transition, and the label on this transition will be the regular expression we want. FindRe, shown in Figure 3.31, implements this algorithm. The algorithm begins by invoking Augment, which introduces new start and accept states. The loop at Marker ① considers each state s of the FA in turn. Transformation $T1$ ensures that each pair of states is connected by at most one transition. This transformation is performed prior to processing s at Marker ③. State s is then eliminated by considering the cross-product of states with edges to and from s. For each such pair of states, transformation $T2$ or $T3$ is applied. State s is then removed at Marker ②, along with all edges to or from state s. When the algorithm terminates, the only states left are *NewStart* and *NewAccept*, introduced by Augment. The regular expression for the FA labels the transition between these two states.

As an example, we find the regular expression corresponding to the FA in Section 3.8.2. The original FA, with a new start state and accepting state added, is shown in Figure 3.32(a). State 1 has a single predecessor, state 0, and a single successor, state 2. Using a $T3$ transformation, we add an arc directly from state 0 to state 2 and remove state 1. This is shown in Figure 3.32(b). State 2 has a single predecessor, state 0, and three successors, states 2, 4, and 5. Using three $T2$ transformations, we add arcs directly from state 0 to states 3, 4, and 5. State 2 is removed. This is shown in Figure 3.32(c).

State 4 has two predecessors, states 0 and 3. It has one successor, state 5. Using two $T2$ transformations, we add arcs directly from states 0 and 3 to state 5. State 4 is removed. This is shown in Figure 3.32(d). Two pairs of transitions are merged using $T1$ transformations to produce the FA in Figure 3.32(e). Finally, state 3 is bypassed with a $T2$ transformation and a pair of transitions is merged with a $T1$ transformation, as shown in Figure 3.32(f). The regular expression we obtain is

$$b^{\star}ab(a \mid b \mid \lambda) \mid b^{\star}aa \mid b^{\star}a$$

By expanding the parenthesized subterm and then factoring a common term, we obtain

$$b^\star aba \mid b^\star abb \mid b^\star ab \mid b^\star aa \mid b^\star a \;\equiv\; b^\star a(ba \mid bb \mid b \mid a \mid \lambda)$$

Careful examination of the original FA verifies that this expression correctly describes it.

3.9 Summary

We have discussed three equivalent and interchangeable mechanisms for defining tokens: the regular expression, the deterministic finite automaton, and the nondeterministic finite automaton. Regular expressions are convenient for programmers because they allow the specification of token structure without regard for implementation considerations. Deterministic finite automata are useful in implementing scanners because they define token recognition simply and cleanly, on a character-by-character basis. Nondeterministic finite automata form a middle ground. Sometimes they are used for definitional purposes, when it is convenient to draw a simple automaton as a “flow diagram” of characters that are to be matched. Sometimes they are directly executed (see Exercise 17), when translation to deterministic finite automata is too costly or inconvenient. Familiarity with all three mechanisms will allow you to use the one best suited to your needs.

```
function FindRE( $N$ ) returns $RegExpr$
    $OrigStates \leftarrow N.States$
    call Augment( $N$ )
    foreach $s \in OrigStates$ do                                        ①
        call Eliminate( $s$ )
        $N.States \leftarrow N.States - \{ s \}$                          ②
    /★  return the regular expression labeling the only remaining transition  ★/
end

procedure Eliminate( $s$ )
    foreach $(x, y) \in N.States \times N.States$ | CountTrans( $x, y$ ) $> 1$ do   ③
        /★  Apply transformation T1 to $x$ and $y$  ★/
    foreach $p \in$ Preds( $s$ ) | $p \neq s$ do
        foreach $u \in$ Succs( $s$ ) | $u \neq s$ do
            if $CountTrans(s, s) = 0$
            then  /★ Apply Transformation T2 to $p$, $s$, and $u$ ★/
            else  /★ Apply Transformation T3 to $p$, $s$, and $u$ ★/
end

function CountTrans( $x, y$ ) returns $Integer$
    return (number of transitions from $x$ to $y$)
end

function Preds( $s$ ) returns $Set$
    return ($\{ p \mid (\exists\, a)(N.T(p, a) = s) \}$)
end

function Succs( $s$ ) returns $Set$
    return ($\{ u \mid (\exists\, a)(N.T(s, a) = u) \}$)
end

procedure Augment( $N$ )
    $OldStart \leftarrow N.StartState$
    $NewStart \leftarrow$ NewState( )
    /★  Define $N.T(NewStart, \lambda) = \{ OldStart \}$  ★/
    $N.StartState \leftarrow NewStart$
    $OldAccepts \leftarrow N.AcceptStates$
    $NewAccept \leftarrow$ NewState( )
    foreach $s \in OldAccepts$ do
        /★  Define $N.T(s, \lambda) = \{ NewAccept \}$  ★/
    $N.AcceptStates \leftarrow \{ NewAccept \}$
end
```

Figure 3.31: An algorithm to generate a regular expression from an FA.

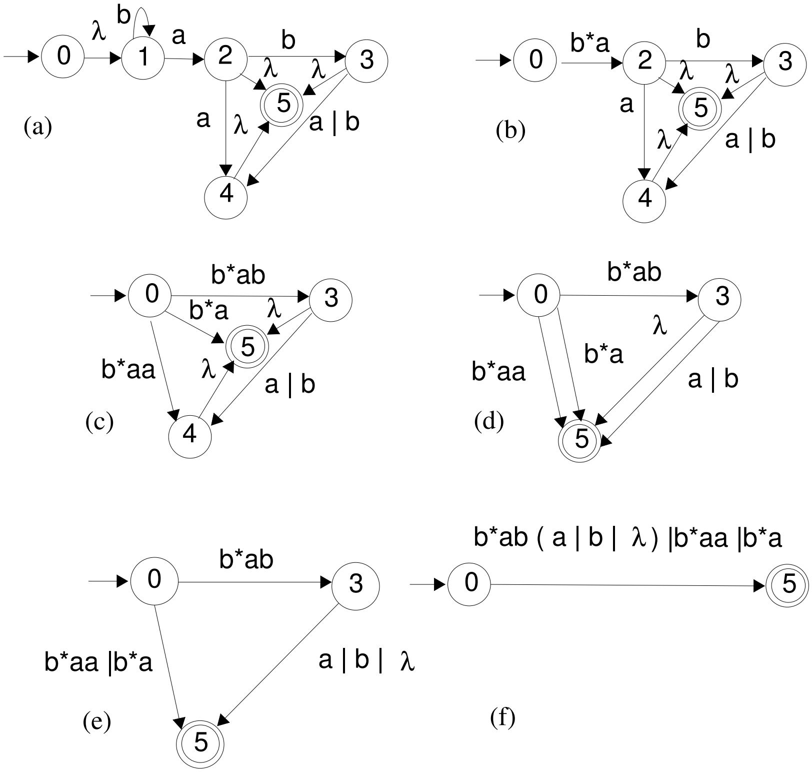


Figure 3.32: Finding a regular expression using FindRe.

Exercises

1. Assume the following text is presented to a C scanner:

```
main(){
    const float payment = 384.00;
    float bal;
    int month = 0;
    bal=15000;
    while (bal>0){
        printf("Month: %2d  Balance: %10.2f\n", month, bal);
        bal=bal-payment+0.015*bal;
        month=month+1;
    }
}
```

   What token sequence is produced? For which tokens must extra information be returned in addition to the token code?

2. How many lexical errors (if any) appear in the following C program? How should each error be handled by the scanner?

```
main(){
    if(1<2.)a=1.0else a=1.0e-n;
    subr(’aa’,"aaaaaa
              aaaaaa");
    /* That’s all

}
```

3. Write regular expressions that define the strings recognized by the FAs in Figure 3.33 on page 107.

4. Write DFAs that recognize the tokens defined by the following regular expressions:

   (a) $(a \mid (bc)^\star d)^+$

   (b) $((0 \mid 1)^\star (2 \mid 3)^+) \mid 0011$

   (c) $(a\ \mathsf{Not}(a))^\star aaa$

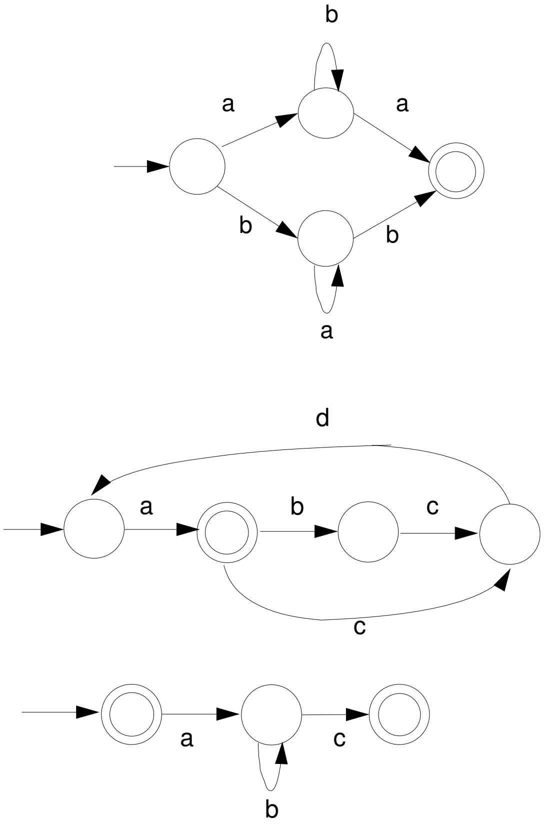


Figure 3.33: FA for Exercise 3.

5. Write a regular expression that defines a C-like, fixed-decimal literal with no superfluous leading or trailing zeros. That is, `0.0`, `123.01`, and `123005.0` are legal, but `00.0`, `001.000`, and `002345.1000` are illegal.

6. Write a regular expression that defines a C-like comment delimited by /* and */. Individual *'s and /'s may appear in the comment body, but the pair */ may not.

7. Define a token class *AlmostReserved* to be those identifiers that are not reserved words but that would be if a single character were changed. Why is it useful to know that an identifier is "almost" a reserved word? How would you generalize a scanner to recognize *AlmostReserved* tokens as well as ordinary reserved words and identifiers?

8. When a compiler is first designed and implemented, it is wise to concentrate on correctness and simplicity of design. After the compiler is fully implemented and tested, you may need to increase compilation speed. How would you determine whether the scanner component of a compiler is a significant performance bottleneck? If it is, what might you do to improve performance (without affecting compiler correctness)?

9. Most compilers can produce a source listing of the program being compiled. This listing is usually just a copy of the source file, perhaps embellished with line numbers and page breaks. Assume you are to produce a prettyprinted listing.

   (a) How would you modify a Lex scanner specification to produce a prettyprinted listing?

   (b) How are compiler diagnostics and line numbering complicated when a prettyprinted listing is produced?

10. For most modern programming languages, scanners require little context information. That is, a token can be recognized by examining its text and perhaps one or two lookahead characters. In Ada, however, additional context is required to distinguish between a single tic (comprising an attribute operator, as in `data'size`) and a tic-character-tic sequence (comprising a quoted character, as in `'x'`). Assume that a Boolean flag `can_parse_char` is set by the parser when a quoted character can be parsed. If the next input character is a tic, `can_parse_char` can be used to control how the tic is scanned. Explain how the `can_parse_char` flag can be cleanly integrated into a Lex-created scanner. The changes you suggest should not unnecessarily complicate or slow the scanning of ordinary tokens.

11. Unlike C, C++, and Java, Fortran generally ignores blanks and therefore may need extensive lookahead to determine how to scan an input line. A typical example of this is `DO 10 I = 1 , 10`, which produces seven tokens, in contrast with `DO 10 I = 1 .  10`, which produces three tokens.

    (a) How would you design a scanner to handle the extended lookahead that Fortran requires?

    (b) Lex contains a mechanism for doing lookahead of this sort. How would you match the identifier (`DO10I`) in this example?

12. Because Fortran generally ignores blanks, a character sequence containing n blanks can be scanned as many as 2^n different ways. Are each of these alternatives equally probable? If not, how would you alter the design you proposed in Exercise 11 to examine the most probable alternatives first?

13. You are to design the ultimate programming language, “Utopia 2010.” You have already specified the language’s tokens using regular expressions and the language’s syntax using a CFG. Now you want to determine those token pairs that require whitespace to separate them (such as `else a`) and those that require extra lookahead during scanning (such as `10.0e-22`). Explain how you could use the regular expressions and CFG to automatically find all token pairs that need special handling.

14. Show that the set $\{[^k]^k \mid k \geq 1\}$ is not regular. *Hint*: Show that no fixed number of FA states is sufficient to exactly match left and right brackets.

15. Show the NFA that would be created for the following expression using the techniques of Section 3.8:

$$(ab^\star c) \mid (abc^\star)$$

Using MakeDeterministic, translate the NFA into a DFA. Using the techniques of Section 3.8.3, optimize the DFA you created into a minimal state equivalent.

16. Consider the following regular expression:

$$(0 \mid 1)^\star 0 (0 \mid 1)(0 \mid 1)(0 \mid 1) \ldots (0 \mid 1)$$

Display the NFA corresponding to this expression. Show that the equivalent DFA is *exponentially* bigger than the NFA you presented.

17. Translation of a regular expression into an NFA is fast and simple. Creation of an equivalent DFA is slower and can lead to a much larger automaton. An interesting alternative is to scan using NFAs, thus obviating the need to ever build a DFA. The idea is to mimic the operation of the Close and MakeDeterministic routines (as defined in Section 3.8.2) while scanning. A set of possible states, rather than a single current state, is maintained. As characters are read, transitions from each state in the current set are followed, thereby creating a new set of states. If any state in the current set is final, then the characters read will comprise a valid token.

    Define a suitable encoding for an NFA (perhaps a generalization of the transition table used for DFAs) and write a scanner driver that can use this encoding by following the set-of-states approach outlined previously. This approach to scanning will surely be slower than the standard approach, which uses DFAs. Under what circumstances is scanning using NFAs attractive?

18. Assume e is any regular expression. $\overline{e}$ represents the set of all strings not in the regular set defined by e. Show that $\overline{e}$ is a regular set.

    Hint: If e is a regular expression, then there is an FA that recognizes the set defined by e. Transform this FA into one that will recognize $\overline{e}$.

19. Let *Rev* be the operator that reverses the sequence of characters within a string. For example, $Rev(abc) = cba$. Let R be any regular expression. $Rev(R)$ is the set of strings denoted by R, with each string reversed. Is $Rev(R)$ a regular set? Why or why not?

20. Prove that the DFA constructed by MakeDeterministic in Section 3.8.2 is equivalent to the original NFA. To do so, you must show that an input string can lead to a final state in the NFA if, and only if, that same string will lead to a final state in the corresponding DFA.

21. You have scanned an integer literal into a character buffer (perhaps `yytext`). You now want to convert the string representation of the literal into numeric (`int`) form. However, the string may represent a value too large to be represented in `int` form. Explain how to convert a string representation of an integer literal into numeric form with full overflow checking.

22. Write Lex regular expressions (using character classes if you wish) that match the following sets of strings:

    (a) The set of all unprintable ASCII characters (those before the blank and the very last character)

    (b) The string `["""]` (that is, a left bracket, three double quotes, and a right bracket)

    (c) The string $x^{12,345}$ (your solution should be far less than 12,345 characters in length)

23. Write a Lex program that examines the words in an ASCII file and lists the ten most frequently used words. Your program should ignore case and should ignore words that appear in a predefined "don't care" list.

    What changes in your program are needed to make it recognize singular and plural nouns (for example, cat and cats) as the same word? How about different verb tenses (walk versus walked versus walking)?

24. Let *Double* be the set of strings defined as $\{s \mid s = ww\}$. *Double* contains only strings composed of two identical repeated pieces. For example, if you have a vocabulary of the ten digits 0 to 9, then the following strings (and many more!) are in *Double*: $11, 1212, 123123, 767767, 98769876, \ldots$.

    Assume you have a vocabulary consisting only of the single letter a. Is *Double* a regular set? Why or why not?

    Assume you now have a vocabulary consisting of the two letters, a and b. Is Double a regular set? Why or why not?

25. Let $Seq(x, y)$ be the set of all strings (of length 1 or more) composed of alternating x's and y's. For example, $Seq(a, b)$ contains a, b, ab, ba, aba, bab, $abab$, $baba$, and so on.

    Write a regular expression that defines $Seq(x, y)$.

    Let S be the set of all strings (of length 1 or more) composed of a's, b's, and c's that start with an a and in which no two adjacent characters are equal. For example, S contains $a, ab, abc, abca, acab, acac, \ldots$ but not $c, aa, abb, abcc, aab, cac, \ldots$. Write a regular expression that defines S. You may use $Seq(x, y)$ within your regular expression if you wish.

26. Let *AllButLast* be a function that returns all of a string but its last character. For example, $AllButLast(abc) = ab$. $AllButLast(\lambda)$ is undefined. Let R be any regular expression that does not generate λ. $AllButLast(R)$ is the set of strings denoted by R, with *AllButLast* applied to each string. Thus, $AllButLast(a^+b) = a^+$. Show that $AllButLast(R)$ is a regular set.

27. Let F be any NFA that contains λtransitions. Write an algorithm that transforms F into an equivalent NFA F' that contains no λ transitions.

    Note: You need not use the subset construction, since you are creating an NFA, not a DFA.

28. Let s be a string. Define $Insert(s)$ to be the function that inserts a # into each possible position in s. If s is n characters long, then $Insert(s)$ returns a set of $n+1$ strings (since there are $n+1$ places, a # may be inserted in a string of length n).

    For example, $Insert(abc) = \{\ \#abc, a\#bc, ab\#c, abc\#\ \}$. *Insert* applied to a set of strings is the union of *Insert* applied to members of the set. Thus, $Insert(ab, de) = \{\ \#ab, a\#b, ab\#, \#de, d\#e, de\#\ \}$.

    Let R be any regular set. Show that $Insert(R)$ is a regular set.

    Hint: Given an FA for R, construct one for $Insert(R)$.

29. Let D be any deterministic finite automaton. Assume that you know D contains exactly n states and that it accepts at least one string of length n or greater. Show that D must also accept at least one string of length $2n$ or greater.

4

Grammars and Parsing

For natural languages such as English or German, we are accustomed to using rules of grammar to define proper sentence structure. Such rules may define phrases in terms of subjects, verbs, and objects. Sentences could then defined in terms of phrases and conjunctions. A properly structured sentence can be diagrammed to show how its components conform to a language's grammar. Grammars can also explain what is absent or superfluous in a malformed sentence. A sentence's **ambiguity** can often be explained by providing multiple diagrams for the same sentence (see Exercises 1 and 2).

Grammars thus serve as a concise definition of how meaningful sentences in a language can be constructed and as a tool for diagnosing malformed sentences. The first test of a sentence's validity is typically its adherence to the language's grammar. Of course, it is possible to construct sentences in a natural language that are grammatically correct but still make no sense. In other words, a natural language's grammar captures a small but important aspect of a sentence's validity with respect to a language.

A compiler's front-end performs several steps to establish the validity of its input. An input stream is scanned for tokens as discussed in Chapter 3. Tokens defined using **regular sets** could be processed by scanners that were constructed *automatically* from the regular-set specifications. Just as regular sets guide the actions of an automatically constructed scanner, so also can the actions of the parsers described in Chapters 5 and 6 be guided by a **grammar** that specifies a programming language's syntax.

Modern programming languages often contain a grammar in their specification as a guide to those who teach, study, or use the language. Based on the analysis discussed in this chapter, such grammars can also participate in the automatic construction of syntax-checking parsers. Programming language rules that are not easily expressed by grammars are enforced during the **semantic analyses** discussed in Chapters 7, 8, and 9. In Chapter 2, we discuss the rudiments of **context-free grammars** (CFGs) and define a simple language using a CFGs. Here, we formalize the definition and notation for CFGss and present algorithms that analyze such grammars in preparation for the parsing techniques covered in Chapters 5 and 6.

4.1 Context-Free Grammars

Formally, a **language** is a set of finite-length strings over a finite alphabet. Because most interesting languages are infinite sets, we cannot define such languages by enumerating their elements. A **context-free grammar** (CFG) is a compact, finite representation of a language, defined by the following four components:

- A finite **terminal alphabet** Σ. This is the set of tokens produced by the scanner. We always augment this set with the token \$, which signifies end-of-input.

- A finite **nonterminal alphabet** N. Symbols in this alphabet are *variables* of the grammar.

- A **start symbol** $\mathsf{S} \in N$ that initiates all *derivations*. S is also called the **goal symbol**.

- A finite set of productions P (sometimes called **rewriting rules**) of the form $\mathsf{A} \rightarrow \mathcal{X}_1 \ldots \mathcal{X}_m$, where $\mathsf{A} \in N$, $\mathcal{X}_i \in N \cup \Sigma$, $1 \le i \le m$, and $m \ge 0$. The only valid production with $m = 0$ is of the form $\mathsf{A} \rightarrow \lambda$, where λ denotes the **empty string**.

These components are often expressed as $\mathsf{G} = (N, \Sigma, P, \mathsf{S})$, which is the formal definition of a CFG. The terminal and nonterminal alphabets must be disjoint (i.e., $\Sigma \cap N = \emptyset$). The **vocabulary** V of a CFG is the set of terminal and nonterminal symbols (i.e., $V = \Sigma \cup N$).

A CFG is essentially a recipe for creating strings. Starting with S, nonterminals are rewritten using the grammar's productions until only terminals remain. A rewrite using the production $\mathsf{A} \rightarrow \alpha$ replaces the nonterminal A with the vocabulary symbols in α. As a special case, a rewrite using the production $\mathsf{A} \rightarrow \lambda$ causes A to be erased. Each rewrite is a step in a **derivation** of the resulting string. The set of terminal strings derivable from S comprises the **context-free language** of grammar G, denoted L(G).

In describing parsers, algorithms, and grammars, consistency is useful in denoting symbols and strings of symbols. We therefore adopt the following notation:

Names Beginning With	Represent Symbols In	Examples
Uppercase	N	A, B, C, Prefix
Lowercase and punctuation	Σ	a, b, c, if, then, (, ;
$\mathcal{X}, \mathcal{Y}$	$N \cup \Sigma$	$\mathcal{X}_i, \mathcal{Y}_3$
Other Greek letters	$(N \cup \Sigma)^\star$	α, γ

Using this notation, we write a production as $\mathsf{A} \rightarrow \alpha$ or $\mathsf{A} \rightarrow \mathcal{X}_1 \ldots \mathcal{X}_m$, depending on whether the detail of the production's **right-hand side** (RHS) is of interest. This format emphasizes that a production's **left-hand side** (LHS) must be a single nonterminal while the RHS is a string of zero or more vocabulary symbols.

There is often more than one way to rewrite a given nonterminal; in such cases, multiple productions share the same LHS symbol. Instead of repeating the LHS symbol, an "or notation" is used.

$$\begin{aligned} \mathsf{A} \rightarrow\ & \alpha \\ \mid\ & \beta \\ & \cdots \\ \mid\ & \zeta \end{aligned}$$

This is an abbreviation for the following sequence of productions:

$$\begin{aligned} \mathsf{A} &\rightarrow \alpha \\ \mathsf{A} &\rightarrow \beta \\ &\cdots \\ \mathsf{A} &\rightarrow \zeta \end{aligned}$$

If $\mathsf{A} \rightarrow \gamma$ is a production, then $\alpha \mathsf{A} \beta \Rightarrow \alpha\gamma\beta$ denotes one step of a derivation using this production. We extend $\Rightarrow$ to $\Rightarrow^+$ (derives in one or more steps) and $\Rightarrow^\star$ (derives in zero or more steps). If $\mathsf{S} \Rightarrow^\star \beta$, then β is said to be a **sentential form** of the CFG. SF(G) denotes the set of sentential forms of grammar G. Thus $\mathsf{L(G)} = \{ w \in \Sigma^\star \mid \mathsf{S} \Rightarrow^+ w \}$. Also, $\mathsf{L(G)} = \mathsf{SF(G)} \cap \Sigma^\star$. That is, the language of G is simply those sentential forms of G that are terminal strings.

Throughout a derivation, if more than one nonterminal is present in a sentential form, then there is a choice as to which nonterminal should be expanded in the next step. Thus to characterize a derivation sequence, we need to specify, at each step, which nonterminal is expanded and which production

$$
\begin{array}{lll}
1 & \mathsf{E} & \rightarrow \mathsf{Prefix}\ (\ \mathsf{E}\ ) \\
2 & & \ \ \mid\ \mathsf{v}\ \ \mathsf{Tail} \\
3 & \mathsf{Prefix} & \rightarrow \mathsf{f} \\
4 & & \ \ \mid\ \lambda \\
5 & \mathsf{Tail} & \rightarrow +\ \mathsf{E} \\
6 & & \ \ \mid\ \lambda
\end{array}
$$

Figure 4.1: A simple expression grammar.

is applied. We can simplify this characterization by adopting a convention such that nonterminals are rewritten in some systematic order. There are two such conventions:

- Leftmost derivation, which expands nonterminals left to right
- Rightmost derivation, which expands nonterminals right to left

4.1.1 Leftmost Derivations

A derivation that always chooses the *leftmost* possible nonterminal at each step is called a **leftmost derivation**. If we know that a derivation is leftmost, we need only specify the productions in the order of their application; the expanded nonterminal is implicit. To denote derivations that are leftmost, we use $\Rightarrow_{\text{lm}}$, $\Rightarrow^{+}_{\text{lm}}$, and $\Rightarrow^{\star}_{\text{lm}}$. A sentential form produced via a leftmost derivation is called a **left sentential form**. The production sequence discovered by a large class of parsers (the **top-down parsers**) is a leftmost derivation. Hence, these parsers are said to produce a **leftmost parse**.

As an example, consider the grammar shown in Figure 4.1, which generates simple expressions (v represents a variable and f represents a function). A leftmost derivation of f (v + v) is as follows:

$$
\begin{array}{lll}
\mathsf{E} & \Rightarrow_{\text{lm}} & \mathsf{Prefix}\ (\ \mathsf{E}\ ) \\
 & \Rightarrow_{\text{lm}} & \mathsf{f}\ (\ \mathsf{E}\ ) \\
 & \Rightarrow_{\text{lm}} & \mathsf{f}\ (\ \mathsf{v}\ \mathsf{Tail}\ ) \\
 & \Rightarrow_{\text{lm}} & \mathsf{f}\ (\ \mathsf{v} + \mathsf{E}\ ) \\
 & \Rightarrow_{\text{lm}} & \mathsf{f}\ (\ \mathsf{v} + \mathsf{v}\ \mathsf{Tail}\ ) \\
 & \Rightarrow_{\text{lm}} & \mathsf{f}\ (\ \mathsf{v} + \mathsf{v}\ )
\end{array}
$$

4.1.2 Rightmost Derivations

An alternative to a leftmost derivation is a **rightmost derivation** (sometimes called a **canonical derivation**). In such derivations, the rightmost possible nonterminal is always expanded. This derivation sequence may seem less intuitive given the English convention of processing information left to right. However, such derivations are produced by an important class of parsers, namely the bottom-up parsers discussed in Chapter 6.

As a bottom-up parser discovers the productions that derive a given token sequence, it traces a rightmost derivation, but the productions are applied in *reverse order*. That is, the last step taken in a rightmost derivation is the first production applied by the bottom-up parser; the first step involving the start symbol is the parser's final production. The sequence of productions applied by a bottom-up parser is called a **rightmost** or **canonical** parse. For derivations that are rightmost, the notation $\Rightarrow_{\text{rm}}$, $\Rightarrow^{+}_{\text{rm}}$, and $\Rightarrow^{\star}_{\text{rm}}$ is used. A sentential form produced via a rightmost derivation is called a **right sentential form**. A rightmost derivation of the grammar shown in Figure 4.1 is as follows.

$$
\begin{array}{lll}
\text{E} & \Rightarrow_{\text{rm}} & \text{Prefix (E)} \\
 & \Rightarrow_{\text{rm}} & \text{Prefix (v Tail)} \\
 & \Rightarrow_{\text{rm}} & \text{Prefix (v + E)} \\
 & \Rightarrow_{\text{rm}} & \text{Prefix (v + v Tail)} \\
 & \Rightarrow_{\text{rm}} & \text{Prefix (v + v)} \\
 & \Rightarrow_{\text{rm}} & \text{f (v + v)}
\end{array}
$$

4.1.3 Parse Trees

A derivation is often represented by a *parse tree* (sometimes called a *derivation tree*). A **parse tree** has the following characteristics:

- It is rooted by the grammar's start symbol S.
- Each node is either a grammar symbol or λ.
- Its interior nodes are nonterminals. An interior node and its children represent the application of a production. That is, a node representing a nonterminal A can have offspring $\mathcal{X}_1, \mathcal{X}_2, \ldots, \mathcal{X}_m$ if, and only if, there exists a grammar production $\text{A} \rightarrow \mathcal{X}_1\ \mathcal{X}_2 \ldots \mathcal{X}_m$. When a derivation is complete, each leaf of the corresponding parse tree is either a terminal symbol or λ.

Figure 4.2 shows the parse tree for f (v + v) using the grammar from Figure 4.1. Parse trees serve nicely to visualize how a string is structured by a grammar. A leftmost or rightmost derivation is essentially a textual representation of a parse tree, but the derivation also conveys the order in which the productions are applied.

A sentential form is derivable from a grammar's start symbol. Hence, a parse tree must exist for every sentential form. Given a sentential form

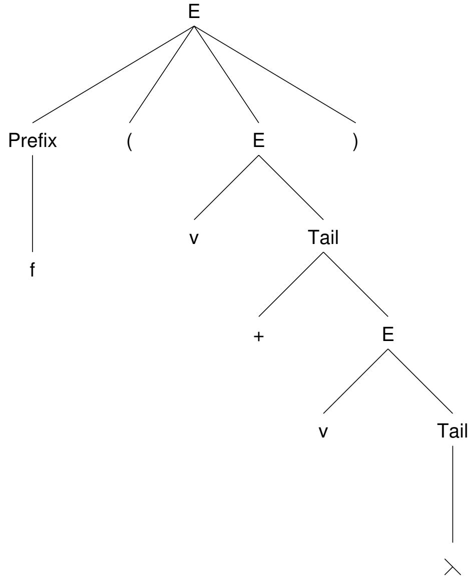


Figure 4.2: The parse tree for f (v + v) .

and its parse tree, a **phrase** of the sentential form is a sequence of symbols descended from a single nonterminal in the parse tree. A **simple** or **prime phrase** is a phrase that contains no smaller phrase. That is, it is a sequence of symbols directly derived from a nonterminal. The **handle** of a sentential form is the leftmost simple phrase. (Simple phrases cannot overlap, so "leftmost" is unambiguous.) Given the parse tree of Figure 4.2 and the sentential form f (v Tail), f and v Tail are simple phrases and f is the handle. Handles are important because they represent individual derivation steps, which can be recognized by various parsing techniques.

4.1.4 Other Types of Grammars

Although CFGs serve well to characterize syntax, most programming languages contain rules that are not expressible using CFGs. For example, the rule that variables must be declared before they are used cannot be expressed because a CFG provides no mechanism for transmitting to the body of a program the exact set of variables that has been declared. In practice, syntactic details that cannot be represented in a CFG are considered part of the static semantics and are checked by semantic routines (along with scope and type rules).

The following grammars are relevant to programming language translation:

- Regular grammars, which are less powerful than CFGs
- Context-sensitive and unrestricted grammars, which are more powerful

Regular Grammars

A CFG that is limited to productions of the form A→a B or C→d is a **regular grammar**. Each rule's RHS consists of either a symbol from $\Sigma \cup \{ \lambda \}$ followed by a nonterminal symbol or just a symbol from $\Sigma \cup \{ \lambda \}$. As the name suggests, a regular grammar defines a regular set (see Exercise 15.) We observed in Chapter 3 that the language $\{ [^i \,]^i \mid i \geq 1 \}$ is not regular. This language is generated by the following CFG:

```
1  S → T
2  T → [ T ]
3    | λ
```

This grammar establishes that the languages definable by regular grammars (regular sets) are a *proper* subset of the context-free languages.

Beyond Context-Free Grammars

CFGs can be generalized to create richer notational mechanisms. A **context-sensitive grammar** requires that nonterminals be rewritten only when they appear in a particular context (for example, $\alpha \mathsf{A} \beta \rightarrow \alpha \delta \beta$), provided the rule never causes the sentential form to contract in length. An **unrestricted** or **type-0 grammar** is the most general. It allows arbitrary patterns to be rewritten.

Although context-sensitive and unrestricted grammars are more powerful than CFGs, they also are far less useful for the following reasons:

- Efficient parsers for such grammars do not exist. Without a parser, a grammar definition cannot participate in the automatic construction of compiler components.
- It is difficult to prove properties about such grammars. For example, it would be daunting to prove that a given type-0 grammar generates the C programming language.

Efficient parsers for many classes of CFGs do exist. Hence, CFGs present a nice balance between generality and practicality.

4.2 Properties of CFGs

CFGs are a notational mechanism for specifying languages. Just as there are many programs that compute the same result, so also there are many grammars that generate the same language. Some are better suited for a particular translation task, as discussed in Chapter 7. Some grammars have one or more of the following problems that preclude their use:

- The grammar may include useless symbols.
- The grammar may allow multiple, distinct derivations (parse trees) for some input string.
- The grammar may include strings that do not belong in the language, or the grammar may exclude strings that are in the language.

In this section, we discuss these problems and their implication for language processing.

4.2.1 Reduced Grammars

A grammar is **reduced** if each of its nonterminals and productions participates in the derivation of some string in the grammar's language. Nonterminals that can be safely removed are called **useless**.

```
1  S → A
2    | B
3  A → a
4  B → B b
5  C → c
```

The above grammar contains two kinds of nonterminals that cannot participate in any derived string:

- With S as the start symbol, the nonterminal C cannot appear in any phrase.
- Any phrase that mentions B cannot be rewritten using the grammar's rules to contain only terminals.

When B, C, and their associated productions are removed, the following reduced grammar is obtained:

```
1  S → A
2  A → a
```

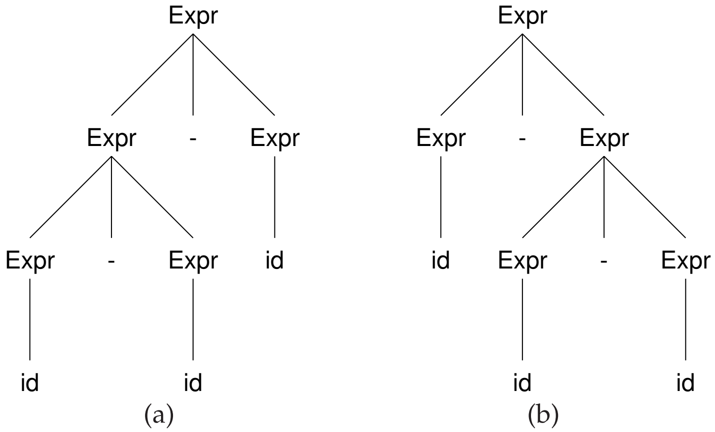


Figure 4.3: Two parse trees for id - id - id.

Exercises 16 and 17 consider how to detect both forms of useless nonterminals. Many parser generators verify that a grammar is in reduced form. An unreduced grammar probably contains errors that result from mistyping of grammar specifications.

4.2.2 Ambiguity

Some grammars allow a derived string to have two or more different parse trees (and thus a nonunique structure). Consider the following grammar, which generates expressions using the infix operator for subtraction.

1 Expr → Expr - Expr
2 | id

This grammar allows two different parse trees for id - id - id, as illustrated in Figure 4.3. The tree in Figure 4.3(a) models the subraction of the third id from the difference of the first two. The tree in Figure 4.3(b) subtracts the difference of the last two id symbols from the first. If the id symbols have values 3, 2, and 1, then tree Figure 4.3(a) evaluates to 0, while tree Figure 4.3(b) evaluates to 2.

Grammars that allow different parse trees for the same terminal string are called **ambiguous**. They are rarely used because a unique structure (i.e., parse tree) cannot be guaranteed for all inputs. Hence, a unique translation, guided by the parse tree structure, may not be obtained.

It seems we need an algorithm that checks an arbitrary CFG for ambiguity. Unfortunately, no algorithm is possible for this in the general case, as the problem is **undecidable** [HU79, Mar03]. For certain grammar classes, successful parser construction by the algorithms we discuss in Chapters 5 and 6 proves a grammar to be unambiguous. However, when such parser construction fails, the grammar may or may not be ambiguous. Section 6.4.1 on page 199 presents some approaches for reasoning about a grammar's ambiguity.

4.2.3 Faulty Language Definition

The most potentially serious flaw that a grammar might have is that it generates the "wrong" language. That is, the terminal strings derivable by the grammar do not correspond *exactly* to the strings present in the desired language. This is a subtle point, because a grammar typically serves as the very definition of a language's syntax.

The correctness of a grammar is usually tested informally by attempting to parse a set of inputs, some of which are supposed to be in the language and some of which are not. One might try to compare for equality the languages defined by a pair of grammars (considering one a standard), but this is rarely done. For some grammar classes, such verification is possible; for others, no comparison algorithm is known. Determining in general whether two CFGs generate the same language is an **undecidable problem**.

4.3 Transforming Extended Grammars

Backus-Naur form (BNF) extends the grammar notation defined in Section 4.1 with syntax for defining optional and repeated symbols.

- Optional symbols are enclosed in square brackets. In the production

  $$\mathsf{A} \rightarrow \alpha \; [\; \mathcal{X}_1 \ldots \mathcal{X}_n \;] \; \beta$$

  the symbols $\mathcal{X}_1 \ldots \mathcal{X}_n$ are entirely present or absent between the symbols of α and β.

- Repeated symbols are enclosed in braces. In the production

  $$\mathsf{B} \rightarrow \gamma \; \{\; \mathcal{X}_1 \ldots \mathcal{X}_m \;\} \; \delta$$

  the entire sequence of symbols $\mathcal{X}_1 \ldots \mathcal{X}_m$ can be repeated zero or more times.

These extensions are useful in representing many programming language constructs. In Java$^{\mathrm{TM}}$, declarations can optionally include modifiers such as `final`, `static`, and `const`. Each declaration can include a *list* of identifiers. A production specifying a Java-like declaration could be as follows:

```
foreach $p \in Prods$ of the form "$\mathsf{A} \rightarrow \alpha$ [ $\mathcal{X}_1 \ldots \mathcal{X}_n$ ] $\beta$" do
    $N \leftarrow$ NewNonTerm( )
    $p \leftarrow$ "$\mathsf{A} \rightarrow \alpha\ N\ \beta$"
    $Prods \leftarrow Prods \cup \{$"$N \rightarrow \mathcal{X}_1 \ldots \mathcal{X}_n$"$\}$
    $Prods \leftarrow Prods \cup \{$"$N \rightarrow \lambda$"$\}$
foreach $p \in Prods$ of the form "$\mathsf{B} \rightarrow \gamma$ { $\mathcal{X}_1 \ldots \mathcal{X}_m$ } $\delta$" do
    $M \leftarrow$ NewNonTerm( )
    $p \leftarrow$ "$\mathsf{B} \rightarrow \gamma\ M\ \delta$"
    $Prods \leftarrow Prods \cup \{$"$M \rightarrow \mathcal{X}_1 \ldots \mathcal{X}_n\ M$"$\}$
    $Prods \leftarrow Prods \cup \{$"$M \rightarrow \lambda$"$\}$
```

Figure 4.4: Algorithm to transform a BNF grammar into standard form.

Declaration → [final] [static] [const] Type identifier { , identifier }

This declaration insists that the modifiers be ordered as shown. Exercises 13 and 14 consider how to specify the optional modifiers in any order.

Although BNF can be useful, algorithms for analyzing grammars and building parsers assume the standard grammar notation as introduced in Section 4.1. The algorithm in Figure 4.4 transforms extended BNF grammars into standard form. For the BNF syntax involving braces, the transformation uses *right* recursion on M to allow zero or more occurrences of the symbols enclosed within braces. This transformation also works using left recursion—the resulting grammar would have generated the same language.

As discussed in Section 4.1, a particular derivation (e.g., leftmost or rightmost) depends on the structure of the grammar. It turns out that right-recursive rules are more appropriate for top-down parsers, which produce leftmost derivations. Similarly, left-recursive rules are more suitable for bottom-up parsers, which produce rightmost derivations.

4.4 Parsers and Recognizers

Compilers are expected to verify the syntactic validity of their inputs with respect to a grammar that defines the programming language's syntax. Given a grammar G and an input string x, the compiler must determine if $x \in$ L(G). An algorithm that performs this test is called a **recognizer**.

For language translation, we must determine not only the string's validity, but also its structure, or **parse tree**. An algorithm for this task is called a **parser**. Generally, there are two approaches to parsing:

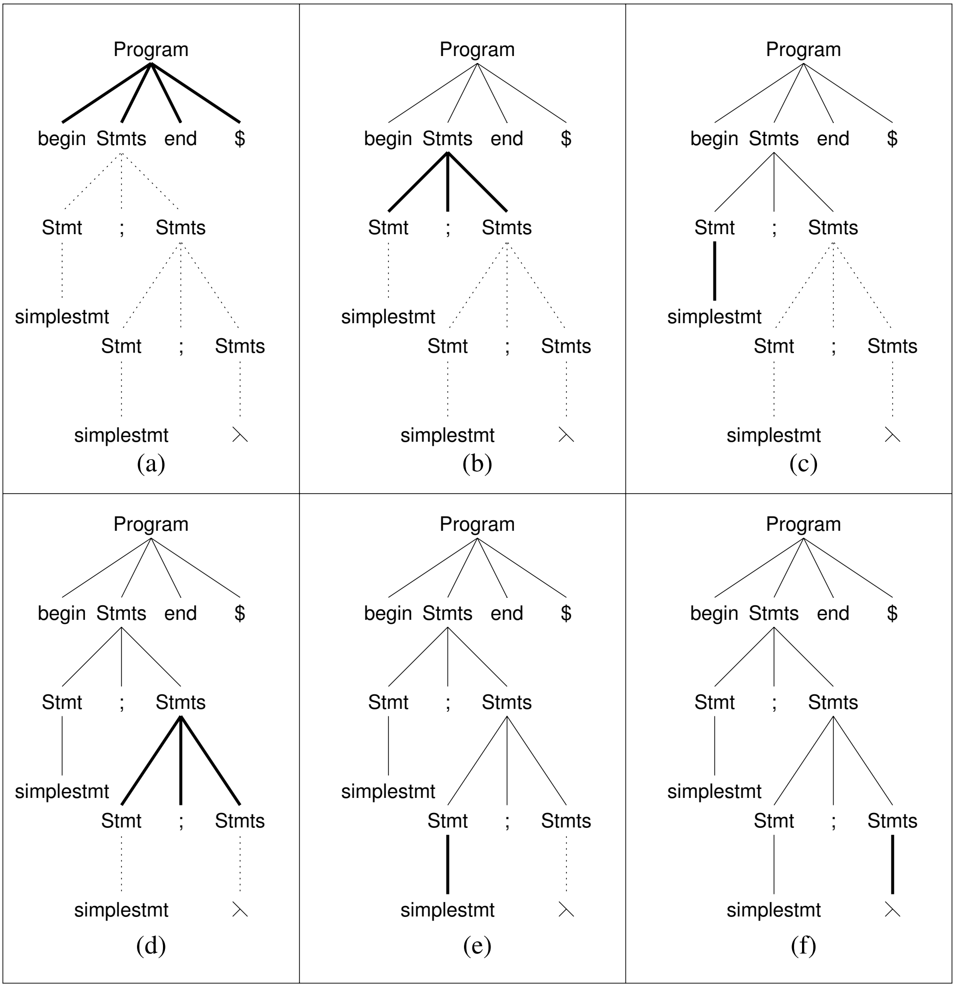


Figure 4.5: Parse of “begin simplestmt ; simplestmt ; end $” using the top-down technique. Legend explained on page 126.

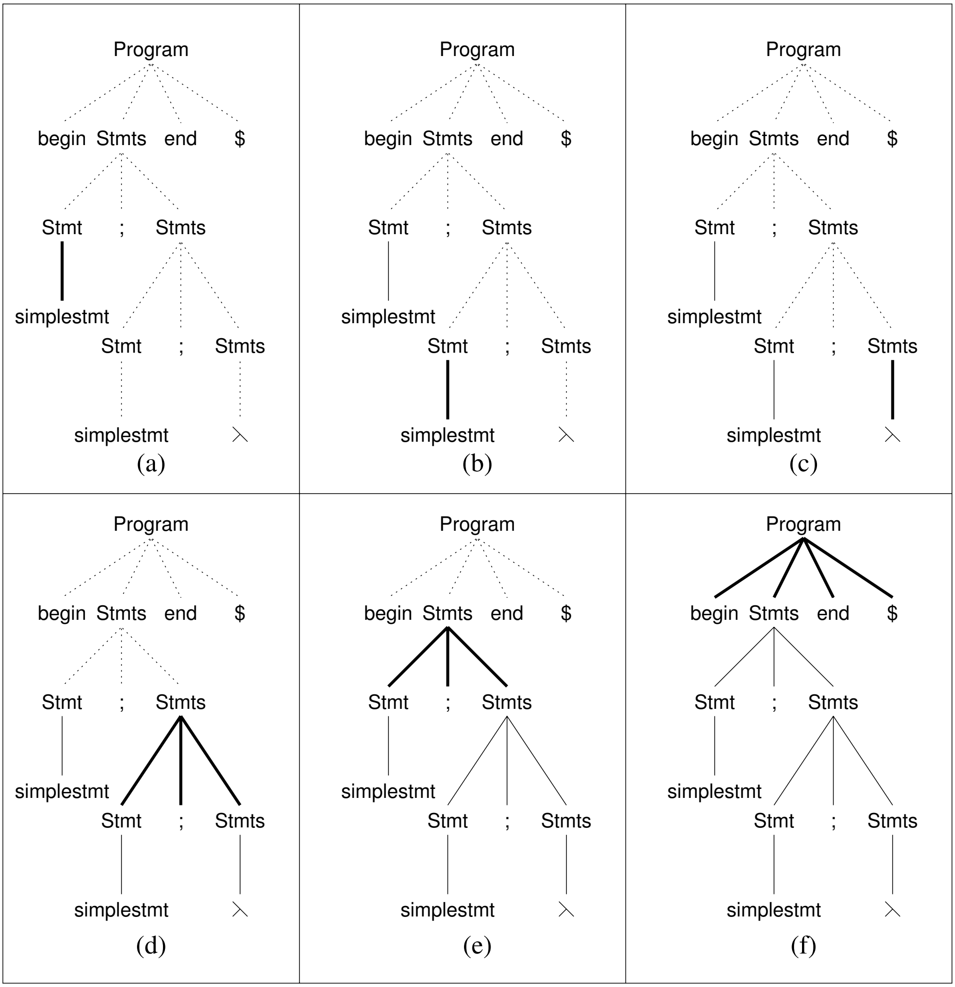


Figure 4.6: Parse of “begin simplestmt ; simplestmt ; end $” using the bottom-up technique. Legend explained on page 126.

- A parser is considered **top-down** if it generates a parse tree by starting at the root of the tree (the start symbol), expanding the tree by applying productions in a depth-first manner. A top-down parse corresponds to a preorder traversal of the parse tree. Top-down parsing techniques are *predictive* in nature because they always predict the production that is to be matched before matching actually begins. The top-down approach includes the recursive-descent parser discussed in Chapter 2.

- The **bottom-up** parsers generate a parse tree by starting at the tree's leaves and working toward its root. A node is inserted in the tree only after its children have been inserted. A bottom-up parse corresponds to a postorder traversal of the parse tree.

The following grammar generates the skeletal block structure of a programming language.

1	Program	→	begin Stmts end $
2	Stmts	→	Stmt ; Stmts
3		|	λ
4	Stmt	→	simplestmt
5		|	begin Stmts end

Using this grammar, Figures 4.5 and 4.6 illustrate a top-down and bottom-up parse of the string begin simplestmt ; simplestmt ; end $. Each box shows one step of the parse, with the particular rule denoted by bold lines between a parent (the rule's LHS) and its children (the rule's RHS). Solid, non-bold lines indicate rules that have already been applied; dashed lines indicate rules that have not yet been applied. For example, Figure 4.5(a) shows the rule Program→begin Stmts end $ applied as the first step of a top-down parse. Figure 4.6(f) shows the same rule applied as the last step of a bottom-up parse.

When specifying a parsing technique, we must state whether a leftmost or rightmost parse will be produced. The best-known and most widely used top-down and bottom-up parsing strategies are called **LL** and **LR**, respectively. These names seem rather arcane, but they reflect how the input is processed and which kind of parse is produced. In both cases, the first character (L) states that the token sequence is processed from left to right. The second letter (L or R) indicates whether a leftmost or rightmost parse is produced. The parsing technique can be further characterized by the number of lookahead symbols (i.e., symbols beyond the current token) that the parser may consult to make parsing choices. LL(1) and LR(1) parsers are the most common, requiring only one symbol of lookahead.

4.5 Grammar Analysis Algorithms

It is often necessary to analyze a grammar to determine if it is suitable for parsing and, if so, to construct tables that can drive a parsing algorithm. In this section, we discuss a number of important analysis algorithms that build upon the basic concepts of grammars and derivations. These algorithms are central to the automatic construction of parsers, as discussed in Chapters 5 and 6.

4.5.1 Grammar Representation

The algorithms presented in this chapter refer to a collection of utilities for accessing and modifying representations of a CFG. The efficiency of these algorithms is affected by the data structures upon which these utilities are built. In this section, we examine how to represent CFGs efficiently. We assume that the implementation programming language offers the following constructs directly or by augmentation:

- A **set** is an unordered collection of distinct entities.
- A **list** is an ordered collection of entities. A entity can appear multiple times in a list.
- An **iterator** is a construct that enumerates the contents of a set or list.

As discussed in Section 4.1, a grammar formally contains two disjoint sets of symbols, Σ and N, which contain the grammar's terminal and nonterminal symbols, respectively. Grammars also contain a designated start symbol and a set of productions. The following observations are relevant to obtaining an efficient representation for grammars:

- Symbols are rarely deleted from a grammar.
- Transformations such as those shown in Figure 4.4 can add symbols and productions to a grammar.
- Grammar-based algorithms typically visit all rules for a given nonterminal or visit all occurrences of a given symbol in the productions.
- Most algorithms process a production's RHS one symbol at a time.

Based on these observations, we represent a production by its LHS symbol and a list of the symbols on its RHS. The empty string λ is not represented explicitly as a symbol. Instead, a production $\mathsf{A} \rightarrow \lambda$ has an empty list of symbols for its RHS. The collection of grammar utilities is as follows.

Grammar(S): Creates a new grammar with start symbol S. The grammar does not yet contain any productions.

Production(A, *rhs*): Creates a new production for nonterminal A and returns a descriptor for the production. The iterator *rhs* supplies the symbols for the production's RHS.

Productions( ): Returns an iterator that visits each of the grammar's productions in no particular order.

Nonterminal(A): Adds A to the set of nonterminals. An error occurs if A is already a terminal symbol. The function returns a descriptor for the nonterminal.

Terminal(x): Adds x to the set of terminals. An error occurs if x is already a nonterminal symbol. The function returns a descriptor for the terminal.

NonTerminals( ): Returns an iterator for the set of nonterminals.

Terminals( ): Returns an iterator for the set of terminal symbols.

IsTerminal($\mathcal{X}$): Returns **true** if $\mathcal{X}$ is a terminal; otherwise, returns **false**.

RHS(p): Returns an iterator for the symbols on the RHS of production p.

LHS(p): Returns the nonterminal defined by production p.

ProductionsFor(A): Returns an iterator that visits each production for nonterminal A.

Occurrences($\mathcal{X}$): Returns an iterator that visits each occurrence of $\mathcal{X}$ in the RHS of all rules.

Production(y): Returns a descriptor for the production $\mathsf{A}\rightarrow\alpha$ where α contains the occurrence y of some vocabulary symbol.

Tail(y): Accesses the symbols appearing after an occurrence. Given a symbol occurrence y in the rule $\mathsf{A}\rightarrow\alpha\ y\ \beta$, Tail($y$) returns an iterator for the symbols in β.

4.5.2 Deriving the Empty String

One of the most common grammar computations determines which nonterminals can derive λ. This information is important because such nonterminals may disappear during a parse and hence must be handled carefully. Determining if a nonterminal can derive λ is not entirely trivial because the derivation can take more than one step:

$$\mathsf{A} \Rightarrow \mathsf{BCD} \Rightarrow \mathsf{BC} \Rightarrow \mathsf{B} \Rightarrow \lambda.$$

```
procedure DerivesEmptyString( )
    foreach A ∈ NonTerminals( ) do
        SymbolDerivesEmpty(A) ← false
    foreach $p$ ∈ Productions( ) do
        RuleDerivesEmpty($p$) ← false
        $Count(p) \leftarrow 0$                                        ①
        foreach $\mathcal{X}$ ∈ RHS($p$) do $Count(p) \leftarrow Count(p) + 1$     ②
        call CheckForEmpty($p$)
    foreach $\mathcal{X}$ ∈ $WorkList$ do                              ③
        $WorkList \leftarrow WorkList - \{\mathcal{X}\}$                ④
        foreach $x$ ∈ Occurrences($\mathcal{X}$) do                   ⑤
            $p$ ← Production($x$)
            $Count(p) \leftarrow Count(p) - 1$
            call CheckForEmpty($p$)
end
procedure CheckForEmpty($p$)
    if $Count(p) = 0$
    then
        RuleDerivesEmpty($p$) ← true                                  ⑥
        A ← LHS($p$)
        if not SymbolDerivesEmpty(A)
        then
            SymbolDerivesEmpty(A) ← true                              ⑦
            $WorkList \leftarrow WorkList \cup \{$A$\}$                ⑧
end
```

Figure 4.7: Algorithm for determining nonterminals and productions that can derive λ.

An algorithm to compute the productions and nonterminals that can derive λ is shown in Figure 4.7. The computation utilizes a *worklist* at Marker ③. A **worklist** is a set that is augmented and diminished as the algorithm progresses. The algorithm is finished when the worklist is empty. Thus, the loop at Marker ③ must account for changes to the set *WorkList*. To prove termination of algorithms that utilize worklists, it must be shown that all worklist elements appear a finite number of times.

In the algorithm shown in Figure 4.7, the worklist contains nonterminals that are discovered to derive λ. The integer $Count(p)$ is initialized at Markers ① and ② to the number of symbols on p's RHS. The count for any production of the form A → λ is 0. Once a production is known to derive λ, its LHS is placed on the worklist at Marker ⑧. When a symbol is taken from the worklist at Marker ④, each occurrence of the symbol is visited at Marker ⑤ and the count

```
function First( $\alpha$ ) returns Set
    foreach A ∈ NonTerminals( ) do  VisitedFirst(A) ← false        ⑨
    ans ← InternalFirst( $\alpha$ )
    return (ans)
end
function InternalFirst( $\mathcal{X}\beta$ ) returns Set
    if $\mathcal{X}\beta = \perp$                                         ⑩
    then  return ($\emptyset$)
    if $\mathcal{X} \in \Sigma$                                          ⑪
    then  return ({ $\mathcal{X}$ })
    /⋆    $\mathcal{X}$ is a nonterminal.                                  ⋆/ ⑫
    ans ← $\emptyset$
    if not VisitedFirst($\mathcal{X}$)
    then
        VisitedFirst($\mathcal{X}$) ← true                                ⑬
        foreach rhs ∈ ProductionsFor($\mathcal{X}$) do
            ans ← ans ∪ InternalFirst( rhs )                      ⑭
    if SymbolDerivesEmpty($\mathcal{X}$)                                  ⑮
    then  ans ← ans ∪ InternalFirst( $\beta$ )
    return (ans)                                                  ⑯
end
```

Figure 4.8: Algorithm for computing First(α).

of the associated production is decremented by 1. This process continues until the worklist is exhausted. The algorithm establishes two structures related to derivations of λ, as follows:

- RuleDerivesEmpty(p) indicates whether or not production p can derive λ. When every symbol in rule p's RHS can derive λ, Marker ⑥ establishes that p can derive λ.
- SymbolDerivesEmpty(A) indicates whether or not the nonterminal A can derive λ. When any production for A can derive λ, Marker ⑦ establishes that A can derive λ.

Both forms of information are useful in the grammar analysis and parsing algorithms discussed in Chapters 4, 5, and 6.

4.5.3 First Sets

A set commonly consulted by parser generators is First(α). This is the set of all terminal symbols that can begin a sentential form derivable from the string of grammar symbols in α. Formally,

$$\mathsf{First}(\alpha) = \{\, \mathsf{a} \in \Sigma \mid \alpha \Rightarrow^{\star} \mathsf{a}\,\beta \,\}$$

Some texts include λ in First(α) if $\alpha \Rightarrow^{\star} \lambda$. Those approaches ultimately require frequent subtraction of λ from symbol sets. We adopt the convention of *never* including λ in First(α), even if $\alpha \Rightarrow^{\star} \lambda$. When the results from the algorithm shown in Figure 4.7 are available, α derives λ if and only if every symbol in α derives λ.

First(α) is computed by scanning α from left to right. If α begins with a terminal symbol a, then clearly First(α) = { a }. If a nonterminal symbol A is encountered, then the grammar productions for A must be consulted. Nonterminals that can derive λ potentially disappear during a derivation, so the computation must account for this as well.

As an example, consider the nonterminals Tail and Prefix from the grammar in Figure 4.1. Each nonterminal has one production that contributes information directly to the nonterminal's First set. Each nonterminal also has a λ-production, which contributes nothing. The solutions are as follows:

$$\begin{array}{rcl} \mathsf{First}(\,\mathsf{Tail}\,) & = & \{\,+\,\} \\ \mathsf{First}(\,\mathsf{Prefix}\,) & = & \{\,\mathsf{f}\,\} \end{array}$$

In some situations, the First set of one symbol can depend on the First sets of other symbols. To compute First(E), the production E → Prefix (E) requires computation of First(Prefix). Because Prefix $\Rightarrow^{\star} \lambda$, First((E)) must also be included. The resulting set: First(E) = { v, f, (}.

The primary computation for the algorithm shown in Figure 4.8 is carried out by the function InternalFirst, whose input argument is the string $\mathcal{X}\beta$. If $\mathcal{X}\beta$ is not empty, then $\mathcal{X}$ is the string's first symbol and β is the rest of the string. InternalFirst then computes its answer as follows:

- The empty set is returned if $\mathcal{X}\beta$ is empty at Marker ⑩. We denote this condition by $\perp$ to emphasize that the empty set is represented by a null list of symbols.

- If $\mathcal{X}$ is a terminal, then First($\mathcal{X}\beta$) is { $\mathcal{X}$ } at Marker ⑪.

- The only remaining possibility is that $\mathcal{X}$ is a nonterminal. If *VisitedFirst*($\mathcal{X}$) is **false**, then the productions for $\mathcal{X}$ are recursively examined for inclusion. Otherwise, $\mathcal{X}$'s productions already participate in the current computation.

- At Marker ⑮ we test if $\mathcal{X}$ can derive λ, as determined previously by the algorithm in Figure 4.7. If $\mathcal{X}$ can derive λ, then we must include all symbols in First(β).

Level	First		*ans*	Marker	Done?	Comment
	$\mathcal{X}$	β			(★=Yes)	
				First(Tail)		
0	Tail	⊥	{ }	⑫		
1	+	E	{ + }	⑪	★	Tail → +E
1	⊥	⊥	{ }	⑩	★	Tail → λ
0			{ + }	⑭		After all rules for Tail
1	⊥	⊥	{ }	⑩	★	Since $\beta = \perp$
0			{ + }	⑮	★	Final answer
				First(Prefix)		
0	Prefix	⊥	{ }	⑫		
1	f	⊥	{ f }	⑪	★	Prefix → f
1	⊥	⊥	{ }	⑩	★	Prefix → λ
0			{ f }	⑭		After all rules for Prefix
1	⊥	⊥	{ }	⑩	★	Since $\beta = \perp$
0			{ f }	⑮	★	Final answer
				First(E)		
0	E	⊥	{ }	⑫		
1	Prefix	(E)	{ }	⑫		E → Prefix (E)
1			{ f }	⑯		Computation shown above
2	(	E)	{ (}	⑪	★	Since Prefix $\Rightarrow^{\star} \lambda$
1			{ f,(}	⑮	★	Results due to E → Prefix (E)
1	v	Tail	{ v }	⑪	★	E → v Tail
1	⊥	⊥	{ }	⑩		Since $\beta = \perp$
0			{ f,(,v }	⑮	★	Final answer

Figure 4.9: First sets for the nonterminals of Figure 4.1.

1 S → A B c
2 A → a
3 | λ
4 B → b
5 | λ

Level	First		*ans*	Marker	Done?	Comment
	$\mathcal{X}$	β			(★=Yes)	
				First(B)		
0	B	⊥	{ }	⑫		
1	b	⊥	{ b }	⑪	★	B → b
1	⊥	⊥	{ }	⑩	★	B → λ
0			{ b }	⑮	★	Final answer
				First(A)		
0	A	⊥	{ }	⑫		
1	a	⊥	{ a }	⑪	★	A → a
1	⊥	⊥	{ }	⑩	★	A → λ
0			{ a }	⑮	★	Final answer
				First(S)		
0	S	⊥	{ }	⑫		
1	A	B c	{ a }	⑯		Computation shown above
2	B	c	{ b }	⑯		Because A $\Rightarrow^{\star} \lambda$; computation shown above
3	c	⊥	{ c }	⑪	★	Because B $\Rightarrow^{\star} \lambda$
2			{ b,c }	⑮	★	
1			{ a,b,c }	⑮	★	
0			{ a,b,c }	⑮	★	

Figure 4.10: A grammar and its First sets.

Figure 4.9 shows the progress of FIRST as it is invoked on the nonterminals of Figure 4.1. The level of recursion is shown in the leftmost column. Each call to FIRST($\mathcal{X}\beta$) is shown with nonblank entries in the $\mathcal{X}$ and β columns. A "★" indicates that the call does not recurse further. Figure 4.10 shows another grammar and the computation of its First sets. For brevity, recursive calls to INTERNALFIRST on null strings are omitted.

Termination of First(A) must be handled properly in grammars where the computation of First(A) appears to depend on First(A), as follows:

$$\begin{array}{c} \mathsf{A} \rightarrow \mathsf{B} \\ \ldots \\ \mathsf{B} \rightarrow \mathsf{C} \\ \ldots \\ \mathsf{C} \rightarrow \mathsf{A} \end{array}$$

In this grammar, First(A) depends on First(B), which depends on First(C), which depends on First(A). In computing First(A), we must avoid endless iteration or recursion. A sophisticated algorithm could preprocess the grammar to determine such cycles of dependence. We leave this as Exercise 19 and present a clearer but slightly less efficient algorithm in Figure 4.8. This algorithm avoids endless computation by remembering which nonterminals have already been visited, as follows:

- First(α) is computed by invoking FIRST(α).
- Before any sets are computed, Marker ⑨ resets *VisitedFirst*(A) for each nonterminal A.
- *VisitedFirst*($\mathcal{X}$) is set at Marker ⑬ to indicate that the productions of A already participate in the computation of First(α).

4.5.4 Follow Sets

Parser-construction algorithms often require the computation of the set of terminals that can *follow* a nonterminal A in some sentential form. Because we augment grammars to contain an end-of-input token ($), every nonterminal except the goal symbol *must* be followed by some terminal. Formally, for A ∈ N,

$$\mathsf{Follow}(\mathsf{A}) = \{\mathsf{b} \in \Sigma \mid \mathsf{S} \Rightarrow^{+} \alpha\ \mathsf{A}\ \mathsf{b}\ \beta\}.$$

Follow(A) provides the **right context** associated with nonterminal A. For example, only those terminals in Follow(A) can occur after a production for A is applied.

```
function Follow( A ) returns Set
    foreach A ∈ NonTerminals( ) do
        VisitedFollow(A) ← false                                  ⑰
    ans ← InternalFollow( A )
    return (ans)
end
function InternalFollow( A ) returns Set
    ans ← ∅
    if not VisitedFolow(A)                                        ⑱
    then
        VisitedFollow(A) ← true                                   ⑲
        foreach a ∈ Occurrences( A ) do                           ⑳
            ans ← ans ∪ First( Tail( a ) )                        ㉑
            if AllDeriveEmpty( Tail( a ) )                        ㉒
            then
                targ ← LHS( Production( a ) )
                ans ← ans ∪ InternalFollow( targ )                ㉓
    return (ans)                                                  ㉔
end
function AllDeriveEmpty( $\gamma$ ) returns Boolean
    foreach $\mathcal{X} \in \gamma$ do
        if not SymbolDerivesEmpty($\mathcal{X}$) or $\mathcal{X} \in \Sigma$
        then return (false)
    return (true)
end
```

Figure 4.11: Algorithm for computing Follow(A).

The algorithm shown in Figure 4.11 computes Follow(A). The primary computation is performed by InternalFollow(A). Each occurrence a of A is visited by the loop at Marker ⑳. Tail(a) is the list of symbols immediately following the occurrence of A.

- Any symbol in First(Tail(a)) can follow A. Marker ㉑ includes such symbols in the returned set.

- Marker ㉒ detects if the symbols in Tail(a) can derive λ. This situation arises when there are no symbols appearing after this occurrence of A or when the symbols appearing after A can each derive λ. In either case, Marker ㉓ includes the Follow set of the current production's LHS.

Many aspects of this algorithm are similar to the First(α) algorithm given in Figure 4.8.

Level	Rule	Marker	Result	Comment
		Follow(B)		
0			Follow(B)	
0	S→A $\underline{\text{B}}$ c	㉑	{c}	
0		㉔	{c}	Returns
		Follow(A)		
0			Follow(A)	
0	S→ $\underline{\text{A}}$ B c	㉑	{b,c}	
0		㉔	{b,c}	Returns
		Follow(S)		
0			Follow(S)	
0		㉔	{ }	Returns

Figure 4.12: Follow sets for the grammar in Figure 4.10. Note that Follow(S) = { } because S does not appear on the RHS of any production.

- Before any sets are computed, Marker ⑰ resets *VisitedFollow*(A) for each nonterminal A.
- *VisitedFollow*(A) is set at Marker ⑲ to indicate that the symbols following A are already participating in this computation.

Figure 4.12 shows the progress of Follow as it is invoked on the nonterminals of Figure 4.10. As another example, Figure 4.13 shows the computation of Follow sets for the grammar in Figure 4.1.

First and Follow sets can be generalized to include strings of length k rather than length 1. First$_k(\alpha)$ is the set of k-symbol terminal prefixes derivable from α. Similarly, Follow$_k$(A) is the set of k-symbol terminal strings that can follow A in some sentential form. First$_k$ and Follow$_k$ are used in the definition of parsing techniques that use k-symbol lookaheads (for example, LL(k) and LR(k)). The algorithms that compute First$_1(\alpha)$ and Follow$_1$(A) can be generalized to compute First$_k(\alpha)$ and Follow$_k$(A) sets (see Exercise 26).

This ends our discussion of CFGs and grammar-analysis algorithms. The First and Follow sets introduced in this chapter play an important role in the automatic construction of LL and LR parsers, as discussed in Chapters 5 and 6, respectively.

Level	Rule	Marker	Result	Comment
		Follow(Prefix)		
0			Follow(Prefix)	
0	$\mathsf{E} \rightarrow \underline{\mathsf{Prefix}}\ (\ \mathsf{E}\ )$	㉑	{ (}	
		Follow(E)		
0			Follow(E)	
0	$\mathsf{E} \rightarrow \mathsf{Prefix}\ (\ \underline{\mathsf{E}}\ )$	㉑	{) }	
0	$\mathsf{Tail} \rightarrow +\ \underline{\mathsf{E}}$	㉓	{ }	
1			Follow(Tail)	
1	$\mathsf{E} \rightarrow \mathsf{v}\ \underline{\mathsf{Tail}}$	㉓	{ }	
2			Follow(E)	
		⑱	{ }	Recursion avoided
1		㉔	{ }	Returns
0		㉔	{) }	Returns
		Follow(Tail)		
0			Follow(Tail)	
0	$\mathsf{E} \rightarrow \mathsf{v}\ \underline{\mathsf{Tail}}$	㉓	{ }	
1			Follow(E)	
1	$\mathsf{E} \rightarrow \mathsf{Prefix}\ (\ \underline{\mathsf{E}}\ )$	㉑	{) }	
1	$\mathsf{Tail} \rightarrow +\ \underline{\mathsf{E}}$	㉓	{ }	
2			Follow(Tail)	
		⑱	{ }	Recursion avoided
1		㉔	{) }	Returns
0		㉔	{) }	Returns

Figure 4.13: Follow sets for the nonterminals of Figure 4.1.

Exercises

1. While ambiguity is avoided in programming languages, (some) humor can be derived from ambiguity in natural languages. For each of the following English sentences, explain why it is ambiguous. First try to determine multiple grammar diagrams for the sentence. If only one such diagram exists, explain why the meaning of the words makes the sentence ambiguous.

   (a) I saw an elephant in my pajamas.

   (b) I cannot recommend this student too highly.

   (c) I saw her duck.

   (d) Students avoid boring professors.

   (e) Milk drinkers turn to powder.

2. In some programming languages, the same symbol can have different meanings in the same statement. For example, PL/I allows the statement

   ```
   IF IF = THEN THEN = ELSE; ELSE ELSE = END; END
   ```

   For the following bizarre English sentence, determine the role of each "buffalo" by analyzing the grammar of the sentence:

   Buffalo buffalo Buffalo buffalo buffalo buffalo Buffalo buffalo.

3. Transform the following grammar into a standard CFG using the algorithm in Figure 4.4:

   ```
   1  S       → Number
   2  Number  →  [ Sign ] [ Digs period ] Digs
   3  Sign    → plus
   4           |  minus
   5  Digs    →  digit { digit }
   ```

4. Design a language and context-free grammar to represent the following languages:

   (a) The set of strings of base-8 numbers

   (b) The set of strings of base-16 numbers

   (c) The set of strings of base-1 numbers

   (d) A language that offers base-8, base-16, and base-1 numbers

5. Describe the language denoted by each of the following grammars:

   (a) $(\{\mathsf{A},\mathsf{B},\mathsf{C}\},\{\mathsf{a},\mathsf{b},\mathsf{c}\},\emptyset,\mathsf{A})$

   (b) $(\{\mathsf{A},\mathsf{B},\mathsf{C}\},\{\mathsf{a},\mathsf{b},\mathsf{c}\},\{\mathsf{A}\rightarrow\mathsf{B\ C}\},\mathsf{A})$

   (c) $(\{\mathsf{A},\mathsf{B},\mathsf{C}\},\{\mathsf{a},\mathsf{b},\mathsf{c}\},\{\mathsf{A}\rightarrow\mathsf{A\ a},\mathsf{A}\rightarrow\mathsf{b}\},\mathsf{A})$

   (d) $(\{\mathsf{A},\mathsf{B},\mathsf{C}\},\{\mathsf{a},\mathsf{b},\mathsf{c}\},\{\mathsf{A}\rightarrow\mathsf{B\ B},\mathsf{B}\rightarrow\mathsf{a},\mathsf{B}\rightarrow\mathsf{b},\mathsf{B}\rightarrow\mathsf{c}\},\mathsf{A})$

6. What are the difficulties associated with constructing a grammar whose generated strings are decimal representations of irrational numbers?

7. A grammar for infix expressions follows:

```
1  Start → E  $
2  E     → T  plus  E
3         |  T
4  T     → T  times  F
5         |  F
6  F     → (  E  )
7         |  num
```

   (a) Show the leftmost derivation of the following string.

   num plus num times num plus num $

   (b) Show the rightmost derivation of the following string.

   num times num plus num times num $

   (c) Describe how this grammar structures expressions, in terms of the precedence and left- or right- associativity of operators.

8. Consider the following two grammars.

   (a)

```
1  Start → E  $
2  E     → (  E  plus  E
3         |  num
```

   (b)

```
1  Start → E  $
2  E     → E  (  plus  E
3         |  num
```

   Which of these grammars, if any, is ambiguous? Prove your answer by showing two distinct derivations of some input string for the ambiguous grammar(s).

9. Compute First and Follow sets for the nonterminals of the following grammar.

1	S	→	a S e
2		|	B
3	B	→	b B e
4		|	C
5	C	→	c C e
6		|	d

10. Compute First and Follow sets for each nonterminal in ac grammar from Chapter 2, reprised as follows.

1	Prog	→	Dcls Stmts $
2	Dcls	→	Dcl Dcls
3		|	λ
4	Dcl	→	floatdcl id
5		|	intdcl id
6	Stmts	→	Stmt Stmts
7		|	λ
8	Stmt	→	id assign Val ExprTail
9		|	print id
10	ExprTail	→	plus Val ExprTail
11		|	minus Val ExprTail
12		|	λ
13	Val	→	id
14		|	num

11. Compute First and Follow sets for each nonterminal in Exercise 3.

12. As discussed in Section 4.3, the algorithm in Figure 4.4 could use left or right recursion to transform a repeated sequence of symbols into standard grammar form. A production of the form $A \to A\ \alpha$ is said to be **left recursive**. Similarly, a production of the form $A \to \beta\ A$ is said to be **right recursive**. Show that any grammar that contains left- and right-recursive rules for the same LHS nonterminal must be ambiguous.

13. Section 4.3 describes extended BNF notation for optional and repeated symbol sequences. Suppose the n grammar symbols $\mathcal{X}_1 \ldots \mathcal{X}_n$ represent a set of n options. What is the effect of the following grammar with regard to how the options can appear?

$$\begin{array}{lcl} \text{Options} & \rightarrow & \text{Options Option} \\ & | & \lambda \\ \text{Option} & \rightarrow & \mathcal{X}_1 \\ & | & \mathcal{X}_2 \\ & & \quad \ldots \\ & | & \mathcal{X}_n \end{array}$$

14. Consider n optional symbols $\mathcal{X}_1 \ldots \mathcal{X}_n$ as described in Exercise 13.

    (a) Devise a CFG that generates any subset of these options. That is, the symbols can occur in any order, any symbol can be missing, and no symbol is repeated.

    (b) How does n (the number of options) affect the size of your grammar?

    (c) How is your solution affected if symbols $\mathcal{X}_i$ and $\mathcal{X}_j$ are present only if $i < j$?

15. Referring to Section 4.1.4, show that regular grammars and finite automata (Chapter 3) have equivalent power by developing

    (a) an algorithm that translates regular grammars into finite automata and

    (b) an algorithm that translates finite automata into regular grammars.

16. Referring to Section 4.2.1, devise an algorithm to detect nonterminals that cannot be reached from a CFG's goal symbol.

17. Referring to Section 4.2.1, devise an algorithm to detect nonterminals that cannot derive any terminal string in a CFG.

18. A CFG is reduced by removing useless terminals and productions. Consider the following two tasks.

    (a) Nonterminals not reachable from the grammar's goal symbol are removed (Exercise 16).

    (b) Nonterminals that derive no terminal string are removed (Exercise 17).

    Does the order of the above tasks matter? If so, which order is preferred?

19. The algorithm presented in Figure 4.8 retains no information between invocations of First. As a result, the solution for a given nonterminal might be computed multiple times.
    (a) Modify the algorithm so it remembers and references valid previous computations of First(A), $A \in N$.
    (b) Frequently an algorithm needs First sets computed for *all* $\mathcal{X} \in N$. Devise an algorithm that efficiently computes First sets for all nonterminals in a grammar. Analyze the efficiency of your algorithm.
    Hint: Consider constructing a directed graph whose vertices represent nonterminals. Let an edge (A, B) represent that First(B) depends on First(A).
    (c) Repeat this exercise for the Follow sets.

20. Prove that First(A) correctly computes First(A) for any $A \in N$.

21. Prove that Follow(A) correctly computes Follow(A) for any $A \in N$.

22. Let G be any CFG and assume $\lambda \notin L(G)$. Show that G can be transformed into a language-equivalent CFG that uses no λ-productions.

23. A **unit production** is a rule of the form $A \rightarrow B$. Show that any CFG that contains unit productions can be transformed into a language-equivalent CFG that uses no unit productions.

24. Some CFGs denote a language with an infinite number of strings; others denote finite languages. Devise an algorithm that determines whether a given CFG generates an infinite language.
    Hint: Use the results of Exercises 22 and 23 to simplify the analysis.

25. Let G be an unambiguous CFG without λ-productions.
    (a) If $x \in L(G)$, show that the number of steps needed to derive x is linear in the length of x.
    (b) Does this linearity result hold if λ-productions are included?
    (c) Does this linearity result hold if G is ambiguous?

26. The algorithms in Figures 4.8 and 4.11 compute First(α) and Follow(A).
    (a) Modify the algorithm in Figure 4.8 to compute $\text{First}_k(\alpha)$.
    Hint: Consider formulating the algorithm so that when $\text{First}_i(\alpha)$ is computed, enough information is retained to compute $\text{First}_{i+1}(\alpha)$.
    (b) Modify the algorithm in Figure 4.11 to compute $\text{Follow}_k(A)$.

5

Top-Down Parsing

Chapter 2 presents a *recursive-descent parser* for the syntax analysis phase of a small compiler. Manual construction of such parsers is both time consuming and error prone, especially when applied at the scale of a real programming language. At first glance, the code for a recursive-descent parser may appear to be written ad hoc. Fortunately, there *are* principles at work. This chapter discusses these principles and their applications in tools that automate the parsing phase of a compiler.

Recursive-descent parsers belong to the more general class of top-down (also called LL) parsers, which were introduced in Chapter 4. In this chapter, we discuss top-down parsers in greater detail, analyzing the conditions under which such parsers can be reliably and automatically constructed from grammars. Our analysis builds on the algorithms and grammar-processing concepts presented in Chapter 4.

Top-down parsers are in theory not as powerful as the bottom-up parsers we study in Chapter 6. However, because of their simplicity, performance, and excellent error diagnostics, top-down parsers have been constructed for many programming languages, almost always using the recursive-descent approach. Such parsers are also convenient for prototyping relatively simple front-ends of larger systems that require a rigorous definition and treatment of the system's input.

5.1 Overview

In this chapter, we study the following two forms of top-down parsers:

- **Recursive-descent parsers** contain a set of mutually recursive procedures that cooperate to parse a string. Code for these procedures can be written directly from a suitable grammar.
- **Table-driven LL parsers** use a generic LL(k) parsing engine and a parse table that directs the activity of the engine. The entries for the parse table are determined by the particular LL(k) grammar. The notation LL(k) is explained below.

Fortunately, **context-free grammars** (CFGs) with certain properties can be used to generate such parsers automatically. Tools that operate in this fashion are generally called **compiler compilers** or **parser generators**. They take a grammar description file as input and attempt to produce a parser for the language defined by the grammar. The term "compiler compiler" applies because the parser generator is *itself a compiler*: it accepts a high-level expression of a program (the grammar definition file) and generates an executable form of that program (the parser). This approach makes parsing one of the easiest and most reliable phases of compiler construction for the following reasons:

- When the grammar serves as a language's definition, parsers can be automatically constructed to perform syntax analysis in a compiler. The rigor of the automatic construction guarantees that the resulting parser is faithful to the language's syntactic specification.
- When a language is revised, updated, or extended, the associated modifications can be applied to the grammar description to generate a parser for the new language.
- When parser construction is successful through the techniques described in this chapter, the grammar is proved unambiguous. While devising an algorithmic test for grammar ambiguity is impossible, parser-construction techniques are of great use to language designers in developing intuition as to why a grammar might be ambiguous.

As discussed in Chapters 2 and 4, every string in a grammar's language can be generated by a derivation that begins with the grammar's start symbol. While it is relatively straightforward to use a grammar's productions to generate sample strings in its language, reversing this process does not seem as simple. That is, given an input string, how can we show why the string is or is not in the grammar's language? This is the *parsing problem*, and in this chapter we consider a parsing technique that is successful with many CFGss. This parsing technique is known by the following names:

- Top-down, because the parser begins with the grammar's start symbol and grows a parse tree from its root to its leaves.
- Predictive, because the parser must predict at each step in the derivation which grammar rule is to be applied next.
- LL(*k*), because these techniques scan the input from left to right (the first "L" of LL), produce a leftmost derivation (the second "L" of LL), and use *k* symbols of lookahead.
- Recursive descent, because this kind of parser can be implemented by a collection of mutually recursive procedures.

In Section 5.2, we identify a subset of CFGss known as the LL(*k*) grammars. In Sections 5.3 and 5.4, we show how to construct recursive-descent and table-driven LL parsers from LL(1) grammars—an efficient subset of the LL(*k*) grammars. For grammars that are not LL(1), Section 5.5 considers grammar transformations that can eliminate non-LL(1) properties. Unfortunately, some languages have no LL(*k*) grammar, as discussed in Section 5.6. Section 5.7 establishes some useful properties of LL grammars and parsers. Parse table representations are considered in Section 5.8. Because parsers are typically responsible for discovering syntax errors in programs, Section 5.9 considers how an LL(*k*) parser might respond to syntactically faulty inputs.

5.2 LL(*k*) Grammars

The following is a reprise from Chapter 2 of the process for constructing a **recursive-descent parser** from a CFGs.

- A **parsing procedure** is associated with each nonterminal A.
- The procedure associated with A is charged with accomplishing one step of a derivation by choosing and applying one of A's productions.
- The parser chooses the appropriate production for A by inspecting the next *k* tokens (terminal symbols) in the input stream. The **Predict set** for production A→α is the set of tokens that trigger application of that production.
- The Predict set for A→α is determined primarily by the detail in α—the **right-hand side** (RHS) of the production. Other CFGs productions may participate in the computation of a production's Predict set.

Generally, the choice of production can be predicated on the next *k* tokens of input, for some constant *k* chosen before the parser is pressed into service.

These k tokens are called the **lookahead** of an $\mathsf{LL}(k)$ parser. If it is possible to construct an $\mathsf{LL}(k)$ parser for a CFGs such that the parser recognizes the CFGs's language, then the CFGs is an **$\mathsf{LL}(k)$ grammar**.

An $\mathsf{LL}(k)$ parser can peek at the next k tokens to decide which production to apply. However, the *strategy* for choosing productions must be established when the parser is constructed. In this section, the strategy is formalized by defining a function called $\mathsf{Predict}_k(p)$. This function considers the grammar production p and computes the set of length-k token strings that predict the application of rule p. We assume henceforth that we have one token of lookahead ($k = 1$). The generalization is left to the reader as Exercise 16. Thus, for rule p, $\mathsf{Predict}(p)$ is the set of terminal symbols (i.e., length-1 strings) that call for applying rule p.

Consider a parser that is presented with the input string $\alpha \mathsf{a} \beta \in \Sigma^\star$. Suppose the parser has constructed the derivation $\mathsf{S} \Rightarrow^\star_{\text{lm}} \alpha\ \mathsf{A} \mathcal{Y}_1 \ldots \mathcal{Y}_n$. At this point, α has been matched and A is the leftmost nonterminal in the derived sentential form. Thus, *some* production for A must be applied to continue the leftmost derivation. Because the input string contains an a as the next input token, the parse must continue with a production for A that derives a as its first terminal symbol.

Recalling the notation from Section 4.5.1 on page 127, we must examine the set of productions

$$P = \{ p \in \textsc{ProductionsFor}(\mathsf{A}) \mid a \in \mathsf{Predict}(p) \}$$

One of the following conditions must be true of the set P and the next input token a:

- P is the empty set. In this case, no production for A can cause the next input token to be matched. The parse cannot continue and a syntax error is issued, with a as the offending token. The productions for A can be helpful in issuing error messages that indicate which terminal symbols could be processed at this point in the parse. Section 5.9 considers error recovery and repair in greater detail.

- P contains more than one production. In this case, the parse could continue, but **nondeterminism** would be required to pursue the independent application of each production in P. For efficiency, we require that our parsers operate deterministically. Thus parser construction must ensure that this case cannot arise.

- P contains exactly one production. In this case, the leftmost parse can proceed deterministically by applying the only production in set P.

A grammar can be analyzed to determine whether each terminal symbol predicts (at most) one of the rules for the nonterminal A. If such analysis holds

```
function Predict($p : \mathsf{A} \rightarrow \mathcal{X}_1 \ldots \mathcal{X}_m$) : Set
    $ans \leftarrow$ First($\mathcal{X}_1 \ldots \mathcal{X}_m$)                          ①
    if RuleDerivesEmpty($p$)                                   ②
    then
        $ans \leftarrow ans \cup$ Follow(A)                             ③
    return ($ans$)
end
```

Figure 5.1: Computation of Predict sets.

for all nonterminal symbols in a grammar, then a deterministic parser can be constructed and the associated grammar is determined to be LL(1).

We next consider a rule p in greater detail and show how to compute Predict(p). Consider a production $p : \mathsf{A} \rightarrow \mathcal{X}_1 \ldots \mathcal{X}_m, m \geq 0$. When $m = 0$, there are no symbols on A's RHS, which is equivalent by convention to the rule $\mathsf{A} \rightarrow \lambda$. As shown in Figure 5.1, the set of symbols that predict rule p is drawn from one or both of the following:

- The set of possible terminal symbols that are first produced in some derivation from $\mathcal{X}_1 \ldots \mathcal{X}_m$
- Those terminal symbols that can follow A in some sentential form

At Marker ①, the algorithm of Figure 5.1 initializes *ans* to **First(**$\mathcal{X}_1 \ldots \mathcal{X}_m$**)**, which is the set of terminal symbols that can appear first (leftmost) in any derivation of $\mathcal{X}_1 \ldots \mathcal{X}_m$. The algorithm for computing this set is given in Figure 4.8 on page 130. Marker ② detects when $\mathcal{X}_1 \ldots \mathcal{X}_m \Rightarrow^\star \lambda$, using the results of the algorithm presented in Figure 4.7 on page 129. RuleDerivesEmpty(p) is true if, and only if, production p can derive λ. In this case, Marker ③ includes those symbols in **Follow(A)**, as computed by the algorithm in Figure 4.11 on page 135. Such symbols can *follow* A after $\mathsf{A} \Rightarrow^\star \lambda$. Thus, the function shown in Figure 5.1 computes the set of length-1 token strings that predict rule p. By convention, λ is *not* a terminal symbol, so it does not participate in any Predict set.

In an LL(1) grammar, the productions for each nonterminal A must have **disjoint predict sets**, as computed with one symbol of lookahead. Most programming languages have LL(1) grammars, but there are some constructs that requires special attention (Section 5.6). However, *not all* CFGss are LL(1). For such grammars, the following may apply:

- More lookahead may be needed, in which case the grammar is LL(k) for some constant $k > 1$.
- A more powerful parsing method may be required. Chapter 6 describes such methods, but they also have limits in their applicability.

```
1  S → A C $
2  C → c
3    | λ
4  A → a B C d
5    | B Q
6  B → b B
7    | λ
8  Q → q
9    | λ
```

Figure 5.2: A CFGs.

Rule	A	$\mathcal{X}_1 \ldots \mathcal{X}_m$	First($\mathcal{X}_1 \ldots \mathcal{X}_m$)	Derives	Follow(A)	Answer
Number				Empty?		
1	S	A C $	a,b,q,c,$	No		a,b,q,c,$
2	C	c	c	No		c
3		λ		Yes	d,$	d,$
4	A	a B C d	a	No		a
5		B Q	b,q	Yes	c,$	b,q,c,$
6	B	b B	b	No		b
7		λ		Yes	q,c,d,$	q,c,d,$
8	Q	q	q	No		q
9		λ		Yes	c,$	c,$

Figure 5.3: Predict calculation for the grammar of Figure 5.2.

- The grammar may be **ambiguous**, allowing multiple, distinct derivations of some string. Such grammars cannot be accommodated by *any* deterministic parsing method.

Finally, as discussed in Section 5.6, there are some languages for which no LL(k) grammar is possible (see also Exercise 26).

We now apply the algorithm in Figure 5.1 to the grammar shown in Figure 5.2. Figure 5.3 shows the Predict calculations. For each production of the form $\mathsf{A} \rightarrow \mathcal{X}_1 \ldots \mathcal{X}_m$, the fourth column shows First($\mathcal{X}_1 \ldots \mathcal{X}_m$). The next column indicates whether $\mathcal{X}_1 \ldots \mathcal{X}_m \Rightarrow^\star \lambda$. The rightmost column shows Predict($\mathsf{A} \rightarrow \mathcal{X}_1 \ldots \mathcal{X}_m$)—the set of symbols that predict the production $\mathsf{A} \rightarrow \mathcal{X}_1 \ldots \mathcal{X}_m$. This set includes First($\mathcal{X}_1 \ldots \mathcal{X}_m$), as well as Follow(A) if $\mathcal{X}_1 \ldots \mathcal{X}_m \Rightarrow^\star \lambda$.

```
function IsLL1( $G$ ) returns Boolean
    foreach $A \in N$ do
        PredictSet ← ∅
        foreach $p \in$ ProductionsFor(A) do
            if Predict( $p$ ) ∩ PredictSet ≠ ∅                    ④
            then return (false)
            PredictSet ← PredictSet ∪ Predict( $p$ )
    return (true)
end
```

Figure 5.4: Algorithm to determine if a grammar G is LL(1).

```
procedure MATCH( ts, token )
    if ts.PEEK( ) = token
    then  call ts.ADVANCE( )
    else  call ERROR( Expected token )
end
```

Figure 5.5: Utility for matching tokens in an input stream.

The algorithm shown in Figure 5.4 determines whether a grammar is LL(1) based on the grammar's Predict sets. The Predict sets for each nonterminal A are checked for intersection. If no two rules for A have any prediction symbols in common, then the grammar is LL(1). The grammar of Figure 5.2 passes this test, and is therefore LL(1).

5.3 Recursive-Descent LL(1) Parsers

We are now prepared to generate the procedures of a recursive-descent parser. The parser's input is a sequence of tokens provided by the stream *ts*. We assume that *ts* offers the following methods:

- PEEK, which examines the next input token without advancing the input
- ADVANCE, which advances the input by one token

The parsers we construct rely on the MATCH method shown in Figure 5.5. This method checks the token stream *ts* for the presence of a particular token.

To construct a recursive-descent parser for an LL(1) grammar, we write a separate procedure for each nonterminal A. If A has rules $p_1, p_2, \ldots, p_n$, then we formulate the procedure shown in Figure 5.6. The code constructed for each p_i is obtained by scanning the RHS of rule p_i (i.e., symbols $\mathcal{X}_1 \ldots \mathcal{X}_m$) from left

```
procedure A(ts)
    switch (...)
        case ts.peek( ) $\in$ Predict($p_1$)
            /*   Code for $p_1$                                  */
        case ts.peek( ) $\in$ Predict($p_i$)
            /*   Code for $p_2$                                  */
        /*   .                                                   */
        /*   .                                                   */
        /*   .                                                   */
        case ts.peek( ) $\in$ Predict($p_n$)
            /*   Code for $p_n$                                  */
        case default
            /*   Syntax error                                    */
end
```

Figure 5.6: A typical recursive-descent procedure. Successful LL(1) analysis ensures that only one of the **case** predicates is **true**.

to right. As each symbol is visited, code is written into the parsing procedure. For productions of the form $A \rightarrow \lambda$, $m = 0$, so there are no symbols to visit. In such cases, the parsing procedure simply returns immediately. In considering each X_i, there are two possible cases, as follows.

- X_i is a terminal symbol. In this case, a call to MATCH(ts, X_i) is written into the parser to insist that X_i is the next symbol in the token stream. If the token is successfully matched, then the token stream is advanced. Otherwise, the input string cannot be in the grammar's language and an error message is issued.

- X_i is a nonterminal symbol. In this case, there is a procedure responsible for continuing the parse by choosing an appropriate production for X_i. Thus, a call to $X_i(ts)$ is written into the parser.

Figure 5.7 shows the parsing procedures created for the LL(1) grammar shown in Figure 5.2. For presentation purposes, the default case (representing a syntax error) is not shown in the parsing procedures of Figure 5.7.

5.4 Table-Driven LL(1) Parsers

The task of creating recursive-descent parsers as presented in Section 5.3 is mechanical and can therefore be automated. However, the size of the parser's

```
procedure S( )
    switch (. . .)
        case ts.PEEK( ) ∈ { a, b, q, c, $ }
            call A( )
            call C( )
            call MATCH($)
end
procedure C( )
    switch (. . .)
        case ts.PEEK( ) ∈ { c }
            call MATCH(c)
        case ts.PEEK( ) ∈ { d, $ }
            return ()
end
procedure A( )
    switch (. . .)
        case ts.PEEK( ) ∈ { a }
            call MATCH(a)
            call B( )
            call C( )
            call MATCH(d)
        case ts.PEEK( ) ∈ { b, q, c, $ }
            call B( )
            call Q( )
end
procedure B( )
    switch (. . .)
        case ts.PEEK( ) ∈ { b }
            call MATCH(b)
            call B( )
        case ts.PEEK( ) ∈ { q, c, d, $ }
            return ()
end
procedure Q( )
    switch (. . .)
        case ts.PEEK( ) ∈ { q }
            call MATCH(q)
        case ts.PEEK( ) ∈ { c, $ }
            return ()
end
```

Figure 5.7: Recursive-descent code for the grammar shown in Figure 5.2. The variable *ts* denotes the token stream produced by the scanner.

code grows with the size of the grammar. Moreover, the overhead of method calls and returns can be a source of inefficiency. In this section we examine how to construct a table-driven LL(1) parser. Actually, the parser itself is standard across all grammars, so we need only provide an adequate parse table.

To make the transition from explicit code to table-driven processing, we use a stack to simulate the actions performed by MATCH and by the calls to the nonterminals' procedures. In addition to the methods typically provided by a stack, we assume that the top-of-stack contents can be obtained nondestructively (without popping the stack) via the method TOS.

In code form, the generic LL(1) parser is given in Figure 5.8. At each iteration of the loop at Marker ⑤, the parser performs one of the following actions:

- If the top-of-stack is a terminal symbol, then MATCH is called. This method, defined in Figure 5.5, ensures that the next token of the input stream matches the top-of-stack symbol. If successful, the call to MATCH advances the token input stream. For the table-driven parser, the matching top-of-stack symbol is popped at Marker ⑨.

- If the top-of-stack is some nonterminal symbol A, then the appropriate production $\mathsf{A} \rightarrow \mathcal{X}_1 \ldots \mathcal{X}_m$ is determined by table lookup at Marker ⑩. If a valid production is found, then APPLY is called to pop the A from the top of the stack (Marker ⑪). The symbols $\mathcal{X}_1 \ldots \mathcal{X}_m$ are then pushed at Marker ⑫ onto the stack starting with $\mathcal{X}_m$, so that the resulting top-of-stack is $\mathcal{X}_1$.

The parse is complete when the end-of-input symbol is matched at Marker ⑧.

Given a CFGs that has passed the IsLL1 test in Figure 5.4, we next examine how to build its LL(1) parse table. The rows and columns of the parse table are labeled by the nonterminals and terminals of the CFGs, respectively. The table, consulted at Marker ⑩ in Figure 5.8, is indexed by the top-of-stack symbol (obtained by the TOS( ) call) and by the next input token (obtained by the *ts*.PEEK( ) call).

Each nonblank entry in a row is a production that has the row's nonterminal as its **left-hand side** (LHS) symbol. A production is typically represented by its rule number in the grammar. The table is used as follows:

- The nonterminal symbol at the top-of-stack determines which row is chosen.

- The next input token (i.e., the **lookahead**) determines which column is chosen.

```
procedure LLparser( $ts$ )
    call push( S )
    $accepted \leftarrow$ false
    while not $accepted$ do                                        (5)
        if TOS( ) $\in \Sigma$                                     (6)
        then
            call match( $ts$, TOS( ) )                             (7)
            if TOS( ) = \$                                         (8)
            then  $accepted \leftarrow$ true
            call pop( )                                            (9)
        else
            $p \leftarrow LLtable$[TOS( ), $ts$.peek( )]           (10)
            if $p = 0$
            then
                call error( Syntax error—no production applicable )
            else  call apply( $p$ )
end
procedure apply( $p : A \rightarrow \mathcal{X}_1 \ldots \mathcal{X}_m$ )
    call pop( )                                                    (11)
    for $i = m$ downto 1 do                                        (12)
        call push( $\mathcal{X}_i$ )
end
```

Figure 5.8: Generic LL(1) parser.

```
procedure FillTable( $LLtable$ )
    foreach A $\in N$ do
        foreach a $\in \Sigma$ do  $LLtable$[A][a] $\leftarrow 0$
    foreach A $\in N$ do
        foreach $p \in ProductionsFor$(A) do
            foreach a $\in$ Predict($p$) do  $LLtable$[A][a] $\leftarrow p$
end
```

Figure 5.9: Construction of an LL(1) parse table.

	Lookahead					
Nonterminal	a	b	c	d	q	$
S	1	1	1		1	1
C			2	3		3
A	4	5	5		5	5
B		6	7	7	7	7
Q			9		8	9

Figure 5.10: LL(1) table. The blank entries should trigger error actions in the parser.

The resulting entry indicates which, if any, production of the CFGs should be applied at this point in the parse.

For practical purposes, the nonterminals and terminals should be mapped to small integers to facilitate table lookup using a two-dimensional array. The procedure for constructing the parse table is shown in Figure 5.9. Upon the procedure's completion, any entry marked 0 will represent a terminal symbol that does not predict any production for the associated nonterminal. Thus, if a 0 entry is accessed during parsing, then the input string contains an error.

Using the grammar shown in Figure 5.2 and its associated Predict sets shown in Figure 5.3, we construct the LL(1) parse table shown in Figure 5.10. The table's contents are the rule numbers for productions as shown in Figure 5.2, with blanks rather than zeros to represent errors.

Finally, using the parse table shown in Figure 5.10, we trace the behavior of an LL(1) parser on the input string a b b d c $ in Figure 5.11.

5.5 Obtaining LL(1) Grammars

It can be difficult for inexperienced compiler writers to create LL(1) grammars. This is because LL(1) requires a unique prediction for each combination of nonterminal and lookahead symbols. It is easy to write productions that violate this requirement.

Fortunately, most LL(1) prediction conflicts can be grouped into two categories: *common prefixes* and *left recursion*. Simple grammar transformations that eliminate common prefixes and left recursion are described below, and these transformations allow us to obtain LL(1) form for most CFGss. However, there are some languages of interest for which no LL(1) grammar can be constructed (see Section 5.6).

Parse Stack	Action	Remaining Input
S		abbdc$
	Apply 1: S→AC$	
$CA		abbdc$
	Apply 4: A→aBCd	
$CdCBa		abbdc$
	Match	
$CdCB		bbdc$
	Apply 6: B→bB	
$CdCBb		bbdc$
	Match	
$CdCB		bdc$
	Apply 6: B→bB	
$CdCBb		bdc$
	Match	
$CdCB		dc$
	Apply 7: B→λ	
$CdC		dc$
	Apply 3: C→λ	
$Cd		dc$
	Match	
$C		c$
	Apply 2: C→c	
$c		c$
	Match	
$		$
	Accept	

Figure 5.11: Trace of an LL(1) parse. The stack is shown in the left column, with top-of-stack as the rightmost character. The input string is shown in the right column, processed from left to right.

```
1  Stmt     → if Expr then StmtList endif
2            | if Expr then StmtList else StmtList endif
3  StmtList → StmtList ; Stmt
4            | Stmt
5  Expr     → var + Expr
6            | var
```

Figure 5.12: A grammar with common prefixes.

```
procedure Factor( )
    foreach $A \in N$ do
        $\alpha \leftarrow LongestCommonPrefix(ProductionsFor(A))$
        while $|\alpha| > 0$ do
            $V \leftarrow$ new NonTerminal( )
            $Productions \leftarrow Productions \cup \{ A \rightarrow \alpha V \}$
            foreach $p \in ProductionsFor(A) \mid RHS(p) = \alpha\beta_p$ do        ⑬
                $Productions \leftarrow Productions - \{ p \}$
                $Productions \leftarrow Productions \cup \{ V \rightarrow \beta_p \}$
            $\alpha \leftarrow LongestCommonPrefix(ProductionsFor(A))$
end
```

Figure 5.13: Factoring common prefixes.

5.5.1 Common Prefixes

In this category of conflicts, two productions for the same nonterminal share a **common prefix** if the productions' RHSs begin with the same string of grammar symbols. For the grammar shown in Figure 5.12, both Stmt productions are predicted by the if token. Even if we allow greater lookahead, the else that distinguishes the two productions can lie arbitrarily far ahead in the input: Expr and StmtList can each generate a terminal string larger than any constant k. The grammar in Figure 5.12 is therefore not LL(k) for any k.

Prediction conflicts caused by common prefixes can be remedied by the simple factoring transformation shown in Figure 5.13. At Marker ⑬ in this algorithm, a production is identified whose RHS shares a common prefix α with other productions. The remainder of the RHS is denoted β_p for production p. With the common prefix factored and placed into a new production for A, each production sharing α is stripped of this common prefix. Applying the algorithm in Figure 5.13 to the grammar in Figure 5.12 produces the grammar in Figure 5.14.

1	Stmt	$\rightarrow$	if Expr then StmtList V_1
2	V_1	$\rightarrow$	endif
3		|	else StmtList endif
4	StmtList	$\rightarrow$	StmtList ; Stmt
5		|	Stmt
6	Expr	$\rightarrow$	var V_2
7	V_2	$\rightarrow$	+ Expr
8		|	λ

Figure 5.14: Factored version of the grammar in Figure 5.12.

```
procedure EliminateLeftRecursion( )
    foreach $A \in N$ do
        if $\exists\, r \in ProductionsFor(A) \mid RHS(r) = A\alpha$
        then
            $X \leftarrow$ new NonTerminal( )
            $Y \leftarrow$ new NonTerminal( )
            foreach $p \in ProductionsFor(A)$ do
                if $p = r$
                then $Productions \leftarrow Productions \cup \{ A \rightarrow X\,Y \}$
                else $Productions \leftarrow Productions \cup \{ X \rightarrow RHS(p) \}$
            $Productions \leftarrow Productions \cup \{ Y \rightarrow \alpha Y, Y \rightarrow \lambda \}$
end
```

Figure 5.15: Eliminating left recursion.

5.5.2 Left Recursion

A production is **left recursive** if its LHS symbol is also the first symbol of its RHS. In Figure 5.14, the production StmtList → StmtList ; Stmt is left-recursive. This definition extends to nonterminals: a nonterminal is left-recursive if it is the LHS symbol of a left-recursive production.

Grammars with left-recursive productions can never be LL(1). To see this, assume that some lookahead symbol t predicts the application of the left-recursive production $A \rightarrow A\beta$. With recursive-descent parsing, the application of this production will cause procedure A to be invoked repeatedly without advancing the input. With the state of the parse unchanged, this behavior will continue indefinitely. Similarly, with table-driven parsing, application of this production will repeatedly push $A\beta$ on the stack without advancing the input.

$$\begin{array}{rlcl}
1 & \mathsf{Stmt} & \rightarrow & \mathsf{if\ Expr\ then\ StmtList\ V_1} \\
2 & \mathsf{V_1} & \rightarrow & \mathsf{endif} \\
3 & & | & \mathsf{else\ StmtList\ endif} \\
4 & \mathsf{StmtList} & \rightarrow & \mathsf{X\ Y} \\
5 & \mathsf{X} & \rightarrow & \mathsf{Stmt} \\
6 & \mathsf{Y} & \rightarrow & \mathsf{;\ Stmt\ Y} \\
7 & & | & \lambda \\
8 & \mathsf{Expr} & \rightarrow & \mathsf{var\ V_2} \\
9 & \mathsf{V_2} & \rightarrow & \mathsf{+\ Expr} \\
10 & & | & \lambda
\end{array}$$

Figure 5.16: LL(1) version of the grammar in Figure 5.14.

Consider the following left-recursive rules.

$$\begin{array}{rlcl}
1 & \mathsf{A} & \rightarrow & \mathsf{A}\ \alpha \\
2 & & | & \beta
\end{array}$$

Each time Rule 1 is applied, an α is generated. The recursion ends when Rule 2 prepends a β to the string of α symbols. Using the regular-expression notation developed in Chapter 3, the grammar generates $\beta\alpha^\star$. The algorithm in Figure 5.15 obtains a grammar that also generates $\beta\alpha^\star$. However, the β is generated *first*. The α symbols are then generated via right recursion. Applying this algorithm to the grammar in Figure 5.14 results in the grammar shown in Figure 5.16. Since X appears as the LHS of only one production, X's unique RHS can be automatically substituted for all uses of X. This allows Rules 4 and 5 of Figure 5.14 to be replaced with StmtList → Stmt Y.

The algorithms presented in Figures 5.13 and 5.15 typically succeed in obtaining an LL(1) grammar. However, some grammars require greater thought to obtain an LL(1) version (some of these are included as exercises at the end of this chapter). All grammars that include the $ (end-of-input) symbol can be rewritten into a form where all right-hand sides begin with a terminal symbol; this form is called **Greibach normal form** (GNF) (see Exercise 17). Once a grammar is in GNF, factoring of common prefixes is straightforward. Surprisingly, even this transformation does not guarantee that a grammar will be LL(1) (see Exercise 18). In fact, as we discuss in the next section, language constructs do exist that have no LL(1) grammar. Fortunately, such constructs are rare in practice and can be handled by modest extensions to the LL(1) parsing technique.

```
1  S    → Stmt $
2  Stmt → if expr then Stmt else Stmt
3        | if expr then Stmt
4        | other
```

Figure 5.17: Grammar for if-then-else.

5.6 A Non-LL(1) Language

Almost all common programming language constructs can be specified by LL(1) grammars. One notable exception, however, is the if-then-else construct present in programming languages such as Java™ and C. The if-then-else language defined in Figure 5.16 has an endif token that *closes* each if. For languages that lack this delimiter, the if-then-else construct is subject to the so-called **dangling else** problem. This occurs when a sequence of nested conditionals contains more thens than elses, which leaves open the correspondence of thens to elses. Programming languages resolve this issue by mandating that each else is matched to its closest, otherwise unmatched then.

We next show that no LL(k) parser can handle languages that embed the if-then-else construct shown in Figure 5.17. This grammar has common prefixes that can be removed by the algorithm in Figure 5.13, but this grammar has a more serious problem. As demonstrated by Exercises 10 and 13, the grammar in Figure 5.17 is **ambiguous** and is therefore not suitable for LL(k) parsing. Recall that an ambiguous grammar can produce at least two distinct parses for some string in the grammar's language. Ambiguity and its possible remediation are considered in greater detail in Chapter 6.

We do not intend to use the grammar of Figure 5.17 for LL(k) parsing. Instead, we study the *language* of this grammar to show that *no* LL(k) grammar exists for this language. In studies of this kind, it is convenient to redact unnecessary detail to expose a language's problematic aspects. In the language defined by the grammar of Figure 5.17, the if expr then Stmt portion serves as an *opening bracket* and the else Stmt portion serves as an *optional closing bracket*. Thus, the language of Figure 5.17 is structurally equivalent to the **dangling bracket language** (DBL) defined as follows:

$$\mathrm{DBL} = \{\, [^i \ \ ]^j \mid i \geq j \geq 0 \,\}.$$

We next show that DBL is not LL(k) for any k.

We can gain some insight into the problem by considering some grammars for DBL. Our first attempt is the grammar shown in Figure 5.18(a), in which CL generates an optional closing bracket. Superficially, the grammar appears to

$$\begin{array}{llll} 1 & \mathsf{S} & \rightarrow & [\ \mathsf{S}\ \mathsf{CL} \\ 2 & & \mid & \lambda \\ 3 & \mathsf{CL} & \rightarrow &] \\ 4 & & \mid & \lambda \end{array}$$

(a)

$$\begin{array}{llll} 1 & \mathsf{S} & \rightarrow & [\ \mathsf{S} \\ 2 & & \mid & \mathsf{T} \\ 3 & \mathsf{T} & \rightarrow & [\ \mathsf{T}\ ] \\ 4 & & \mid & \lambda \end{array}$$

(b)

Figure 5.18: Attempts to create an LL(1) grammar for DBL.

be LL(1) because it is free of left recursion and common prefixes. However, the ambiguity present in the grammar of Figure 5.17 is retained in this grammar. Any sentential form containing CL CL can generate the terminal] two ways, depending on which CL generates the] and which generates λ. Thus, the string [[ ] has two distinct parses.

To resolve the ambiguity, we create a grammar that follows the Java and C convention: Each] is matched with the *nearest unmatched* [. This approach results in the grammar shown in Figure 5.18(b). This grammar generates zero or more unmatched opening brackets followed by zero or more pairs of matching brackets. In fact, this grammar is parsable using most bottom-up techniques (such as SLR(1), which is discussed in Chapter 6). While this grammar is factored and is not left-recursive, it is not LL(1) because the [token is in the predict sets for both rules for S (Rules 1 and 2 of Figure 5.18(b)). The following analysis explains why this grammar is not LL(k) for any k:

$$\begin{array}{rcl} [& \in & \mathsf{Predict}(\mathsf{S} \rightarrow [\ \mathsf{S}) \\ [& \in & \mathsf{Predict}(\mathsf{S} \rightarrow \mathsf{T}) \\ [\,[& \in & \mathsf{Predict}_2(\mathsf{S} \rightarrow [\ \mathsf{S}) \\ [\,[& \in & \mathsf{Predict}_2(\mathsf{S} \rightarrow \mathsf{T}) \\ & \cdots & \\ [^k & \in & \mathsf{Predict}_k(\mathsf{S} \rightarrow [\ \mathsf{S}) \\ [^k & \in & \mathsf{Predict}_k(\mathsf{S} \rightarrow \mathsf{T}) \end{array}$$

In particular, when an LL parser sees only open brackets, it cannot decide whether to predict a matched or an unmatched open bracket. Bottom-up parsers have an advantage here because they can delay applying a production until an entire RHS is matched. On the other hand, top-down methods cannot delay. Rather, they must predict a production based on the first (or first k) symbols derivable from a RHS. To parse languages containing if-then-else constructs, the ability to postpone segments of the parse is crucial.

1	S	→	Stmt $
2	Stmt	→	if expr then Stmt V
3		|	other
4	V	→	else Stmt
5		|	λ

	Lookahead					
Nonterminal	if	expr	then	else	other	$
S	1				1	
Stmt	2				3	
V				$\boxed{4,5}$		5

Figure 5.19: Ambiguous grammar for if-then-else and its LL(1) table. The ambiguity is resolved by favoring Rule 4 over Rule 5 in the boxed entry.

Our analysis shows that LL(1) parser generators cannot automatically create parsers from grammars that embed the if-then-else construct. This shortcoming can be handled by using grammars that lead to LL(1) conflicts. Such conflicts are then resolved by hand to obtain the desired effect. Factoring the grammar in Figure 5.17 yields the ambiguous grammar and (correspondingly nondeterministic) parse table shown in Figure 5.19. As expected, the else symbol predicts multiple productions as seen in Rules 4 and 5. Since the else should match the closest then, we resolve the conflict in favor of Rule 4. Favoring Rule 5 would defer consumption of the else. Moreover, the parse table entry for nonterminal V and terminal else is Rule 4's only legitimate chance to appear in the parse table. If this rule is absent from the parse table, then the resulting LL(1) parser could never match *any* else. We therefore insist that rule V → else Stmt be predicted for V when the lookahead is else. The parse table or recursive-descent code can be modified manually to achieve this effect. Some parser generators offer mechanisms for establishing priorities when conflicts arise.

5.7 Properties of LL(1) Parsers

We can establish the following useful properties for LL(1) parsers:

- A correct, leftmost parse is constructed.

  This follows from the fact that LL(1) parsers simulate a leftmost derivation. Moreover, the algorithm in Figure 5.4 finds a CFGs to be LL(1) only if the Predict sets of a nonterminal's productions are disjoint. Thus, the LL(1) parser traces the unique, leftmost derivation of an accepted string.

- All grammars in the LL(1) class are unambiguous.

  If a grammar is ambiguous, then some string has two or more distinct leftmost derivations. When two such derivations are compared, there must be a nonterminal A for which at least two different productions could be applied to obtain the different derivations. In other words, with a lookahead token of x, a derivation could continue by applying $A \rightarrow \alpha$ or $A \rightarrow \beta$. It follows that $x \in$ Predict($A \rightarrow \alpha$) and $x \in$ Predict($A \rightarrow \beta$). Thus, the test at Marker ④ in Figure 5.4 determines that such a grammar is not LL(1).

- All table-driven LL(1) parsers operate in linear time and space with respect to the length of the parsed input. (Exercise 14 examines whether recursive-descent parsers are equally efficient.)

  Consider the number of actions that can be taken by an LL(1) parser when the token x is presented as lookahead. Some number of productions will be applied before x is either matched or found to be in error.

  - Suppose a grammar is λ-free. In this case, no production can be applied twice without advancing the input. Otherwise, the cycle involving the same production would continue to be applied indefinitely. This condition should have been reported as an error when the LL(1) parser was constructed.
  - If the grammar does include λ, then the number of nonterminals that could pop from the stack because of the application of λ-rules is proportional to the length of the input. Exercise 15 explores this point in more detail.

  Thus, each input token induces a bounded number of parser actions. It follows that the parser operates in linear time.

  The LL(1) parser consumes space for the **lookahead buffer** and for the parse stack. The lookahead buffer is of constant size, but the stack grows and contracts during parsing. However, the maximum stack used during any parse is proportional to the length of the parsed input, for either of the following reasons:

  - The stack grows only when a production is applied of the form $A \rightarrow \alpha$. As argued previously, no production could be applied twice without advancing the input and, correspondingly, decreasing the stack size. If we regard the number and size of a grammar's productions to be bounded by some constant, then each input token contributes to a constant increase in stack size.

– If the parser's stack grew superlinearly, then the the parser would require more than linear time just to push entries on the stack.

5.8 Parse Table Representation

Many entries in the parse tables of Figures 5.10 and 5.19 are blank. In an array implementation, such entries would be filled by an otherwise unused integer such as zero. If a parser accesses a zero entry while parsing some input string then the string is determined to contain an syntax error. With respect to the non-zero entries, LL(1) parse tables tend to be sparsely populated because the Predict sets for most productions are small relative to the size of the grammar's terminal vocabulary. For example, an LL(1) parser was constructed for a subset of Ada using a grammar that contained 70 terminals and 138 nonterminals. Of the 9660 potential LL(1) parse table entries, only 629 (6.5%) allowed the parse to continue.

In some parse tables, blanks are not prevalent, but a single action is repeated across many columns. For example, action 1 is predicted for nonterminal S in the parse table of Figure 5.10 for all possible lookahead symbols except d.

Given such statistics, it makes sense to view a row's most popular entry as a **default**. We then strive to represent the **nondefault** entries efficiently. Generally, consider a two-dimensional parse table with N rows, M columns, and E nondefault entries. The parse table constructed in Section 5.4 occupies space proportional to $N \times M$. Especially when $E \ll N \times M$, our goal is to represent the parse table using space proportional to E. Although modern workstations are equipped with ample storage to handle LL(1) tables for any practical LL(1) grammar, most computers operate more efficiently when storage accesses exhibit greater locality. A smaller parse table loads faster and makes better use of high-speed storage. Thus, it is worthwhile to consider sparse representations for LL(1) parse tables. However, any increase in space efficiency must not adversely affect the efficiency of accessing the parse table.

We next consider strategies for decreasing the storage required to represent parse tables. The table shown in Figure 5.20 serves as an example for the techniques presented below. In a table used for LL(1) parsing, the table entries (L, P, Q, etc.) would be integers denoting a grammar rule. Similar tables are used for the bottom-up parsing methods presented in Chapter 6. Although the table's entries encode information differently for bottom-up parsing, the space-reducing techinques presented below are equally applicable to such tables.

	Column				
Row	1	2	3	4	5
1	L			P	
2		Q			R
3			U		
4	W	X			
5		Y		Z	

Figure 5.20: Sparse table T.

5.8.1 Compaction

We begin by considering **compaction** methods that convert a table T into a representation devoid of default entries. Such methods operate as follows.

1. The nondefault entries of T are stored in compacted form.
2. A mapping is provided from the index pair (i, j) to the set $E \cup \{default\}$.
3. The LL(1) parser is modified. Wherever the parser accesses $T[i, j]$, the mapping is applied to (i, j) and the compacted form supplies the contents of $T[i, j]$.

Binary Search

The compacted form can be achieved by listing the nondefault entries in order of their appearance in T, scanning from left to right, top to bottom. For the original table shown in Figure 5.20, the resulting compact table using binary search is shown in Figure 5.21. If row r of the compact table contains the nondefault entry $T[i, j]$, then row r also contains i and j, which are necessary for key comparison when the table is searched. We save space if $3 \times E < N \times M$, assuming each table entry takes one unit of storage. Because the data is sorted by row and column, the compact table can be accessed by **binary search**. Given E nondefault entries, each access takes $O(\log(E))$ time.

Hash Table

The compact table shown in Figure 5.22 uses $|E| + 1$ slots and stores $T[i, j]$ at a location determined by **hashing** i and j, using the hash function

$$h(i, j) = (i \times j) \bmod (|E| + 1)$$

	T's Nondefault	From *T*'s	
Index	Contents	Row	Column
0	L	1	1
1	P	1	4
2	Q	2	2
3	R	2	5
4	U	3	3
5	W	4	1
6	X	4	2
7	Y	5	2
8	Z	5	4

Figure 5.21: Compact version of the table in Figure 5.20 using binary search. Only the boxed information is stored in the compact table.

To create the compact table, we process the nondefault entries of T in any order. The nondefault entry at $T[i, j]$ is stored in the compact table at $h(i, j)$ if that position is unoccupied. Otherwise, we search forward in the table, storing $T[i, j]$ at the next available slot. This method of handling **collisions** in the compact table is called **linear resolution**. Because the compact table contains $|E| + 1$ slots, one slot is always free after all nondefault entries are hashed. The vacant slot avoids an infinite loop when searching the compact table for a default entry.

Hash performance can be improved by allocating more slots in the compact table and by choosing a hash function that results in fewer collisions. Because the nondefault entries of T are known in advance, both goals can be achieved by using **perfect hashing** [Spr77, CLRS01]. With this technique, each nondefault entry $T[i, j]$ maps to one of $|E|$ slots using the key (i, j). A nondefault entry is detected when the perfect hash function returns a value greater than $|E|$.

5.8.2 Compression

Compaction reduces the storage requirements of a parse table by eliminating default entries. However, the indices of a nondefault entry must be stored in the compact table to facilitate nondefault entry lookup. As shown in Figures 5.21 and 5.22, a given row or column index can be repeated multiple times. We next examine a **compression** method that tries to eliminate such redundancy and take advantage of default entries.

	T's Nondefault	From *T*'s		Hashes
Index	Contents	Row	Column	to
0	R	2	5	10 ≡ 0
1	L	1	1	1
2	Y	5	2	10 ≡ 0
3	Z	5	4	20 ≡ 0
4	P	1	4	4
5	Q	2	2	4
6	W	4	1	4
7				
8	X	4	2	8
9	U	3	3	9

Figure 5.22: Compact version of the table in Figure 5.20 using hashing. Only the boxed information is stored in the compact table.

The compression algorithm we study is called **double-offset indexing**. The algorithm, shown in Figure 5.23, operates as follows:

- The algorithm initializes a vector *V* at Marker ⑭. Although the vector could hold $N \times M$ entries, the final size of the vector is expected to be closer to $|E|$. The entries of *V* are initialized to the parse table's default value.

- Marker ⑮ considers the rows of *T* in an arbitrary order.

- When row *i* is considered, a shift value for the row is computed by the FindShift method. The shift value, retained in $R[i]$, records the amount by which an index into row *i* is shifted to find its entry in vector *V*. Method Fits checks to be certain that, when shifted, row *i* fits into *V* without any collision with the nondefault entries already established in *V*.

- The size of *V* is reduced at Marker ⑯ by removing all default values at *V*'s high end.

To use the compressed tables, entry $T[i, j]$ is found by inspecting *V* at location $l = R[i] + j$. If the row recorded at $V.fromrow[l]$ is *i*, then the table entry at $V.entry[l]$ is the nondefault table entry from $T[i, j]$. Otherwise, $T[i, j]$ has the default value.

We illustrate the effectiveness of the algorithm in Figure 5.23 by applying it to the sparse table shown in Figure 5.20. Suppose the rows are considered

```
procedure Compress( )
    for $i = 1$ to $N \times M$ do                                         ⑭
        $V.entry[i] \leftarrow default$
    foreach $row \in \{1, 2, \ldots, N\}$ do                               ⑮
        $R[row] \leftarrow$ FindShift($row$)
        for $j = 1$ to $M$ do
            if $T[row, j] \neq default$
            then
                $place \leftarrow R[row] + j$
                $V.entry[place] \leftarrow T[row, j]$
                $V.fromrow[place] \leftarrow row$
    call Trunc($V$)                                                        ⑯
end
function FindShift($row$) returns Integer
    return $\left( \min_{shift=-M+1}^{N \times M - M} \text{Fits}(row, shift) \right)$
end
function Fits($row, shift$) returns Boolean
    for $j = 1$ to $M$ do
        if $T[row, j] \neq default$ and not RoomInV($shift + j$)           ⑰
        then return (false)
    return (true)
end
function RoomInV($where$) returns Boolean
    if $where \geq 1$
    then
        if $V.entry[where] = default$
        then return (true)
    return (false)
end
procedure Trunc($V$)
    for $i = N \times M$ downto 1 do
        if $V.entry[i] \neq default$
        then
            /*  Retain $V[1 \ldots i]$                                 */
            return ()
end
```

Figure 5.23: Compression algorithm.

in order $1, 2, 3, 4, 5$. The resulting structures, shown in Figure 5.24, can be explained as follows:

Row 1: This row cannot be negatively shifted because it has an entry in column 1. Thus, $R[1]$ is 0 and $V[1 \ldots 5]$ represents row 1, with nondefault entries at index 1 and 4.

Row 2: This row can merge into V without shifting, because its nondefault values (columns 2 and 5) can be accommodated at 2 and 5, respectively.

Row 3: Similarly, row 3 can be accommodated by V without any shifting.

Row 4: When this row is considered, the first slot of V that can accommodate its leftmost column is slot 6. Thus, $R[4] = 5$ and row 4's nondefault entries are placed at 6 and 7.

Row 5: Finally, columns 2 and 4 of row 5 can be accommodated at 8 and 10, respectively. Thus, $R[5] = 6$.

As suggested by the pseudocode at Marker ⑮, rows can be presented to FindShift in any order. However, the size of the resulting compressed table can depend on the order in which rows are considered. Exercises 20 and 21 explore this point further. In general, finding a row ordering that achieves maximum compression is an **NP-complete** problem [GJ79]. This means that the best-known algorithms for obtaining optimal compression would have to try all row permutations. However, compression heuristics work well in practice. When compression is applied to the Ada LL(1) parse table mentioned previously, the number of entries drops from 9660 to 660. This result is only 0.3% off from the 629 nondefault entries in the original table.

5.9 Syntactic Error Recovery and Repair

A compiler should produce a useful set of diagnostic messages when presented with a faulty input. Thus, when a single error is detected, it is usually desirable to continue processing the input to detect additional errors. Generally, parsers can continue syntax analysis using one of the following approaches:

- With **error recovery**, the parser attempts to ignore the current error. The parser enters a configuration where it is able to continue processing the input.
- **Error repair** is more ambitious. The parser attempts to correct the syntactically faulty program by modifying the input to obtain an acceptable parse.

In this section, we explore each of these approaches in turn. We then examine error detection and recovery for LL(1) parsers.

R

Row	Shift
i	*R*[*i*]
1	0
2	0
3	0
4	5
5	6

V

Index	Entry	From
		Row
1	L	1
2	Q	2
3	U	3
4	P	1
5	R	2
6	W	4
7	X	4
8	Y	5
9		
10	Z	5

Figure 5.24: Compression of the table in Figure 5.20. Only the boxed information is actually stored in the compressed structures.

5.9.1 Error Recovery

With **error recovery**, we try to reset the parser so that the remaining input can be parsed. This process may involve modifying the parse stack and remaining input. Depending on the success of the recovery process, subsequent syntax analysis may be accurate. Unfortunately, it is more often the case that faulty error recovery causes errors to **cascade** throughout the remaining parse. For example, consider the C fragment `a=func c+d)`. If error recovery continues the parse by predicting a Statement after the `func`, then another syntax error is found at the parenthesis. A single syntax error has been amplified by error recovery by issuing two error messages.

The primary measure of quality in an error-recovery process is how few false or cascaded errors it induces. Normally, semantic analysis and code generation are disabled upon error recovery because there is no intention to execute the code of a syntactically faulty program.

A simple form of error recovery is often called **panic mode**. In this approach, the parser skips input tokens until it finds a frequently occurring delimiter (e.g., a semicolon). The parser then continues by expecting those nonterminals that derive strings that can follow the delimiter.

5.9.2 Error Repair

With **error repair**, the parse attempts to repair the syntactically faulty program by modifying the parsed or (more commonly) the unparsed portion of the

```
1  S → V
2    | W
3  V → v A b
4  W → w A c
5  A → λ
```

Figure 5.25: An LL(1) grammar.

program. The compiler does not presume to know or to suggest an appropriate revision of the faulty program. The purpose of error repair is to analyze the offending input more carefully so that better diagnostics can be issued.

Algorithms for error recovery and error repair can exploit the fact that LL(1) parsers have the **correct-prefix** property: For each state entered by such parsers, there is a string of tokens that could result in a successful parse. Consider the input string α x β, where token x causes an LL(1) parser to detect a syntax error. The correct-prefix property means that there is at least one string $\alpha\ \gamma \neq \alpha$ x β that can be accepted by the parser.

What can a parser do to repair the faulty input? The following options are possible:

- Modification of α
- Insertion of text δ to obtain $\alpha\ \delta$ x β
- Deletion of x to obtain $\alpha\ \beta$

These options are not equally attractive. The correct-prefix property implies that α is at least a portion of a syntactically correct program. Thus, most error recovery methods do not modify α except in special situations. One notable case is **scope repair**, where nesting brackets may be inserted or deleted to match the corresponding brackets in x β.

Insertion of text must also be done carefully. In particular, error repair based on insertion must ensure that the repaired string will not continually grow so that parsing can never be completed. Some languages are **insert correctable**. For such languages, it is always possible to repair syntactic faults by insertion. Deletion is a drastic alternative to insertion, but it does have the advantage of making progress through the input.

5.9.3 Error Detection in LL(1) Parsers

The recursive-descent and table-driven LL(1) parsers constructed in this chapter are based on Predict sets. These sets are, in turn, based on First and Follow information that is computed globally for a grammar. In particular, recall that the production $A \to \lambda$ is predicted by the symbols in Follow(A).

Suppose that A occurs in the productions $V \to v\ A\ b$ and $W \to w\ A\ c$, as shown in Figure 5.25. For this grammar, the production $A \to \lambda$ is predicted by the symbols in Follow(A) = $\{b, c\}$. Examining the grammar in greater detail, we see that the application of $A \to \lambda$ should be followed only by b if the derivation stems from V. However, if the derivation stems from W, then A should be followed only by c. As described in this chapter, LL(1) parsing cannot distinguish between contexts calling for application of $A \to \lambda$. If the next input token is b or c, the production $A \to \lambda$ is applied, even though the next input token may not be acceptable. If the wrong symbol is present, the error is detected later, when matching the symbol after A in $V \to v\ A\ b$ or $W \to w\ A\ c$. Exercise 23 considers how such errors can be caught sooner by *full* LL(1) parsers, which are more powerful than the *strong* LL(1) parsers defined in this chapter.

5.9.4 Error Recovery in LL(1) Parsers

The LR(1) parsers described in Chapter 6 are formally more powerful than the LL(1) parsers. However, the continued popularity of LL(1) parsers can be attributed, in part, to their superior error diagnosis and error recovery. Because of the predictive nature of an LL(1) leftmost parse, the parser can easily extend the parsed portion of a faulty program into a syntactically valid program. When an error is detected, the parser can produce messages informing the programmer of what tokens were *expected* so that the parse could have continued.

A simple and uniform approach to error recovery in LL(1) parsers is discussed by Wirth [Wir76]. When applied to recursive-descent parsers, the parsing procedures described in Section 5.3 are augmented with an extra parameter that receives a set of terminal symbols. Consider the parsing procedure A(*ts*, *termset*) associated with some nonterminal A. When A is called during operation of the recursive-descent parser, any symbol passed via *termset* can legitimately serve as the lookahead symbol when *this* instance of A returns. For example, consider the grammar and Wirth-style parsing procedures shown in Figure 5.26. Error recovery is placed in E so that if an a is not found, the input is advanced until a symbol is found that *can* follow E. The set of symbols passed to E includes those symbols passed to S as well as a closing bracket (if called from Marker ⑱) or a closing parenthesis (if called from Marker ⑲). If E detects an error, then the input is advanced until a symbol in *termset* is

```
1  S → [ E ]
2    | ( E )
3  E → a
```

```
procedure S( ts, termset )
    switch ()
        case ts.peek( ) $\in$ { [ }
            call match( [ )
            call E( ts, termset $\cup$ { ] } )                    (18)
            call match( ] )
        case ts.peek( ) $\in$ { ( }
            call match( ( )
            call E( ts, termset $\cup$ { ) } )                    (19)
            call match( ) )
end
procedure E( ts, termset )
    if ts.peek( ) = a
    then  call match( ts, a )
    else
        call error( Expected an a )
        while ts.peek( ) $\notin$ termset  do  call ts.advance( )
end
```

Figure 5.26: A grammar and its Wirth-style, error-recovering parser.

found. Because end-of-input can follow S, every *termset* includes $. In the worst case, the input program is advanced until $, at which point all pending parsing procedures can exit.

Summary This concludes our study of LL parsers. Given an LL(1) grammar, we have studied how to construct recursive-descent or table-driven LL(1) parsers. Grammars that are not LL(1) can often be converted to LL(1) form by eliminating left recursion and by factoring common prefixes. Some programming language constructs are inherently non-LL(1). Intervention by the compiler writer can often resolve the conflicts that arise in such cases. Alternatively, more powerful parsing methods can be considered, such as those presented in Chapter 6.

Exercises

1. For each of the following grammars, determine whether or not the grammar is LL(1):

(a)
$$\begin{array}{llll}
1 & \mathsf{S} & \rightarrow & \mathsf{A\ B\ c} \\
2 & \mathsf{A} & \rightarrow & \mathsf{a} \\
3 & & | & \lambda \\
4 & \mathsf{B} & \rightarrow & \mathsf{b} \\
5 & & | & \lambda
\end{array}$$

(b)
$$\begin{array}{llll}
1 & \mathsf{S} & \rightarrow & \mathsf{A\ b} \\
2 & \mathsf{A} & \rightarrow & \mathsf{a} \\
3 & & | & \mathsf{B} \\
4 & & | & \lambda \\
5 & \mathsf{B} & \rightarrow & \mathsf{b} \\
6 & & | & \lambda
\end{array}$$

(c)
$$\begin{array}{llll}
1 & \mathsf{S} & \rightarrow & \mathsf{A\ B\ B\ A} \\
2 & \mathsf{A} & \rightarrow & \mathsf{a} \\
3 & & | & \lambda \\
4 & \mathsf{B} & \rightarrow & \mathsf{b} \\
5 & & | & \lambda
\end{array}$$

(d)
$$\begin{array}{llll}
1 & \mathsf{S} & \rightarrow & \mathsf{a\ S\ e} \\
2 & & | & \mathsf{B} \\
3 & \mathsf{B} & \rightarrow & \mathsf{b\ B\ e} \\
4 & & | & \mathsf{C} \\
5 & \mathsf{C} & \rightarrow & \mathsf{c\ C\ e} \\
6 & & | & \mathsf{d}
\end{array}$$

2. Consider the following grammar, which is already suitable for LL(1) parsing:

$$\begin{array}{llll}
1 & \mathsf{Start} & \rightarrow & \mathsf{Value\ \$} \\
2 & \mathsf{Value} & \rightarrow & \mathsf{num} \\
3 & & | & \mathsf{lparen\ Expr\ rparen} \\
4 & \mathsf{Expr} & \rightarrow & \mathsf{plus\ Value\ Value} \\
5 & & | & \mathsf{prod\ Values} \\
6 & \mathsf{Values} & \rightarrow & \mathsf{Value\ Values} \\
7 & & | & \lambda
\end{array}$$

(a) Construct First and Follow sets for each nonterminal in the grammar.

(b) Construct the Predict sets for the grammar.

(c) Construct a recursive-descent parser based on the grammar.

(d) Add code into the parser to compute sums and products as indicated by the grammar.

Note that a sum always involves exactly two Values, while a product is formed over 0 or more Values.

(e) Build an LL(1) parse table based on the grammar.

3. Construct the LL(1) parse table for the following grammar:

1	Expr	→	– Expr
2		|	(Expr)
3		|	Var ExprTail
4	ExprTail	→	– Expr
5		|	λ
6	Var	→	id VarTail
7	VarTail	→	(Expr)
8		|	λ

4. Trace the operation of an LL(1) parser for the grammar of Exercise 3 on the following input:

id – –id ((id))

5. Transform the following grammar into LL(1) form using the techniques presented in Section 5.5:

1	DeclList	→	DeclList ; Decl
2		|	Decl
3	Decl	→	IdList : Type
4	IdList	→	IdList , id
5		|	id
6	Type	→	ScalarType
7		|	array (ScalarTypeList) of Type
8	ScalarType	→	id
9		|	Bound . . Bound
10	Bound	→	Sign intconstant
11		|	id
12	Sign	→	+
13		|	–
14		|	λ
15	ScalarTypelist	→	ScalarTypeList , ScalarType
16		|	ScalarType

6. Run your solution to Exercise 5 through any LL(1) parser generator to verify that it is actually LL(1). How do you know that your solution generates the same language as the original grammar?

7. Show that every regular language can be defined by an LL(1) grammar.

8. A grammar is said to have **cycles** if it contains a nonterminal A such that A $\Rightarrow^+$ A (this derivation notation, covered in Chapter 4, means that A derives itself using at least one step). Show that an LL(1) grammar must not have cycles.

9. Recall that an LL(k) grammar allows *k* tokens of lookahead. Construct an LL(2) parser for the following grammar:

```
1  Stmt   → id ;
2         | id ( IdList ) ;
3  IdList → id
4         | id , IdList
```

10. Show the two distinct parse trees that can be constructed for

    if expr then if expr then other else other

    using the grammar given in Figure 5.17. For each parse tree, explain the correspondence of then and else.

11. In Section 5.7, it is established that LL(1) parsers operate in linear time. That is, when parsing an input, the parser requires *on average* only a constant-bounded amount of time per input token.

    Is it ever the case that an LL(1) parser requires more than a constant-bounded amount of time to accept some particular symbol? In other words, can we bound by a constant the time interval between successive calls to the scanner to obtain the next token?

12. Design an algorithm that reads an LL(1) parse table and produces the corresponding recursive-descent parser.

13. An **ambiguous grammar** can produce two distinct parses for some string in the grammar's language. Explain why an ambiguous grammar is never LL(*k*) for any *k*, even if the grammar is free of common prefixes and left recursion.

14. Section 5.7 argues that table-driven LL(1) parsers operate in linear time and space. Explain why this claim does or does not hold for recursive-descent LL(1) parsers.

15. Explain why the number of nonterminals that can pop from an LL(1) parse stack is not bounded by a grammar-specific constant.

16. Design an algorithm that computes $\mathsf{Predict}_k$ sets for a CFGs.

17. As discussed in Section 5.5, a grammar is in GNF if all productions are of the form $\mathsf{A} \rightarrow \mathsf{a}\alpha$, where a is a terminal symbol and α is a string of zero or more grammar (i.e., terminal or nonterminal) symbols.

    Let G be a grammar that does not generate λ. Design an algorithm to transform G into GNF.

18. If we construct a GNF version of a grammar using the algorithm developed in Exercise 17, the resulting grammar is free of left recursion. However, the resulting grammar can still have common prefixes that prevent it from being LL(1). If we apply the algorithm presented in Figure 5.13 of Section 5.5.1, the resulting grammar will be free of left recursion and common prefixes. Show that the absence of common prefixes and left recursion in an unambiguous grammar does not necessarily make a grammar LL(1).

19. Section 5.7 and Exercises 14 and 15 examine the efficiency of LL(1) parsers.

    (a) Analyze the efficiency of *operating* a table-driven LL(k) parser, assuming an LL(k) table has already been constructed. Your answer should be formulated in terms of the length of the parsed input.

    (b) Analyze the efficiency of *constructing* an LL(k) parse table. Your answer should be formulated in terms of the size of the grammar (the space necessary to represent its vocabularies and productions).

    (c) Analyze the efficiency of operating a recursive-descent LL(k) parser.

20. Apply the table compression algorithm in Figure 5.23 to the table shown in Figure 5.20, presenting rows in the order $1, 5, 2, 4, 3$. Compare the success of compression with the result presented in Figure 5.24.

21. Although table-compression is an NP-complete problem, explain why the following heuristic works well in practice:

    > Rows are considered in order of decreasing density of nondefault entries. (That is, rows with the greatest number of nondefault entries are considered first.)

    Apply this heuristic to the table shown in Figure 5.20 and describe the results.

22. A sparse array can be represented as a vector of rows, with each row represented as a *list* of nondefault column entries. Thus, the nondefault entry at $T[i, j]$ would appear as an element of list $R[i]$. The element would contain both its column identification (j) and the nondefault entry ($T[i, j]$).

    (a) Express the table shown in Figure 5.20 using this format.

    (b) Compare the effectiveness of this representation with those given in Section 5.8. Consider both the savings in space and any increase or decrease in access time.

23. Section 5.9.3 contains an example where the production $\mathsf{A} \rightarrow \lambda$ is applied using an invalid lookahead token. With Follow sets computed globally for a given grammar, the style of LL(1) parsing described in this chapter is known as **strong LL**(1). A **full LL**(1) parser applies a production only if the next input token is valid. Design an algorithm for constructing full LL(1) parse tables.

    Hint: If a grammar contains n occurrences of the nonterminal A, then consider *splitting* this nonterminal so that each occurrence is a unique symbol. Thus, A is split into $\mathsf{A}_1, \mathsf{A}_2, \ldots, \mathsf{A}_n$. Each new nonterminal has productions similar to A, but the context of each nonterminal can differ.

24. Consider the following grammar:

```
1  S → V
2    | W
3  V → v A b
4  W → w A c
5  A → λ
```

    Is this grammar LL(1)? Is the grammar *full* LL(1), as defined in Exercise 23?

25. Section 5.9.4 describes an error recovery method that relies on dynamically constructed sets of Follow symbols. Compare these sets with the Follow information computed for full LL(1) in Exercise 23.

26. As pointed out in Section 5.6, there are some languages that are not LL(k) for any k. In other words, given k tokens of lookahead, where k can be chosen as any integer constant, there is no top-down parsing technique that can recognize the language.

    Using the alphabet $\{ a, b \}$, devise such a language and explain why no LL(k) grammar exists for that language.

6

Bottom-Up Parsing

Because of their power, efficiency, and ease of construction, bottom-up parsers are commonly used in the syntax-checking phase of a compiler. Grammar features that are problematic for top-down parsing (Chapter 5), such as left-recursive productions and common prefixes, can typically be accommodated without issue in bottom-up parsing. For example, the grammar shown in Figure 5.12 on page 156 is sufficiently clear to serve as a definition of its language's syntax. However, due to common prefixes and left-recursive rules, that grammar is not suitable for top-down parsing. When those problems are addressed, the grammar shown in Figure 5.16 on page 158 is obtained. Unfortunately, that grammar does not clearly articulate the language's syntax.

It turns out that the original grammar in Figure 5.12, while unsuitable for top-down parsing, is usable as is for bottom-up parsing. In fact, bottom-up parsers can handle the largest class of grammars that allow parsing to proceed **deterministically** (i.e., without backtracking). For many programming languages, grammars suitable for bottom-up parsing serve as the very definition of the language's syntax.

Given a suitable grammar, top-down parsers can be constructed automatically using the techniques described in Chapter 5. This chapter discusses analogous techniques and tools for automatically constructing bottom-up parsers. These **parser generators** or **compiler compilers** are useful not only because they automatically construct tables that drive bottom-up parsing, but also because they are powerful diagnostic tools for developing or modifying grammars.

When language extensions are considered (e.g., C to C++), the syntax modifications are usually prototyped using the language's grammar. Analysis performed by the parser generator can reveal problems with the proposed syntax extensions. By proceeding carefully, a language designer can ensure that old programs retain their meaning in the extended language.

6.1 Overview

In Chapter 5, we learned how to construct top-down (also called LL) parsers based on **context-free grammars** (CFGs) that had certain properties. The fundamental concern of an LL parser is which production to choose in expanding a given nonterminal. This choice is based on the parser's current state and on a peek at the unconsumed portion of the parser's input string. The derivations and parse trees produced by LL parsers are constructed as follows: the leftmost nonterminal is expanded at each step, and the parse tree grows systematically—top-down, from left to right. The LL parser begins with the tree's root, which is labeled with the grammar's goal symbol. Suppose that A is the next nonterminal to be expanded, and that the parser chooses the production $A \rightarrow \gamma$. In the parse tree, the node corresponding to this A is supplied with children that are labeled with the symbols in γ.

In this chapter, we study bottom-up (also called LR) parsers, whose operation can be compared with top-down parsers as follows:

- A bottom-up parser begins with the parse tree's leaves and moves toward its root. A top-down parser moves the parse tree's root toward its leaves.
- A bottom-up parser traces a *rightmost* derivation *in reverse*. A top-down parser traces a leftmost derivation.
- A bottom-up parser uses a grammar rule to replace the rule's **right-hand side** (RHS) with its **left-hand side** (LHS). A top-down parser does the opposite, replacing a rule's LHS with its RHS.

Figures 4.5 and 4.6 illustrate the differences between a top-down and a bottom-up parse. The style of parsing considered in this chapter is known by the following names:

- **Bottom-up**, because the parser works its way from the terminal symbols to the grammar's goal symbol
- **Shift-reduce**, because the two most prevalent actions taken by the parser are to *shift* symbols onto the parse stack and to *reduce* a string of such symbols located at the top-of-stack to one of the grammar's nonterminals

- **LR**(k), because such parsers scan the input from the left (the "L" in LR) producing a rightmost derivation (the "R" in LR) in reverse, using k symbols of lookahead

Unfortunately, the term LR denotes both the generic bottom-up parsing engine as well as a particular technique for constructing the engine's tables. It should be clear in context which meaning is intended.

In an LL parser, each state is committed to expand a particular nonterminal. On the other hand, an LR parser can concurrently anticipate the eventual success of multiple nonterminals. This flexibility makes LR parsers more general than LL parsers.

Tools for the automatic construction of LR parsers are available for a variety of platforms, including ML, Java$^{\text{TM}}$, C, and C++. Suppose a parser generator for a platform t emits a program p based on a supplied grammar g. All parsers generated by this parser generator are compiled using t. Thus, while p is compiled by t, the resulting program will parse input according to grammar g. For example, yacc[1] is a popular parser generator that emits C code. If yacc is given a grammar for the syntax of Fortran, then the resulting parser is compiled using C. However, the resulting parser performs syntax analysis for Fortran. The syntax of most modern programming languages is defined by grammars that are suitable for automatic parser generation using LR techniques.

The basic properties and actions of a generic LR parser are introduced in Sections 6.1 and 6.2. Section 6.3 presents the most basic table-construction method for LR parsers. Section 6.4 considers problems that prevent automatic LR parser construction. Sections 6.5.1, 6.5.2, and 6.5.4 discuss table-building algorithms of increasing sophistication and power. Of particular interest is the LALR(1) technique covered in Section 6.5.2, which is used in most LR parser generators. The formal definition of most modern programming languages includes an LALR(1) grammar to specify the language's syntax.

6.2 Shift-Reduce Parsers

In this section, we examine the operation of an LR parser, assuming that an LR parse table has already been constructed to guide the parser's actions. The reader may be understandably curious about how the table's entries are determined. However, table-construction techniques are best considered after obtaining a solid understanding of an LR parser's operation.

We describe the operation of an LR parser informally in Sections 6.2.1 and 6.2.2. Section 6.2.3 describes a generic LR parsing engine whose actions are guided by the parse table defined in Section 6.2.4. Section 6.2.5 presents LR(k) parsing more formally.

[1] The tool yacc's name stands for *yet another compiler compiler*.

6.2.1 LR Parsers and Rightmost Derivations

One method of understanding an LR parse is to appreciate that such parses construct rightmost derivations in reverse. Given a grammar and a rightmost derivation of some string in its language, the sequence of productions applied by an LR parser is the sequence used by the rightmost derivation, but played backwards. Figure 6.2 shows a grammar and the rightmost derivation of a string in the grammar's language. The language is suitable for expressing sums in a prefix (Lisp-like) notation. Each step of the derivation is annotated with the production number used at that step. For this example, the derivation of the string plus num num $ is achieved by applying Rules 1, 2, 3, and 3.

A bottom-up (LR) parse is accomplished by playing this sequence backwards: Rules 3, 3, 2, and 1. In contrast to LL parsing, an LR parser finds the RHS of a production and replaces it with the production's LHS. First, the leftmost num is **reduced** to an E by the rule E → num. This rule is applied again to obtain plus E E $. The sum is then reduced by E → plus E E to obtain E $. This can then be reduced by Rule 1 to the goal symbol Start.

6.2.2 LR Parsing as Knitting

Section 6.2.1 presents the order in which productions are applied to perform a bottom-up parse. We next examine *how* the RHS of a production is found so that a reduction can occur. The actions of an LR parser are somewhat analogous to **knitting**. Figure 6.1 illustrates this by showing a parse in progress for the grammar and string of Figure 6.2. The right needle contains the currently unprocessed portion of the string: num $. The left needle is the parser's stack, plus num, which represents the processed portion of the input string.

A **shift** operation transfers a symbol from the right needle to the left needle. When a **reduction** by the rule $A \rightarrow \gamma$ is performed, the symbols in γ must occur at the sharp end of the left needle—that is, at the *top* of the parse stack. Reduction by $A \rightarrow \gamma$ removes the symbols in γ and prepends the LHS symbol A to the unprocessed input of the right needle. A is then treated as an input symbol to be shifted onto the left needle. To illustrate the parse tree under construction, Figure 6.1 shows the symbols in γ as children of A.

We now follow the parse that is illustrated in Figure 6.1. In Figure 6.1(a), the left needle shows that two shifts have been performed. With plus num on the left needle, it is time to reduce by E → num. Figure 6.1(b) shows the effect of this reduction, with the resulting E prepended to the input. This same sequence of activities is repeated to obtain the state shown in Figure 6.1(c) where the left needle contains plus E E. When reduced by E → plus E E, we obtain Figure 6.1(d). The resulting E $ is shifted (Figure 6.1(e)) and reduced by Start → E $ (Figure 6.1(f)). The parse is accepted when the Start symbol is moved from the right needle to the left needle (not shown).

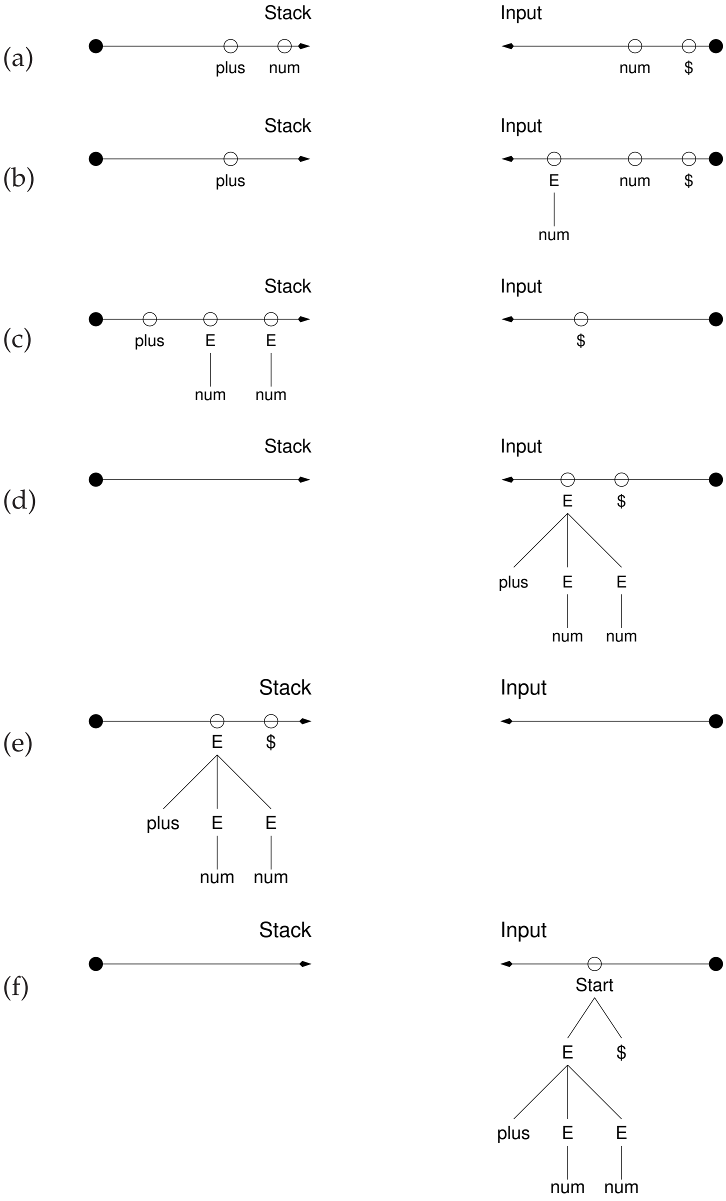


Figure 6.1: Bottom-up parsing resembles knitting.

1	Start	→	E $
2	E	→	plus E E
3		|	num

Rule	Derivation
1	$\text{Start} \Rightarrow_{\mathrm{rm}} \text{E \$}$
2	$\Rightarrow_{\mathrm{rm}} \text{plus E E \$}$
3	$\Rightarrow_{\mathrm{rm}} \text{plus E num \$}$
3	$\Rightarrow_{\mathrm{rm}} \text{plus num num \$}$

Figure 6.2: Grammar and rightmost derivation of plus num num $.

Based on the input string and the sequence of shift and reduce actions, symbols transfer back and forth between the needles. The artifact of the knitting is the parse tree, shown on the last needle if the input string is accepted.

6.2.3 LR Parsing Engine

Before considering an example in some detail, we present a simple driver for our shift-reduce parser in Figure 6.3. The parsing engine is driven by a table, whose entries are discussed in Section 6.2.4. The table is indexed at Marker ① using the parser's *current state* and the next (as yet, unprocessed) input symbol. The **current state** of the parser is defined by the contents of the parser's stack. To avoid rescanning the stack's contents prior to each parser action, state information is computed and stored with each symbol shifted onto the stack. Thus, Marker ① need only consult the state information associated with the stack's topmost symbol. The parse table calls for a shift or reduce as follows:

- Marker ② performs a shift of the next input symbol to state *s*.
- A reduction occurs at Markers ④ and ⑤. The RHS of a production is popped off the stack and its LHS symbol is prepended to the input.

The parser continues to perform shift and reduce actions until one of the following situations occurs:

- The input is reduced to the grammar's goal symbol at Marker ③. The input string is **accepted**.
- No valid action is found at Marker ①. In this case, the input string has a syntax error.

```
call Stack.PUSH( StartState )
accepted ← false
while not accepted do
    action ← Table[Stack.TOS( )][InputStream.PEEK( )]            ①
    if action = shift s
    then
        call Stack.PUSH( s )                                      ②
        if s ∈ AcceptStates                                       ③
        then  accepted ← true
        else  call InputStream.ADVANCE( )
    else
        if action = reduce A→γ
        then
            call Stack.POP( |γ| )                                 ④
            call InputStream.PREPEND( A )                         ⑤
        else
            call ERROR( )                                         ⑥
```

Figure 6.3: Driver for a bottom-up parser.

6.2.4 The LR Parse Table

We have seen that an LR parse constructs a rightmost derivation in reverse. Each reduction step in the LR parse uses a grammar rule such as A→γ to replace γ by A. A sequence of **sentential forms** is thus constructed, beginning with the input string and ending with the grammar's goal symbol.

Given a sentential form, the **handle** is defined as the sequence of symbols that will next be replaced by reduction. The difficulties lie in identifying the handle and in knowing which production to employ in the reduction (should there be multiple productions with the same RHS). These activities are choreographed by the parse table.

A suitable parse table for the grammar in Figure 6.4 is shown in Figure 6.5. This same grammar appears in Figure 5.2 on page 148 to illustrate top-down parsing. Readers familiar with top-down parsing can use this grammar to compare the methods.

To conserve space, shift and reduce actions are distinguished graphically in our parse tables:

- A shift to State *s* is denoted by $\boxed{s}$.
- Reduction by rule *r* is indicated by an unboxed entry of *r*.
- Blank entries are error actions.

1	Start	→ S $
2	S	→ A C
3	C	→ c
4		| λ
5	A	→ a B C d
6		| B Q
7	B	→ b B
8		| λ
9	Q	→ q
10		| λ

Rule	Derivation
1	Start $\Rightarrow_{\text{rm}}$ S $
2	$\Rightarrow_{\text{rm}}$ A C $
3	$\Rightarrow_{\text{rm}}$ A c $
5	$\Rightarrow_{\text{rm}}$ a B C d c $
4	$\Rightarrow_{\text{rm}}$ a B d c $
7	$\Rightarrow_{\text{rm}}$ a b B d c $
7	$\Rightarrow_{\text{rm}}$ a b b B d c $
8	$\Rightarrow_{\text{rm}}$ a b b d c $

Figure 6.4: Grammar and rightmost derivation of a b b d c $.

The parser accepts when the Start symbol is shifted in the parser's starting state.

Using the table in Figure 6.5, Figures 6.6 and 6.7 show the steps of a bottom-up parse. For pedagogical purposes, each stack cell is shown as two elements:

$$\boxed{\begin{matrix} a \\ n \end{matrix}}$$

The bottom element n is the parser state entered when the cell is pushed. The top symbol a is the symbol causing the cell to be pushed. The parsing engine described in Figure 6.3 keeps track only of the state.

The reader should verify that the reductions taken in Figures 6.6 and 6.7 trace a rightmost derivation in reverse. Moreover, shift actions are essentially implied by the inability to perform a useful reduction. The shifted tokens must make progress toward developing a handle. Tokens are therefore shifted until a handle appears at the top of the parse stack, at which time the next reduction in the reverse derivation can be applied.

State	a	b	c	d	q	$	Start	S	A	B	C	Q
0	$\boxed{3}$	$\boxed{2}$	8		8	8	accept	$\boxed{4}$	$\boxed{1}$	$\boxed{5}$		
1			$\boxed{11}$			4					$\boxed{14}$	
2		$\boxed{2}$	8	8	8	8				$\boxed{13}$		
3		$\boxed{2}$	8	8						$\boxed{9}$		
4						$\boxed{8}$						
5			10		$\boxed{7}$	10						$\boxed{6}$
6			6			6						
7			9			9						
8						1						
9			$\boxed{11}$	4							$\boxed{10}$	
10				$\boxed{12}$								
11				3		3						
12			5			5						
13			7	7	7	7						
14						2						

Figure 6.5: Parse table for the grammar shown in Figure 6.4.

Of course, the parse table plays a central role in determining the shifts and reductions that are necessary to recognize a valid string. For example, the rule C → λ could be applied at any time, but the parse table calls for this only in certain states and only when certain tokens are next in the input stream.

6.2.5 LR(k) Parsing

The concept of LR parsing was introduced by Knuth [Knu65], whose famous series entitled *The Art of Computer Programming* [Knu73a, Knu73b, Knu73c] began as introductory material for a textbook on compiler construction. As is the case with LL parsers, LR parsers are parameterized by the number of lookahead symbols that are consulted to determine the appropriate parser action. An LR(k) parser can peek at the next k tokens. This notion of "peeking" and the term LR(0) are confusing, because even an LR(0) parser must refer to

0	Initial Configuration	a b b d c $
0 a 3	shift a	b b d c $
0 a 3 b 2	shift b	b d c $
0 a 3 b 2 b 2	shift b	d c $
0 a 3 b 2 b 2	Reduce λ to B	B d c $
0 a 3 b 2 b 2 B 13	shift B	d c $
0 a 3 b 2	Reduce b B to B	B d c $
0 a 3 b 2 B 13	shift B	d c $
0 a 3	Reduce b B to B	B d c $
0 a 3 B 9	shift B	d c $
0 a 3 B 9	Reduce λ to C	C d c $
0 a 3 B 9 C 10	shift C	d c $
0 a 3 B 9 C 10 d 12	shift d	c $
0	Reduce a B C d to A	A c $

(continue to Figure 6.7)

Figure 6.6: Bottom-up parse of a b b d c $.

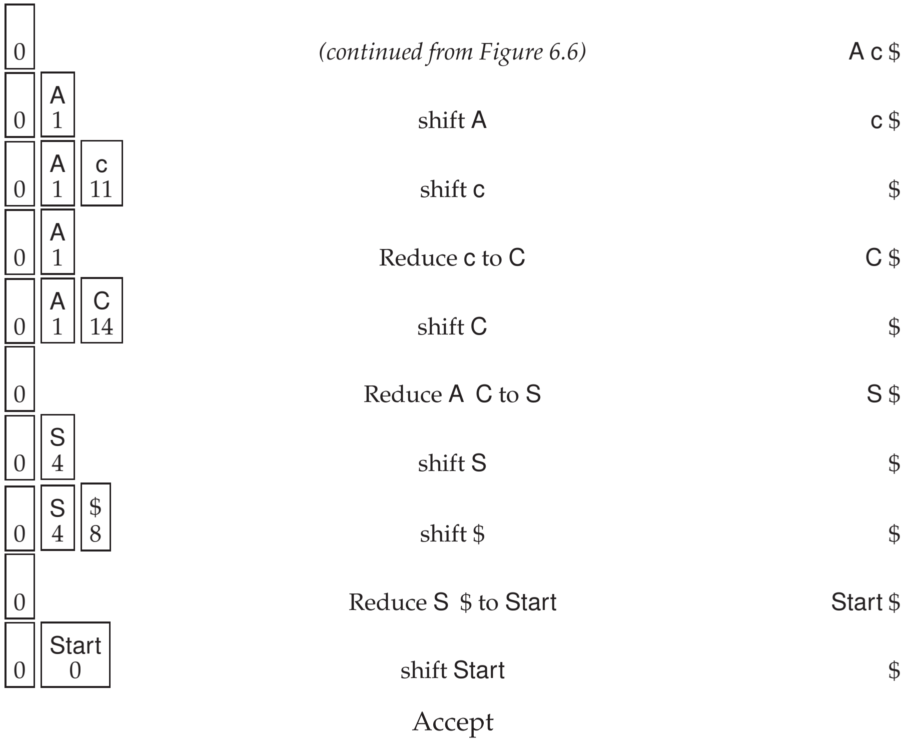


Figure 6.7: Continued bottom-up parse of a b b d c $.

the next input token, for the purpose of indexing the parse table to determine the appropriate action. The "0" in LR(0) refers not to the lookahead at parse-time, but rather to the lookahead used in *constructing* the parse table. At parse-time, LR(0) and LR(1) parsers index the parse table using one token of lookahead; for $k \geq 2$, an LR(k) parser uses k tokens of lookahead.

The number of columns in an LR(k) parse table grows dramatically with k. For example, an LR(3) parse table is indexed by the parse state to select a row, and by the next 3 input tokens to select a column. If the terminal alphabet has n symbols, then the number of distinct three-token sequences is n^3. More generally, an LR(k) table has n^k columns for a token alphabet of size n. To keep the size of parse tables within reason, most parser generators are limited to one token of lookahead. Some parser generators do make selected use of extra lookahead where such information is helpful.

Most of this chapter is devoted to the problems of constructing LR parse tables. Before we consider such techniques, it is instructive to formalize the definition of LR(k) in terms of the properties an LR(k) parser must possess. All **shift-reduce parsers** operate by shifting symbols and examining lookahead information until the end of the handle is found. Then the handle is reduced to a nonterminal, which replaces the handle on the stack. An LR(k) parser, guided by its parse table, must decide whether to shift or reduce, knowing only the symbols already shifted (left context) and the next k lookahead symbols (right context).

A grammar is LR(k) if, and only if, it is possible to construct an LR parse table such that k tokens of lookahead allows the parser to recognize *exactly* those strings in the grammar's language. An important property of an LR parse table is that each cell accommodates only one entry. In other words, the LR(k) parser is **deterministic—**exactly one action can occur at each step.

We next formalize the properties of an LR(k) grammar, using the following definitions and notation from Chapter 4:

- If $\mathsf{S} \Rightarrow^{\star} \beta$, then β is a **sentential form** of a grammar with goal symbol S.
- $\mathsf{First}_k(\alpha)$ is the set of length-k terminal prefixes that can be derived from α.

Assume that in some LR(k) grammar there are two sentential forms $\alpha\beta w$ and $\alpha\beta y$ with $w, y \in \Sigma^{\star}$. These sentential forms share a common prefix $\alpha\beta$. Furthermore, with the prefix $\alpha\beta$ on the stack, assume that their k-token lookahead sets are identical: $\mathsf{First}_k(w) = \mathsf{First}_k(y)$. Suppose the parse table calls for reduction by $\mathsf{A} \rightarrow \beta$ given the left context of $\alpha\beta$ and the k-token lookahead present in w; this results in $\alpha\mathsf{A}w$. With the same lookahead information in y, the LR(k) parser must make the same decision: $\alpha\beta y$ becomes $\alpha\mathsf{A}y$. Formally, a grammar is LR(k) if, and only if, the following conditions imply $\alpha\mathsf{A}y = \gamma\mathsf{B}x$.

- $\mathsf{S} \Rightarrow^{\star}_{\mathrm{rm}} \alpha\mathsf{A}w \Rightarrow_{\mathrm{rm}} \alpha\beta w$
- $\mathsf{S} \Rightarrow^{\star}_{\mathrm{rm}} \gamma\mathsf{B}x \Rightarrow_{\mathrm{rm}} \alpha\beta y$
- $\mathsf{First}_k(w) = \mathsf{First}_k(y)$

This implication allows reduction by $\mathsf{A} \rightarrow \beta$ whenever $\alpha\beta$ is on top-of-stack and the k-symbol lookahead is $\mathsf{First}_k(w)$. In other words, LR(k) parsers are defined such that they can always determine the correct reduction given the following:

- The left context up to the end of the handle
- The next k symbols of the input

This definition is instructive in that it defines the minimum properties a grammar must possess to be parsable by LR(k) techniques. It does not tell us how to *build* a suitable LR(k) parser; in fact, the primary contribution of Knuth's early work [Knu65] was an algorithm for LR(k) construction. We begin with the simplest LR(0) parser, which lacks sufficient power for most applications. After examining problems that arise in LR(0) construction we turn to the more powerful LR(1) parsing method and its variants.

When LR parser construction fails, the associated grammar may be *ambiguous* (as discussed in Section 6.4.1). For other grammars, the parser may require more information about the unconsumed input string (lookahead). In fact, some grammars require an *unbounded* amount of lookahead (Section 6.4.2). In either case, the parser-generator identifies *inadequate* parsing states that are useful for resolving the problem. While it can be shown that there can be no algorithm to determine if a grammar is ambiguous, Section 6.4 describes techniques that work well in practice.

6.3 LR(0) Table Construction

The table-construction methods discussed in this chapter analyze a grammar to devise a parse table suitable for use in the generic parser presented in Figure 6.3. Each symbol in the terminal and nonterminal alphabets corresponds to a column of the table. The analysis proceeds by exploring the state-space of the parser. Each state corresponds to a row of the parse table. Because the state-space is necessarily finite, this exploration must terminate at some point.

After the parse table's rows have been determined, analysis then attempts to fill in the cells of the table. Because we are interested only in *deterministic* parsers, each table cell can hold one entry. An important outcome of the LR construction methods is the determination of **inadequate states**—states that lack sufficient information to place at most one parsing action in each column.

We begin by considering LR(0) table construction for the grammar shown in Figure 6.2. In constructing the parse table, we are required to consider the parser's progress in recognizing a rule's **right-hand side** (RHS). For example, consider the rule E → plus E E. Prior to reducing the RHS to E, each component of the RHS must be found. A plus must be identified, then two Es must be found. Once these three symbols are on top-of-stack, then it is possible for the parser to apply the reduction and replace the three symbols with the **left-hand side** (LHS) symbol E.

To keep track of the parser's progress, we introduce the notion of an **LR(0) item**—a grammar production with a **bookmark** that indicates the current progress through the production's RHS. The bookmark is analogous to the "progress bar" present in many applications, which indicates the completed fraction of a task. Figure 6.8 shows the progress of the bookmark symbol (•) through all of the possible LR(0) items for the production E → plus E E. A **fresh** item has its marker at the extreme left, as in E → • plus E E. When the marker is at the extreme right, as in E → plus E E •, we say the item is **reducible**. A rule of the form A → λ deserves special consideration. The

LR(0) item	Progress of rule in this state
E→ • plus E E	Beginning of rule
E→plus • E E	Processed a plus, expect an E
E→plus E • E	Expect another E
E→plus E E •	Handle on top-of-stack, ready to reduce

Figure 6.8: LR(0) items for production E→plus E E.

symbol λ denotes that there is *nothing* on this rule's RHS. We make this clear when representing such rules as items. For A→λ, the only possible item is the reducible A→ • .

We now define a **parser state** as a set of LR(0) items. While each state is formally a set, we drop the usual braces notation and simply list the the set's elements (items). The LR(0) construction algorithm is shown in Figure 6.9.

The start state for our parser—nominally state 0—is formed at Marker ⑦ by including fresh items for each of the grammar's goal-symbol productions. For our example grammar in Figure 6.2, we initialize the start state with Start→ • E \$. The algorithm maintains *WorkList*—a set of states that need to be processed by the loop at Marker ⑧. Each state formed during processing is passed through AddState, which determines at Marker ⑨ if the set of items has already been identified as a state. If not, then a new state is constructed at Marker ⑩. The resulting state is added to *WorkList* at Marker ⑪. The state's row in the parse table is initialized at Marker ⑫.

The processing of a state begins when the loop at Marker ⑧ extracts a state *s* from the *WorkList*. When ComputeGoto is called on state *s*, the following steps are performed:

1. The **closure** of state *s* is computed at Marker ⑰. If a nonterminal B appears just after the bookmark symbol (•), then in state *s* we can process a B once one has been found. Transitions from state *s* must include actions that can lead to discovery of a B. Closure in Figure 6.10 returns a set that includes its supplied set of items along with fresh items for B's rules. The addition of fresh items can trigger the addition of still more fresh items. Because the computed answer is a set, no item is added twice. The loop at Marker ⑭ continues until nothing new is added. Thus, this loop eventually terminates.

2. Marker ⑱ determines transitions from *s*. When a new state is added during LR(0) construction, Marker ⑫ sets all actions for this state as error. Transitions are defined for each grammar symbol $\mathcal{X}$ that appears after the bookmark. ComputeGoto in Figure 6.10 defines a transition at Marker ⑳ from *s* to a (potentially new) state that reflects the parser's

```
function ComputeLR0( Grammar ) returns ( Set, State )
    States ← $\emptyset$
    StartItems ← $\{$ Start → $\bullet$ RHS( $p$ ) | $p \in$ ProductionsFor( Start ) $\}$    ⑦
    StartState ← AddState( States, StartItems )
    while ( $s$ ← WorkList.ExtractElement( ) ) $\neq \perp$ do                  ⑧
        call ComputeGoto( States, $s$ )
    return ( ( States, StartState ) )
end
function AddState( States, items ) returns State
    if items $\notin$ States                                                  ⑨
    then
        $s$ ← newState( items )                                               ⑩
        States ← States $\cup \{ s \}$
        WorkList ← WorkList $\cup \{ s \}$                                     ⑪
        Table[ $s$ ][ $\star$ ] ← error                                        ⑫
    else   $s$ ← FindState( items )
    return ( $s$ )
end
function AdvanceDot( state, $\mathcal{X}$ ) returns Set
    return ( $\{$ A → $\alpha\mathcal{X} \bullet \beta$ | A → $\alpha \bullet \mathcal{X}\beta \in$ state $\}$ )   ⑬
end
```

Figure 6.9: LR(0) construction.

progress after shifting across *every* item in this state with $\mathcal{X}$ after the bookmark. All such items indicate transition to the same state since the parsers we construct must operate deterministically. In other words, the parse table has only one entry for a given state and symbol.

We now construct an LR(0) parse table for the grammar shown in Figure 6.2. In Figure 6.11 each state is shown as a separate box. The **kernel** of state s is the set of items explicitly represented in the state. We use the convention of drawing a line within a state to separate the kernel and closure items, as in States 0, 1, and 5. In the other states, no item contains a $\bullet$ before a nonterminal, so no closure items are indicated for those states. Next to each item in each state is the state number reached by shifting the symbol next to the item's bookmark.

In Figure 6.11 the transitions are also shown with labeled edges between the states. If a state contains a reducible item, then the state is double-boxed. The edges and double-boxed states emphasize that the basis for LR parsing is a **deterministic finite automaton** (DFA), called the **characteristic finite-state machine** (CFSM).

A **viable prefix** of a right sentential form is any prefix that does not extend beyond its handle. Formally, a CFSM recognizes its grammar's viable

```
function CLOSURE( $state$ ) returns $Set$
    $ans \leftarrow state$
    repeat                                                          (14)
        $prev \leftarrow ans$
        foreach $A \rightarrow \alpha \bullet B\gamma \in ans$ do                            (15)
            foreach $p \in$ PRODUCTIONSFOR( $B$ ) do
                $ans \leftarrow ans \cup \{ B \rightarrow \bullet \mathrm{RHS}(p) \}$                 (16)
    until $ans = prev$
    return ( $ans$ )
end
procedure COMPUTEGOTO( $States, s$ )
    $closed \leftarrow$ CLOSURE( $s$ )                                        (17)
    foreach $\mathcal{X} \in (N \cup \Sigma)$ do                                     (18)
        $RelevantItems \leftarrow$ ADVANCEDOT( $closed, \mathcal{X}$ )                   (19)
        if $RelevantItems \neq \emptyset$
        then
            $Table[s][\mathcal{X}] \leftarrow$ shift ADDSTATE( $States, RelevantItems$ )  (20)
end
```

Figure 6.10: LR(0) closure and transitions.

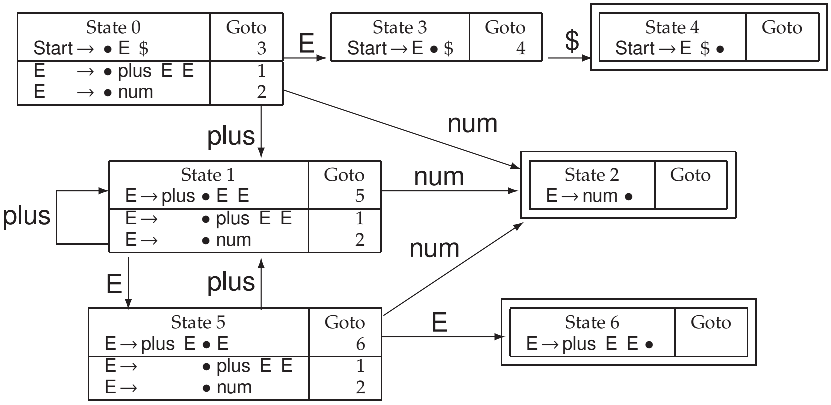


Figure 6.11: LR(0) computation for Figure 6.2, shown as a characteristic finite-state machine. State 0 is the initial state, and the double-boxed states are accept states.

Sentential Prefix	Transitions	Resulting Sentential Form
		plus plus num num num $
plus plus num	States 1, 1, and 2	plus plus E num num $
plus plus E num	States 1, 1, 5, and 2	plus plus E E num $
plus plus E E	States 1, 1, 5, and 6	plus E num $
plus E num	States 1, 5, and 2	plus E E $
plus E E	States 1, 5, and 6	E $
E $	States 1, 3, and 4	Start

Figure 6.12: Processing of plus plus num num num $ by the LR(0) machine in Figure 6.11.

prefixes. Each transition shifts the symbols of a valid sentential form. When the automaton arrives in a double-boxed state, it has processed a viable prefix that ends with a handle. The handle is the RHS of the (unique) reducible item in the state. At this point, a reduction can be performed. The sentential form produced by the reduction can be processed anew by the CFSM. This process can be repeated until the grammar's goal symbol is shifted (successful parse) or the CFSM blocks (an input error).

For the input string plus plus num num num $, Figure 6.12 shows the results of repeatedly presenting the (reverse) derived sentential forms to the CFSM. This approach serves to illustrate how a CFSM recognizes viable prefixes. However, it is unnecessary to make repeated passes over an input string's sentential forms. For example, each of the repeated passes over the input string's first two tokens (plus plus) causes the parser to enter State 1. Because the CFSM is deterministic, processing a given sequence of vocabulary symbols always has the same effect. Thus, the parsing algorithm given in Figure 6.3 does not make repeated passes over the derived sentential forms. Instead, the parse state is recorded after each shift so that the current parse state is always associated with whatever symbol happens to be on top of stack. As reductions eat into the stack, the symbol exposed at the new top-of-stack bears the current state, which is the state the CFSM would reach if it rescanned the entire prefix of the current sentential form up to, and including, the current top-of-stack symbol.

If a grammar is LR(0), then the construction discussed in this section has the following properties (refer to Figure 6.11 as a reference):

- Given a syntactically correct input string, the CFSM will block only in double-boxed states, which call for a reduction. The CFSM clearly shows that no progress can occur unless a reduction takes place.

```
procedure CompleteTable( Table, grammar )
    call ComputeLookahead( )
    foreach state ∈ Table do
        foreach rule ∈ Productions(grammar) do
            call TryRuleInState( state, rule )
    call AssertEntry( StartState, GoalSymbol, accept )                ㉑
end
procedure AssertEntry( state, symbol, action )
    if Table[state][symbol] = error                                   ㉒
    then   Table[state][symbol] ← action
    else
        call ReportConflict( Table[state][symbol], action )           ㉓
end
```

Figure 6.13: Completing an LR(0) parse table.

- There is at most one item present in any double-boxed state—the rule that should be applied upon entering the state. Upon reaching such states, the CFSM has completely processed a rule. The associated item is **reducible**, with the marker moved as far right as possible.

- If the CFSM's input string is syntactically invalid, then the parser will enter a state such that the offending terminal symbol cannot be shifted.

The table established during LR(0) construction at Marker ⑳ in Figure 6.10 is almost suitable for parsing by the algorithm in Figure 6.3. Each state is a row of the table, and the columns represent grammar symbols. Entries are present only where the LR(0) construction allows transition between states—these are the shift actions. To complete the table we apply the algorithm in Figure 6.13, which establishes the appropriate reduce actions.

For LR(0), the decision to call for a reduce is reflected in the code of Figure 6.14; arrival in a double-boxed state signals a reduction irrespective of the next input token. As reduce actions are inserted, AssertEntry reports any **conflicts** that arise when a given state and grammar symbol call for multiple parsing actions. Marker ㉒ allows an action to be asserted only if the relevant table cell was previously undefined (cells are initialized to the value of error at Marker ⑫). Finally, Marker ㉑ calls for acceptance when the goal symbol is shifted in the table's start state. Given the construction in Figure 6.11 and the grammar in Figure 6.2, LR(0) analysis yields the parse table is shown in Figure 6.15.

```
procedure ComputeLookahead( )
    /★  Reserved for the LALR($k$) computation given in Section 6.5.2  ★/
end
procedure TryRuleInState($s, r$)
    if LHS($r$)$\rightarrow$RHS($r$) $\bullet$ $\in s$
    then
        foreach $\mathcal{X} \in (\Sigma \cup N)$ do call AssertEntry($s, \mathcal{X}$, reduce r )
end
```

Figure 6.14: LR(0) version of TryRuleInState.

State	num	plus	$	Start	E
0	$\boxed{2}$	$\boxed{1}$		accept	$\boxed{3}$
1	$\boxed{2}$	$\boxed{1}$			$\boxed{5}$
2	reduce 3				
3			$\boxed{4}$		
4	reduce 1				
5	$\boxed{2}$	$\boxed{1}$			$\boxed{6}$
6	reduce 2				

Figure 6.15: LR(0) parse table for the grammar in Figure 6.2.

6.4 Conflict Diagnosis

Sometimes LR construction is not successful, even for simple languages and grammars. In the following sections we consider table-construction methods that are more powerful than LR(0), thereby accommodating a much larger class of grammars. This section examines why **conflicts** arise during LR table construction and develops approaches for understanding and resolving such conflicts.

The generic LR parser shown in Figure 6.3 is *deterministic*. Given a parse state and an input symbol, the parse table can specify exactly one action to be performed by the parser—shift, reduce, accept, or error. In Chapter 3 we tolerated nondeterminism in the scanner specification because we knew of an efficient algorithm for transforming a nondeterministic DFA into a deterministic DFA. Unfortunately, no such algorithm is possible for stack-based parsing engines. Some CFGss cannot be parsed deterministically. In such cases, perhaps there is another grammar that generates the same language and for which a deterministic parser can be constructed. There are **context-free languages** (CFLs) that provably cannot be parsed using the (deterministic) LR method (see Exercise 11). However, programming languages are typically designed to be parsed deterministically.

A parse table **conflict** arises when the table-construction method cannot decide between multiple alternatives for some table-cell entry. We then say that the associated state (row of the parse table) is **inadequate** for that method. An inadequate state for a weaker table-construction algorithm can sometimes be resolved by a stronger algorithm. For example, the grammar of Figure 6.4 is not LR(0) since a mix of shift and reduce actions can be seen in State 0. However, the table-construction algorithms introduced in Section 6.5.1 resolve the LR(0) conflicts for this grammar.

If we consider the possibilities for multiple table-cell entries, only the following two cases are troublesome for LR(k) parsing:

- **shift/reduce conflicts** exist in a state when table construction cannot use the next k tokens to decide whether to shift the next input token or call for a reduction. The bookmark symbol must occur before a terminal symbol t in one of the state's items, so that a shift of t could be appropriate. The bookmark symbol must also occur at the end of some other item, so that a reduction in this state is also possible.

- **reduce/reduce conflicts** exist when table construction cannot use the next k tokens to distinguish between multiple reductions that could be applied in the inadequate state. Of course, a state with such a conflict must have at least two reducible items.

Other combinations of actions in a table cell do not make sense. For example, it cannot be the case that some terminal t could be shifted but also cause an error. Additionally, there cannot be a shift/shift error: if a state admits the shifting of terminal symbols t and u, then the target state for the two shifts is different, and there is no conflict. Exercise 13 considers the impossibility of a shift/reduce conflict on a nonterminal symbol.

Although the table-construction methods we discuss in the following sections vary in power, each is capable of reporting conflicts that render a state inadequate. Conflicts arise for one of the following reasons:

- The grammar is **ambiguous**. No (deterministic) table-construction method can resolve conflicts that arise due to ambiguity. Ambiguous grammars are considered in Section 6.4.1, but here we summarize some approaches for addressing the ambiguity.

  If a grammar is ambiguous, then some input string has at least two distinct parse trees. The following must then be considered:

  - If both parse trees are desirable (as in Exercise 38), then the grammar's language contains a **pun**. While puns may be tolerable in natural languages, they are undesirable in the design of computer languages. A program specified in a computer language should have an unambiguous interpretation.
  - If only one tree has merit, then the grammar can often be modified to eliminate the ambiguity. While there are inherently ambiguous languages (see Exercise 14), computer languages are not designed with this property.

- The grammar is not ambiguous, but the current table-building approach could not resolve the conflict. In this case, the conflict might disappear if one or more of the following approaches is taken:

  - The current table-construction method is given more lookahead.
  - A more powerful table-construction method is used.

  It is possible that no amount of lookahead or table-building power can resolve the conflict, even if the grammar is unambiguous. We consider such a grammar in Section 6.4.2 and in Exercise 37.

When an LR(k) construction algorithm develops an inadequate state, it is an unfortunate but important fact that it is impossible to decide automatically which of the above problems afflicts the grammar. This follows from the impossibility of an algorithm to determine if an arbitrary CFGs is ambiguous [HU79, GJ79]. It is therefore also impossible to determine generally whether a bounded amount of lookahead can resolve an inadequate state.

As a result, human (rather than mechanical) reasoning is required to understand and repair grammars for which conflicts arise. Sections 6.4.1 and 6.4.2 develop intuition and strategies for such reasoning.

6.4.1 Ambiguous Grammars

Consider the grammar and its LR(0) construction shown in Figure 6.16. The grammar generates sums of numbers using the familiar *infix* notation. In the LR(0) construction, all states are adequate except State 5. In this state a plus can be shifted to arrive in State 3. However, State 5 also allows reduction by the rule E → E plus E. This inadequate state exhibits a shift/reduce conflict for LR(0). To resolve this conflict it must be decided how to fill in the LR parse table for State 5 and the symbol plus. Unfortunately, this grammar is ambiguous, so a unique entry cannot be determined.

While there is no automatic method for determining if an arbitrary grammar is ambiguous, the inadequate states can provide valuable assistance in

```
1  Start → E $
2  E     → E plus E
3        | num
```

State 0	Goto
Start → • E $	2
E → • E plus E	2
E → • num	1

State 1	Goto
E → num •	

State 2	Goto
E → E • plus E	3
Start → E • $	4

State 3	Goto
E → E plus • E	5
E → • E plus E	5
E → • num	1

State 4	Goto
Start → E $ •	

State 5	Goto
E → E plus E •	
E → E • plus E	3

Figure 6.16: An ambiguous expression grammar.

finding a string with multiple derivations—should one exist. Recall that a parser state represents transitions made by the CFSM when recognizing viable prefixes. The bookmark symbol shows the progress made thus far. Symbols appearing after the bookmark are symbols that can be shifted to make progress toward a successful parse. While our ultimate goal is the discovery of an input string with multiple derivations, we begin by trying to find an ambiguous sentential form. Once identified, the sentential form can easily be extended into a terminal string by replacing nonterminals using the grammar's productions.

Using State 5 in Figure 6.16 as an example, the steps taken to understand conflicts are as follows:

1. Using the parse table or CFSM, determine a sequence of vocabulary symbols that cause the parser to move from the start state to the inadequate state. For Figure 6.16, the simplest such sequence is E plus E, which passes through States 0, 2, 3, and 5. Thus, in State 5 we have E plus E on the top-of-stack. One option is a reduction by E → E plus E. However, with the item E → E • plus E, it is also possible to shift a plus and then an E.

2. If we line up the dots of these two items, we obtain a snapshot of what is on the stack upon arrival in this state and what may be successfully shifted in the future. Here we obtain the sentential form prefix E plus E • plus E. The shift/reduce conflict tells us that there are two potentially successful parses. We therefore try to construct two derivation trees for E plus E plus E, one assuming the reduction at the bookmark symbol and one assuming the shift. Completing either derivation may require extending this sentential prefix so that it becomes a sentential

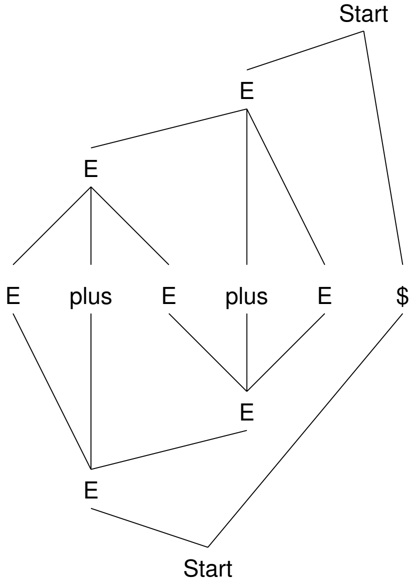


Figure 6.17: Two derivations for E plus E plus E $. The parse tree on top favors reduction in State 5; the parse tree on bottom favors a shift.

form: a string of vocabulary symbols derivable (in two different ways) from the goal symbol.

For our example, E plus E plus E is almost a complete sentential form. We need only append $ to obtain E plus E plus E $.

To emphasize that the derivations are constructed for the same string, Figure 6.17 shows the derivations above and below the sentential form. If the reduction is performed, then the early portion of E plus E plus E $ is structured under a nonterminal; otherwise, the input string is shifted so that the latter portion of the sentential form is reduced first. The parse tree that favors the reduction in State 5 corresponds to a **left-associative grouping** for addition, while the shift corresponds to a **right-associative grouping**.

Having analyzed the ambiguity in the grammar of Figure 6.16, we next eliminate the ambiguity by creating a grammar that favors left-association—the reduction instead of the shift. Such a grammar and its LR(0) construction are shown in Figure 6.18. The grammars in Figures 6.16 and 6.18 generate the same language. In fact, the language is regular, denoted by the regular expression num (plus num)* $. So we see that even simple languages can have ambiguous grammars. In practice, diagnosing ambiguity can be more

```
1  Start → E $
2  E     → E plus num
3        | num
```

State 0	Goto
Start → • E $	2
E → • E plus num	2
E → • num	1

State 1	Goto
E → num •	

State 2	Goto
E → E • plus num	3
Start → E • $	4

State 3	Goto
E → E plus • num	5

State 4	Goto
Start → E $ •	

State 5	Goto
E → E plus num •	

Figure 6.18: Unambiguous grammar for infix sums and its LR(0) construction.

difficult. In particular, finding the ambiguous sentential form may require significant extension of a viable prefix. Exercises 37 and 38 provide practice in finding and fixing a grammar's ambiguity.

6.4.2 Grammars that are not LR(*k*)

Figure 6.19 shows a grammar and a portion of its LR(0) construction for a language similar to infix addition, where expressions end in either a or b. The complete LR(0) construction is left as Exercise 15. State 2 contains a reduce/reduce conflict. In this state, it is not clear whether num should be reduced to an E or an F. The viable prefix that takes us to State 2 is simply num. To obtain a sentential form, this must be extended either to num a $ or num b $. If we use the former sentential form, then F cannot be involved in the derivation. Similarly, if we use the latter sentential form, E is not involved. Thus, progress past num cannot involve more than one derivation, and the grammar is not ambiguous.

Since LR(0) construction failed for the grammar in Figure 6.19, we could try a more ambitious table-construction method from among those discussed in Sections 6.5.1, 6.5.2, and 6.5.4. It turns out that none can succeed. All LR(*k*) constructions analyze grammars using *k* lookahead symbols. If a grammar is LR(*k*), then there is some value of *k* for which all states are adequate in the LR(*k*) construction described in Section 6.5.4. The grammar in Figure 6.19 is not LR(*k*) for any *k*. To see this, consider the following rightmost derivation of a sufficiently long string num plus ... plus num a:

```
1  Start  → Exprs $
2  Exprs  → E a
3         | F b
4  E      → E plus num
5         | num
6  F      → F plus num
7         | num
```

State 0	Goto
Start → • Exprs $	1
Exprs → • E a	4
Exprs → • F b	3
E → • E plus num	4
E → • num	2
F → • F plus num	3
F → • num	2

State 2	Goto
E → num • F → num •	

Figure 6.19: A grammar that is not LR(k).

$$
\begin{aligned}
\text{Start} &\Rightarrow_{\text{rm}} \text{Exprs \$} \\
&\Rightarrow_{\text{rm}} \text{E a \$} \\
&\Rightarrow_{\text{rm}} \text{E plus num a \$} \\
&\Rightarrow_{\text{rm}}^{\star} \text{E plus ... plus num a \$} \\
&\Rightarrow_{\text{rm}} \text{num plus ... plus num a \$}
\end{aligned}
$$

A bottom-up parse must play the above derivation backwards. Thus, the first few steps of the parse will be:

0	Initial Configuration	num plus . . . plus num a $
0 │ num 2	shift num	plus . . . plus num a $

With num on top-of-stack, we are in State 2. A deterministic, bottom-up parser must decide at this point whether to reduce num to an E or an F. If the decision were delayed, then the reduction would have to take place in the middle of the stack, and this is not allowed. The information needed to resolve the reduce/reduce conflict appears just before the $ symbol. Unfortunately, the relevant a or b could be arbitrarily far ahead in the input, because strings derived from E or F can be arbitrarily long.

In summary, simple grammars and languages can have subtle problems that prohibit generation of a bottom-up parser. It follows that a top-down parser would also fail on such grammars (Exercise 16). The LR(0) construction

1	Start	→	E $
2	E	→	E plus num
3		|	E times num
4		|	num

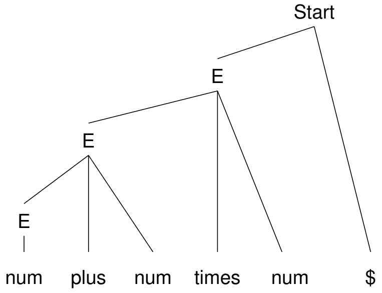


Figure 6.20: Expressions with sums and products.

can provide important clues for diagnosing a grammar's inadequacies; understanding and resolving such conflicts requires human intelligence. Also, the LR(0) construction forms the basis of the more advanced constructions considered next.

6.5 Conflict Resolution and Table Construction

While LR(0) construction succeeded for the grammar in Figure 6.18, most grammars require some lookahead during table construction. Sections 6.5.1, 6.5.2, and 6.5.4 consider methods that, while based on the LR(0) construction, use increasingly sophisticated *lookahead techniques* to resolve conflicts. Section 6.5.1 presents the SLR(k) construction, which is simple but does not work as well as the LALR(k) construction introduced in Section 6.5.2. The most powerful technique, LR(k), is presented in Section 6.5.4.

6.5.1 SLR(k) Table Construction

The SLR(k) (Simple LR with k tokens of lookahead) method attempts to resolve inadequate states using grammar analysis methods presented in Chapter 4. To demonstrate the SLR(k) construction, we require a grammar that is not LR(0). We begin by extending the grammar in Figure 6.18 to accommodate expressions involving sums and products. Figure 6.20 shows such a grammar along with a parse tree for the string num plus num times num $. Exercise 17

1	Start	→	E $
2	E	→	E plus T
3		|	T
4	T	→	T times num
5		|	num

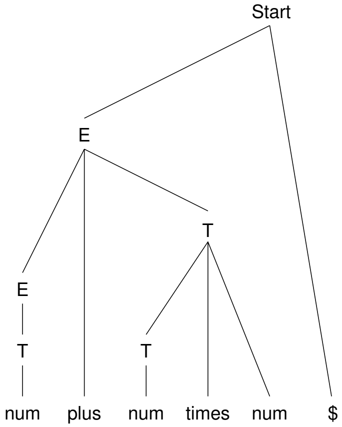


Figure 6.21: Grammar for sums of products.

shows that this grammar is LR(0). However, it does not produce the parse trees that structure the expressions appropriately. The parse tree shown in Figure 6.20 structures the computation by adding the first two nums and then multiplying that sum by the third num. As such, the input string $3+4*7$ would produce a value of 49 if evaluation were guided by the computation's parse tree.

A common convention in mathematics is that multiplication has precedence over addition. Thus, the computation $3 + 4 * 7$ should be viewed as adding 3 to the product $4 * 7$, resulting in the value 31. Such conventions are typically adopted in programming language design, in an effort to simplify program authoring and readability. We therefore seek a parse tree that appropriately structures expressions involving multiplication and addition.

To develop the grammar that achieves the desired effect, we first observe that a string in the language of Figure 6.20 should be regarded as a *sum of products*. The grammar in Figure 6.18 generates sums of nums. A common technique to expand a language involves replacing a terminal symbol in the grammar by a nonterminal whose role in the grammar is equivalent. To produce a sum of Ts rather than a sum of nums, we need only replace num with T to obtain the rules for E shown in Figure 6.21. To achieve a sum of products, each T can now derive a product, with the simplest product consisting of a single num. Thus, the rules for T are based on the rules for E, substituting times for plus. Figure 6.21 shows a parse tree for the input string from Figure 6.20, with multiplication having precedence over addition.

Figure 6.22 shows a portion of the LR(0) construction for our precedence-respecting grammar. States 1 and 6 are inadequate for LR(0) because in each of these states, there is the possibility of shifting a times or applying a reduction to E. Figure 6.22 shows a sequence of parser actions for the sentential form E plus num times num \$, leaving the parser in State 6.

Consider the shift/reduce conflict of State 6. To determine if the grammar in Figure 6.21 is ambiguous, we turn to the methods described in Section 6.4. We proceed by assuming the shift and reduce are *each* possible given the sentential form E plus T times num \$.

- If the shift is taken, then we can continue the parse in Figure 6.22 to obtain the parse tree shown in Figure 6.21.
- Reduction by rule E→E plus T yields E times num \$, which causes the CFSM in Figure 6.22 to block in State 3 with no progress possible. If we try to reduce using T→num, then we obtain E times T \$, which can be further reduced to E times E \$. Neither of these phrases can be further reduced to the goal symbol.

Thus, E times num \$ is not a valid sentential form for this grammar and a reduction in State 6 for this sentential form is inappropriate.

With the item E→E plus T • in State 6, reduction by E→E plus T must be appropriate under some conditions. If we examine the sentential forms E plus T \$ and E plus T plus num \$, we see that the E→E plus T must be applied in State 6 when the next input symbol is plus or \$, but not times. LR(0) could not selectively call for a reduction in any state; however, methods that can consult lookahead information in TryRuleInState can resolve this conflict.

Consider the sequence of parser actions that could be applied between a reduction by E→E plus T and the next shift of a terminal symbol. Following the reduction, E must be shifted onto the stack. At this point, assume terminal symbol plus is the next input symbol. If the reduction to E can lead to a successful parse, then plus can appear next to E in *some* valid sentential form. An equivalent statement is plus ∈ Follow(E), using the Follow computation from Chapter 4.

SLR(k) parsing uses Follow$_k$(A) to call for a reduction to A in any state containing a reducible item for A. Algorithmically, we obtain SLR(k) by performing the LR(0) construction in Figure 6.9; the only change is to the method TryRuleInState, whose SLR(1) version is shown in Figure 6.23. For our example, States 1 and 6 are resolved by computing Follow(E) = { plus, \$ }. The SLR(1) parse table that results from this analysis is shown in Figure 6.24.

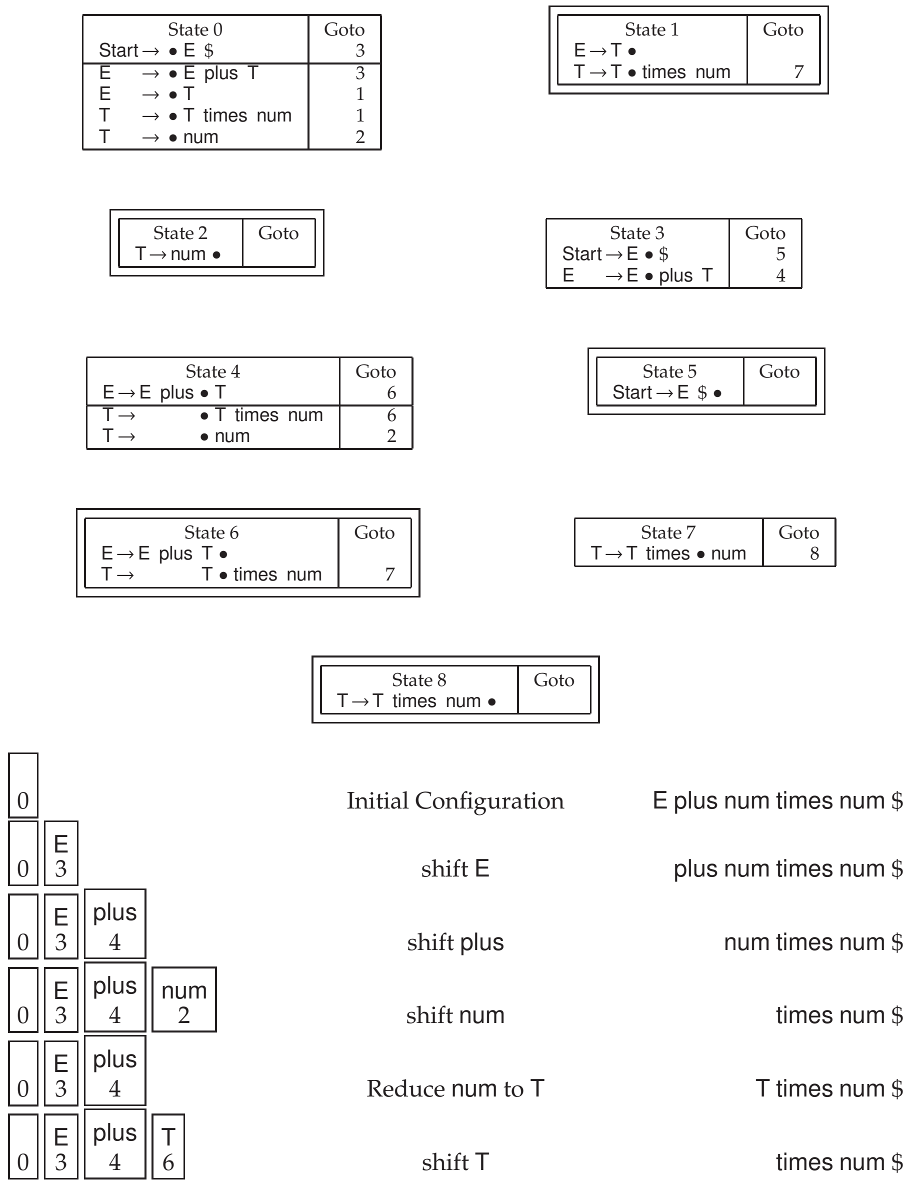


Figure 6.22: LR(0) construction and parse leading to inadequate State 6.

```
procedure TryRuleInState($s, r$)
    if LHS($r$)$\rightarrow$RHS($r$) $\bullet \in s$
    then
        foreach $\mathcal{X} \in$ Follow(LHS($r$)) do
            call AssertEntry($s, \mathcal{X}$, reduce r)
end
```

Figure 6.23: SLR(1) version of TryRuleInState.

State	num	plus	times	$	Start	E	T
0	$\boxed{2}$				accept	$\boxed{3}$	$\boxed{1}$
1		3	$\boxed{7}$	3			
2		5	5	5			
3		$\boxed{4}$		$\boxed{5}$			
4	$\boxed{2}$						$\boxed{6}$
5				1			
6		2	$\boxed{7}$	2			
7	$\boxed{8}$						
8		4	4	4			

Figure 6.24: SLR(1) parse table for the grammar in Figure 6.21.

6.5.2 LALR(k) Table Construction

SLR(k) attempts to resolve LR(0) inadequate states using Follow_k information. Such information is computed by considering *all* of a grammar's rules. Sometimes SLR(k) construction fails only because the Follow_k information is not *rule specific*. Consider the grammar and its partial LR(0) construction shown in Figure 6.25. This grammar generates the language $\{\,\mathsf{a}, \mathsf{ab}, \mathsf{ac}, \mathsf{xac}\,\}$. The grammar is not ambiguous because each of these strings has a unique derivation. However, State 3 has an LR(0) shift/reduce conflict. SLR(k) tries to resolve the shift/reduce conflict by computing

$$\mathsf{Follow}_k(\,\mathsf{A}\,) = \{\,\mathsf{b}\$^{k-1}, \mathsf{c}\$^{k-1}, \$^{k}\,\}$$

In other words, A can be followed in some sentential form by any number of \$ (end-of-string) symbols, possibly prefaced by b or c. This information is insufficient to resolve the shift/reduce conflict in State 3. Based on SLR(1) analysis, the symbol c could signal a shift to State 6 or a reduction by $\mathsf{A}\rightarrow\mathsf{a}$.

If we examine States 0 and 3 more carefully, we see that it is not possible for c to occur after the expansion of A in State 3. In the closure of State 0, a fresh item for A was created by the item $\mathsf{S}\rightarrow\ \bullet\ \mathsf{A}\ \mathsf{B}$. Following the shift of A, only b or \$ can occur—there is no sentential form A c \$.

Given the above analysis, we can address the shift/reduce conflict by modifying the grammar to have two "versions" of A. Using A_1 and A_2, the resulting SLR(1) grammar is shown in Figure 6.26. Here, State 2 is resolved since $\mathsf{Follow}(\,\mathsf{A}_1\,) = \{\,\$, \mathsf{b}\,\}$.

To summarize, SLR(k) has difficulty with the grammar in Figure 6.25 because SLR's Follow sets are computed using all of a grammar's rules. Copying productions and renaming nonterminals can cause the Follow computation to become more production-specific, as in Figure 6.26. However, this is tedious and the resulting grammar is more difficult to understand and maintain. In this section we consider LALR(k) (<u>L</u>ookahead <u>A</u>head LR with k tokens of lookahead) parsing, which offers a more specialized computation of the symbols that can follow a nonterminal. The term LALR is not particularly informative—SLR and LR also use lookahead. However, LALR offers superior lookahead analysis for constructing the bottom-up parsing table.

Like SLR(k), LALR(k) is based on the LR(0) construction given in Section 6.3. Thus, an LALR(k) table has the same number of rows (states) as does an LR(0) table for the same grammar. While LR(k) (discussed in Section 6.5.4) offers more powerful lookahead analysis, this is achieved at the expense of introducing (typically, many) more states.

1	Start	→	S $
2	S	→	A B
3		|	a c
4		|	x A c
5	A	→	a
6	B	→	b
7		|	λ

State 0	Goto
Start → • S $	4
S → • A B	2
S → • a c	3
S → • x A c	1
A → • a	3

State 3	Goto
S → a • c	6
A → a •	

Figure 6.25: A grammar that is not SLR(k).

1	Start	→	S $
2	S	→	A_1 B
3		|	a c
4		|	x A_2 c
5	A_1	→	a
6	A_2	→	a
7	B	→	b
8		|	λ

State 0	Goto
Start → • S $	3
S → • A_1 B	4
S → • a c	2
S → • x A_2 c	1
A_1 → • a	2

State 2	Goto
S → a • c	8
A_1 → a •	

Figure 6.26: An SLR(1) grammar for the language defined in Figure 6.25.

```
procedure TryRuleInState($s, r$)
    if $\text{LHS}(r) \rightarrow \text{RHS}(r) \bullet \ \in s$
    then
        foreach $\mathcal{X} \in \Sigma$ do
            if $\mathcal{X} \in \mathit{ItemFollow}((s, \text{LHS}(r) \rightarrow \text{RHS}(r) \bullet))$
            then call AssertEntry($s, \mathcal{X}$, reduce $r$)
end
```

Figure 6.27: LALR(1) version of TryRuleInState.

Due to its balance of power and efficiency, LALR(1) is the most popular LR table-building method. To obtain LALR(1), we redefine the following two methods from Figure 6.14:

TryRuleInState In the LALR(1) version shown in Figure 6.27, a reduce action is asserted only for those symbols that can occur after the reduction, according to *ItemFollow*. Exercise 26 investigates the relationship between an *ItemFollow* set and Follow as computed for SLR.

ComputeLookahead Figure 6.28 contains code to build and evaluate a *lookahead propagation graph*. The *ItemFollow*((*state*, *item*)) sets keep track of the symbols that can follow the *item* when its reduction occurs (i.e., when the bookmark is fully to the right). The details of the propagation graph are discussed in Section 6.5.3.

6.5.3 LALR Propagation Graph

We have not formally named each LR(0) item, but an item occurs at most once in any state. Thus, the pair $(s, \mathsf{A} \rightarrow \alpha \bullet \beta)$ suffices to identify an item $\mathsf{A} \rightarrow \alpha \bullet \beta$ that occurs in state s. For each valid state and item pair, Marker 24 in Figure 6.28 creates a vertex v in the LALR **propagation graph**. Each item in an LR(0) construction is represented by a vertex in this graph. The *ItemFollow* sets are initially empty, except for the augmenting item $\mathsf{Start} \rightarrow \bullet \mathsf{S}\ \$$ in the LR(0) start-state. Edges are placed in the graph between items i and j when the symbols that follow the reducible form of item i should be included in the corresponding set of symbols for item j. For the purposes of lookahead analysis, the input string can be considered to end with an arbitrary number of \$ (end-of-input) symbols. Marker 25 forces the entire program, derived from any rule for Start, to be followed by \$.

Marker 26 of the algorithm in Figure 6.28 considers items of the form $\mathsf{A} \rightarrow \alpha \bullet \mathsf{B}\gamma$ in state s. This generic item indicates that the bookmark is just before the symbol B, with grammar symbols in α appearing before the bookmark, and grammar symbols in γ appearing after B. Note that α or γ could be absent, in which case $\alpha = \lambda$ or $\gamma = \lambda$, respectively. The symbol B is always present unless the grammar rule is $\mathsf{A} \rightarrow \lambda$. The lookahead computation is specifically concerned with $\mathsf{A} \rightarrow \alpha \bullet \mathsf{B}\gamma$ when B is a nonterminal, because Closure in Figure 6.10 adds items to state s for each of B's productions. The symbols that can follow B depend on γ, which is either absent or present in the item. Also, even when γ is present, it is possible that $\gamma \Rightarrow^{\star} \lambda$. The algorithm in Figure 6.28 takes these cases into account as follows:

- For the item $\mathsf{A} \rightarrow \alpha \bullet \mathsf{B}\gamma$, any symbol in First($\gamma$) can follow each closure item $\mathsf{B} \rightarrow \bullet \, \delta$. This is true even when γ is absent: in such cases, First(λ) = $\emptyset$. Thus, Marker ㉘ places symbols from First(γ) in the *ItemFollow* sets for each $\mathsf{B} \rightarrow \bullet \, \delta$ in state s.

  ItemFollow sets are useful only when the bookmark progresses to the point of a reduction. $\mathsf{B} \rightarrow \bullet \, \delta$ is only the *promise* of a reduction to B once δ has been found. Thus, the lookahead symbols must accompany the bookmark's progress across δ so they are available for $\mathsf{B} \rightarrow \delta \bullet$ in the appropriate state. Such migration of lookahead symbols is represented by propagation edges.

- Edges must be placed in the propagation graph when the symbols that are associated with one item should be added to the symbols associated with another item. The two situations that call for the addition of propagation edges are as follows:

  - As described above, lookahead symbols introduced by Marker ㉘ are useful only when the bookmark has advanced to the end of the rule. In LR(0), the CFSM contains an edge from state s to state t when the advance of the bookmark symbol for an item in state s creates an item in state t. For lookahead propagation in LALR, the edges are more specific—Marker ㉗ places edges in the propagation graph between *items*, not states.

    Specifically, an edge is placed from an item $\mathsf{A} \rightarrow \alpha \bullet \mathsf{B}\gamma$ in state s to the item $\mathsf{A} \rightarrow \alpha\mathsf{B} \bullet \gamma$ in state t obtained by advancing the bookmark, if t is the CFSM state reached by processing a B in state s.

  - Consider again the item $\mathsf{A} \rightarrow \alpha \bullet \mathsf{B}\gamma$ and the closure items introduced when B is a nonterminal. When $\gamma \Rightarrow^{\star} \lambda$, either because γ is absent or because the string of symbols in γ can derive λ, then any symbol that can follow A can also follow B. Thus, Marker ㉙ places a propagation edge from the item for A to the item for B.

  The edges placed by these steps are used at Marker ㉚ to affect the appropriate *ItemFollow* sets. The loop at Marker ㉚ continues until no changes are observed in any *ItemFollow* set. This loop must eventually terminate because the lookahead sets are increased only by symbols drawn from a finite alphabet (Σ).

```
procedure ComputeLookahead( )
    call BuildItemPropGraph( )
    call EvalItemPropGraph( )
end
procedure BuildItemPropGraph( )
    foreach $s \in States$ do
        foreach $item \in state$ do
            $v \leftarrow Graph$.AddVertex( $(s, item)$ )                        (24)
            $ItemFollow(v) \leftarrow \emptyset$
    foreach $p \in$ ProductionsFor( Start ) do
        $ItemFollow((StartState, \mathsf{Start} \rightarrow \bullet \mathrm{RHS}(p))) \leftarrow \{\$\}$      (25)
    foreach $s \in States$ do
        foreach $\mathsf{A} \rightarrow \alpha \bullet \mathsf{B}\gamma \in s$ do                             (26)
            $v \leftarrow Graph$.FindVertex( $(s, \mathsf{A} \rightarrow \alpha \bullet \mathsf{B}\gamma)$ )
            call $Graph$.AddEdge( $v, (Table[s][B], \mathsf{A} \rightarrow \alpha\mathsf{B} \bullet \gamma)$ )   (27)
            foreach $(w \leftarrow (s, \mathsf{B} \rightarrow \bullet \delta)) \in Graph.Vertices$ do
                $ItemFollow(w) \leftarrow ItemFollow(w) \cup \mathsf{First}(\gamma)$          (28)
                if AllDeriveEmpty( $\gamma$ )                                 (29)
                then call $Graph$.AddEdge( $v, w$ )
end
procedure EvalItemPropGraph( )
    repeat                                                              (30)
        $changed \leftarrow$ false
        foreach $(v, w) \in Graph.Edges$ do
            $old \leftarrow ItemFollow(w)$
            $ItemFollow(w) \leftarrow ItemFollow(w) \cup ItemFollow(v)$
            if $ItemFollow(w) \neq old$
            then $changed \leftarrow$ true
    until not $changed$
end
```

Figure 6.28: LALR(1) version of ComputeLookahead.

State		LR(0) Item	Goto State	Prop Edges Placed by Step		Initialize *ItemFollow*	
				㉗	㉙	First(γ)	㉘
0	1	Start→ • S $	4	13		$	2,3,4
	2	S→ • A B	2	8	5	b	5
	3	S→ • a c	3	11			
	4	S→ • x A c	1	6			
	5	A→ • a	3	12			
1	6	S→x • A c	9	18		c	7
	7	A→ • a	10	19			
2	8	S→A • B	8	17	9,10		
	9	B→ • b	7	16			
	10	B→ •					
3	11	S→a • c	6	15			
	12	A→a •					
4	13	Start→S • $	5	14			
5	14	Start→S $ •					
6	15	S→a c •					
7	16	B→b •					
8	17	S→A B •					
9	18	S→x A • c	11	20			
10	19	A→a •					
11	20	S→x A c •					

Figure 6.29: LALR(1) analysis of the grammar in Figure 6.25.

Now consider the grammar in Figure 6.25 and its LALR(1) construction shown in Figure 6.29. The items listed under the column for Marker ㉗ are the targets of edges placed in the propagation graph to carry symbols to the point of reduction. For example, consider Items 6 and 7. For the item S→x • A c, we have γ = c. Thus, when the item A→ • a is generated in Item 7, c can follow the reduction to A. Marker ㉘ therefore adds c directly to Item 5's *ItemFollow* set. However, c is not useful until it is time to apply the reduction A→a. Thus, propagation edges are placed between Items 7 and 19 by Marker ㉗.

In most cases, lookahead is either *generated* (when First(γ) $\neq \emptyset$) or *propagated* (when $\gamma = \lambda$). However, it is possible that First(γ) $\neq \emptyset$ and $\gamma \Rightarrow^\star \lambda$, as in Item 2. Here, γ = B; we have First(B) = $\{ b \}$ but we also have B $\Rightarrow^\star \lambda$. Thus, Marker ㉘ causes b to contribute to Item 5's *ItemFollow* set. Additionally, the

Item	Prop To	Initial	Pass 1
1	13	$	
2	5,8	$	
3	11	$	
4	6	$	
5	12	b	$
6	18		$
7	19	c	
8	9,10,17		$
9	16		$
10			$
11	15		$
12			b $
13	14		$
14			$
15			$
16			$
17			$
18	20		$
19			c
20			$

Figure 6.30: Iterations for LALR(1) follow sets.

ItemFollow set at Item 2 is forwarded to Item 5 by a propagation edge placed by Marker ㉙. Finally, the lookahead present at Item 2 must make its way to Item 17 where it can be consulted when the reduction S → A B is applied. Thus, propagation edges are placed by Marker ㉗ between Items 2 and 8 and between Items 8 and 17.

Constructing the propagation graph is only half of the process we need to compute lookahead sets. Once LALR(1) construction has established the propagation graph and has initialized the *ItemFollow* sets, as shown in Figure 6.29, the propagation graph can be evaluated. EvalItemPropGraph in Figure 6.28 evaluates the graph by iteratively propagating lookahead information along the graph's edges until no new information appears.

In Figure 6.30 we trace the progress of this algorithm on our example. The "Initial" column shows the lookahead sets established by Marker ㉘. The loop at Marker ㉚ unions lookahead sets as determined by the propagation graph's edges. For the example we have considered thus far, the loop at Marker ㉚ converges after a single pass. As written, the algorithm requires a second pass to detect that no lookahead sets change after the first pass. We do not show

```
1 Start → S $
2 S     → x C1 y1 Cn yn
3        | A1
4 A1    → b1 C1
5        | a1
6 An    → bn Cn
7        | an
8 C1    → An
9 Cn    → A1
```

Figure 6.31: LALR(1) analysis: grammar.

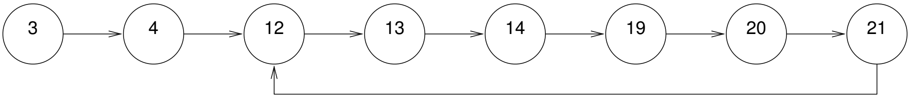


Figure 6.32: Embedded propagation subgraph.

the second pass in Figure 6.30.

The loop at Marker ㉚ continues until no *ItemFollow* set is changed from the previous iteration. The number of iterations prior to convergence depends on the structure of the propagation graph. The graph with edges specified in Figure 6.30 is *acyclic*—such graphs can be evaluated completely in a single pass.

In general, multiple passes can be required for convergence. We illustrate this using Figure 6.33, which records how an LALR(1) propagation graph is constructed for the grammar shown in Figure 6.31. Figure 6.34 shows the progress of the loop at Marker ㉚. The lookahead sets for a given item are the union of the symbols displayed in the three rightmost columns. For this example, the sets converge after two passes, but a third pass is necessary to detect this. Two passes are necessary, because the propagation graph embeds the graph shown in Figure 6.32. This graph contains a cycle with one "retreating" backedge. Information cannot propagate from item 20 to 12 in a single pass. Exercise 28 explores how to extend the grammar in Figure 6.31 so that convergence can require *any number* of iterations. In practice, LALR(1) lookahead computations converge quickly, usually in one or two passes.

In summary, LALR(1) is a powerful parsing method and is the basis for most bottom-up parser generators. To achieve greater power, more lookahead can be applied, but this is rarely necessary. LALR(1) grammars are available for all popular programming languages.

State	LR(0) Item		Goto State	Prop Edges Placed by Step		Initialize *ItemFollow*	
				㉗	㉙	First(γ)	㉘
0	1	Start→ •S $	3	11		$	2,3
	2	S→ •x C1 y1 Cn yn	1	6			
	3	S→ •A1	5	16	4,5		
	4	A1→ •b1 C1	4	12			
	5	A1→ •a1	2	10			
1	6	S→x •C1 y1 Cn yn	13	27		y1	7
	7	C1→ •An	7	18	8,9		
	8	An→ •bn Cn	8	19			
	9	An→ •an	9	23			
2	10	A1→a1 •					
3	11	Start→S •$	12	26			
4	12	A1→b1 •C1	6	17	13		
	13	C1→ •An	7	18	14,15		
	14	An→ •bn Cn	8	19			
	15	An→ •an	9	23			
5	16	S→A1 •					
6	17	A1→b1 C1 •					
7	18	C1→An •					
8	19	An→bn •Cn	10	24	20		
	20	Cn→ •A1	11	25	21,22		
	21	A1→ •b1 C1	4	12			
	22	A1→ •a1	2	10			
9	23	An→an •					
10	24	An→bn Cn •					
11	25	Cn→A1 •					
12	26	Start→S $ •					
13	27	S→x C1 •y1 Cn yn	14	28			
14	28	S→x C1 y1 •Cn yn	15	32		yn	29
	29	Cn→ •A1	11	25	30,31		
	30	A1→ •b1 C1	4	12			
	31	A1→ •a1	2	10			
15	32	S→x C1 y1 Cn •yn	16	33			
16	33	S→x C1 y1 Cn yn •					

Figure 6.33: LALR(1) analysis for the grammar in Figure 6.31.

Item	Prop To	Initial	Pass 1	Pass 2
1	11	$		
2	6	$		
3	4,5,16	$		
4	12		$	
5	10		$	
6	27		$	
7	8,9,18	y1		
8	19		y1	
9	23		y1	
10			$ y1 yn	
11	26		$	
12	13,17		$ y1 yn	
13	14,15,18		$	y1 yn
14	19		$	y1 yn
15	23		$	y1 yn
16			$	
17			$	y1 yn
18			y1 $	yn
19	20,24		y1 $	yn
20	21,22,25		y1 $	yn
21	12		y1 $	yn
22	10		y1 $	yn
23			y1 $	yn
24			y1 $	yn
25			y1 $ yn	
26			$	
27	28		$	
28	32		$	
29	25,30,31	yn		
30	12		yn	
31	10		yn	
32	33		$	
33			$	

Figure 6.34: Iterations for LALR(1) follow sets.

```
1  Start → S  $
2  S     → lp  M  rp
3         | lb  M  rb
4         | lp  U  rb
5         | lb  U  rp
6  M     → expr
7  U     → expr
```

Figure 6.35: A grammar that is not LALR(k).

6.5.4 LR(k) Table Construction

In this section we describe an LR table-construction method that accommodates all deterministic, context-free languages. While that may seem attractive, LR(k) parsing is not very practical because even LR(1) tables (k = 1) are typically much larger than the LR(0) tables upon which SLR(k) and LALR(k) parsing are based. Moreover, it is rare that LR(1) can handle a grammar for which LALR(1) construction fails. We present such a grammar in Figure 6.35, but such grammars do not arise very often in practice. When LALR(1) fails, it is typically for one of the following reasons:

- The grammar is ambiguous—LR(k) cannot help.
- More lookahead is needed—LR(k) can help (for $k > 1$). However, LALR(k) might suffice in such cases.
- No amount of lookahead suffices—LR(k) cannot help.

The grammar in Figure 6.35 allows strings generated by the nonterminal M to be surrounded by *matching* parentheses (lp and rp) or braces (lb and rb). The grammar also allows S to generate strings with *unmatched* punctuation. The unmatched expressions are generated by the nonterminal U. The grammar can easily be expanded by replacing the terminal expr with a nonterminal that derives arithmetic expressions, such as E in the grammar of Figure 6.21. While M and U generate the same terminal strings, the grammar distinguishes between them so that a semantic action can report the mismatched punctuation—using reduction by U→expr.

A portion of the LALR(1) analysis of the grammar in Figure 6.35 is shown in Figure 6.37; the complete analysis is left for Exercise 29. Consider the lookaheads that will propagate into State 6. For Item 14, which calls for the reduction M→expr, rp is sent to Item 8 and then to Item 14. Also, rb is sent to Item 12 and then to Item 14. Thus, *ItemFollow*(14) = { rb, rp }. Similarly,

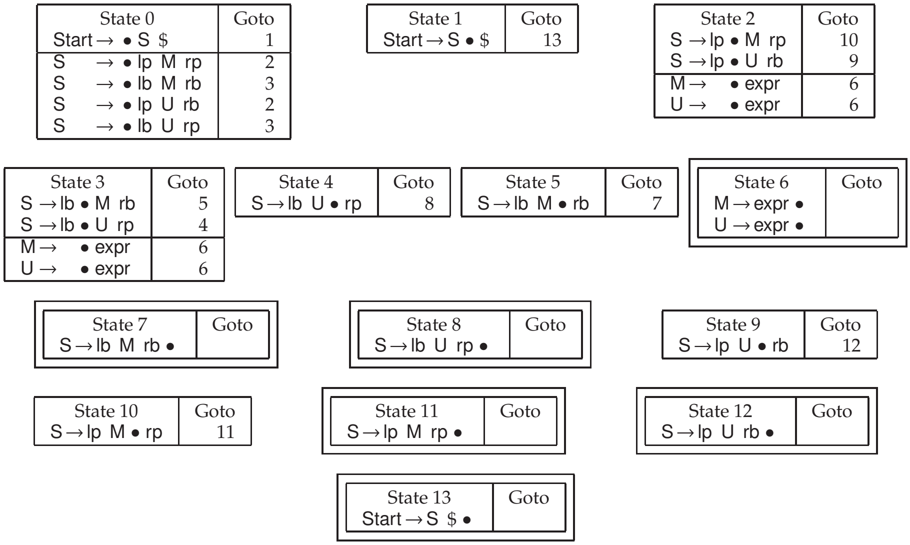


Figure 6.36: LR(0) construction.

we compute *ItemFollow*(15) = { rb, rp }. Thus, State 6 contains a reduce/reduce conflict. For LALR(1), the rules M → expr and U → expr can each be followed by either rp or rb.

Because LALR(1) is based on LR(0), there is exactly one state with the kernel of State 6. Thus, States 2 and 3 must share State 6 when shifting an expr. If only we could split State 6, so that State 2 shifts to one version and State 3 shifts to the other, then the lookaheads in each state could resolve the conflict between M → expr and U → expr. The LR(1) construction causes such splitting, because a state is uniquely identified not only by its kernel from LR(0) but also its lookahead information.

SLR(k) and LALR(k) supply information to LR(0) states to help resolve conflicts. In LR(k), such information is part of the items themselves. For LR(k), we extend an item's notation from $A \rightarrow \alpha \bullet \beta$ to $[\, A \rightarrow \alpha \bullet \beta \,, w \,]$. For LR(1), w is a (terminal) symbol that can follow A when this item becomes reducible. For LR(k), $k \geq 0$, w is a k-length string that can follow A after reduction. If symbols x and y can both follow A when $A \rightarrow \alpha \bullet \beta$ becomes reducible, then the corresponding LR(1) state contains both $[\, A \rightarrow \alpha \bullet \beta \,, x \,]$ and $[\, A \rightarrow \alpha \bullet \beta \,, y \,]$.

Notice how nicely the notation for LR(k) generalizes LR(0). For LR(0), w must be a 0-length string. The only such string is λ, which provides no information at a possible point of reduction, since λ does not occur as input.

State		LR(0) Item	Goto State	Prop Edges Placed by Step		Initialize *ItemFollow*	
				㉗	㉙	First(γ)	㉘
0	1	Start→ • S $	1	??		$	2,3,4,5
	2	S→ • lp M rp	2	6			
	3	S→ • lb M rb	3	10			
	4	S→ • lp U rb	2	7			
	5	S→ • lb U rp	3	11			
2	6	S→lp • M rp	10	??		rp	8
	7	S→lp • U rb	9	??		rb	9
	8	M→ • expr	6	14			
	9	U→ • expr	6	15			
3	10	S→lb • M rb	5	??		rb	12
	11	S→lb • U rp	4	??		rp	13
	12	M→ • expr	6	14			
	13	U→ • expr	6	15			
6	14	M→expr •					
	15	U→expr •					

Figure 6.37: Partial LALR(1) analysis. Notice the propagation of rp and rb to Item 14 from Items 8 and 12, respectively. Item 15 suffers a similar fate from Items 13 and 9. This leads to a reduce/reduce conflict between M→expr and U→expr on rp and rb in State 6.

The full LR(1) construction for the grammar in Figure 6.35 is given in Figure 6.40). For now, consider two of the construction's LR(1) items:

[S→lp • M rp , $] and [M→expr • , rp]

The first item is not ready for reduction, but indicates that $ will follow the reduction to S when the item eventually becomes reducible (State 11 of Figure 6.40). The second item calls for a reduction by rule M→expr when rp is the next input token.

In LR(k), a state is a set of LR(k) items, and construction of the CFSM is basically the same as with LR(0). States are represented by their kernel items, and new states are generated as needed. Figure 6.38 presents an LR(1) construction algorithm in terms of modifications to the LR(0) algorithm shown in Figures 6.9 and 6.10. At Marker ㉛, any symbol that can follow B due to the presence of γ is considered; when $\gamma \Rightarrow^{\star} \lambda$, then any symbol a that

```
Marker ⑦: We initialize StartItems by including LR(1) items that have $ as
    the follow symbol:
        $StartItems \leftarrow \{ [\,\mathsf{Start} \rightarrow \bullet\, \mathrm{RHS}(p)\,, \$\,] \mid p \in \textsc{ProductionsFor}(\mathsf{Start}) \}$

Marker ⑬: We augment the LR(0) item so that AdvanceDot returns the
    appropriate LR(1) items:
        return ($\{ [\,\mathsf{A} \rightarrow \alpha\mathcal{X} \bullet \beta\,, \mathrm{a}\,] \mid [\,\mathsf{A} \rightarrow \alpha \bullet \mathcal{X}\beta\,, \mathrm{a}\,] \in state \}$)

Marker ⑮: This entire loop is replaced by the following:
        foreach $[\,\mathsf{A} \rightarrow \alpha \bullet \mathsf{B}\gamma\,, \mathsf{a}\,] \in ans$ do
            foreach $p \in \textsc{ProductionsFor}(B)$ do
                foreach $b \in \mathsf{First}(\gamma\mathsf{a})$ do                 ㉛
                    $ans \leftarrow ans \cup \{ [\,\mathsf{B} \rightarrow \bullet\, \mathrm{RHS}(p)\,, b\,] \}$
```

Figure 6.38: Modifications to Figures 6.9 and 6.10 to obtain an LR(1) parser

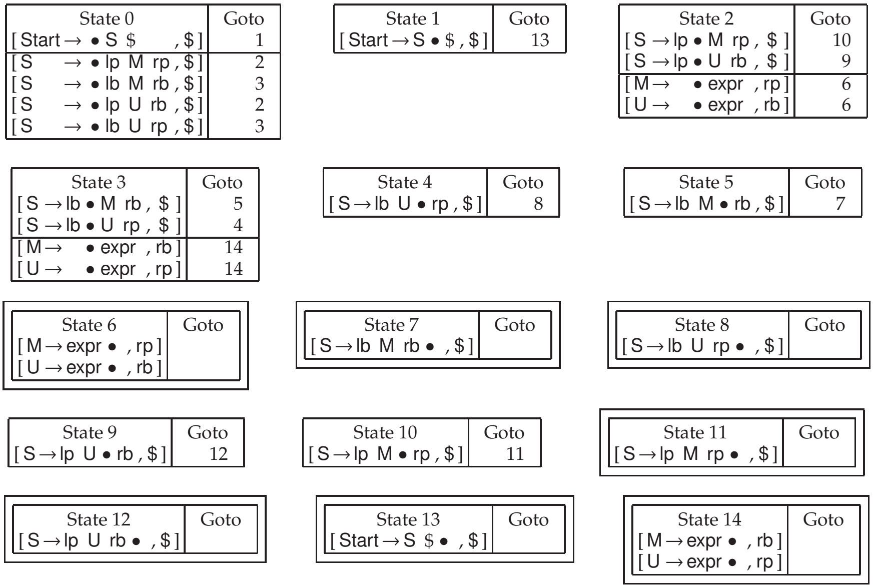
```
procedure TryRuleInState($s, r$)
    if $[\,\mathrm{LHS}(r) \rightarrow \mathrm{RHS}(r) \bullet\,, w\,] \in s$
    then call AssertEntry($s, w,$ reduce r)
end
```

Figure 6.39: LR(1) version of TryRuleInState.

can follow A can also follow B. Thus, Marker ㉛ considers each symbol in First(γa). The current state receives an item for reach rule for B and each possible follow symbol. Figure 6.14 shows TryRuleInState—the LR(0) method for determining if a state calls for a particular reduction. The LR(1) version of TryRuleInState is shown in Figure 6.39.

Figure 6.40 shows the LR(1) construction for the grammar in Figure 6.35. States 6 and 14 would be merged under LR(0). For LR(1), these states differ by the lookaheads associated with the reducible items. Thus, LR(1) is able to resolve what would have been a reduce/reduce conflict under LR(0).

The number of states (such as States 6 and 14) that split during LR(1) construction is usually much larger. Instead of constructing a full LR(1) parse table, one could begin with LALR(1), which is based on the LR(0) construction. States could then be split selectively. As discussed in Exercise 35, LR(k) can resolve only the reduce/reduce conflicts that arise during LALR(k) construction. A shift/reduce conflict in LALR(k) will also be present in the corresponding LR(k) construction. Exercise 36 considers how to split LR(0) states on demand



State 0	Goto
[Start → • S $, $]	1
[S → • lp M rp , $]	2
[S → • lb M rb , $]	3
[S → • lp U rb , $]	2
[S → • lb U rp , $]	3

State 1	Goto
[Start → S • $, $]	13

State 2	Goto
[S → lp • M rp , $]	10
[S → lp • U rb , $]	9
[M → • expr , rp]	6
[U → • expr , rb]	6

State 3	Goto
[S → lb • M rb , $]	5
[S → lb • U rp , $]	4
[M → • expr , rb]	14
[U → • expr , rp]	14

State 4	Goto
[S → lb U • rp , $]	8

State 5	Goto
[S → lb M • rb , $]	7

State 6	Goto
[M → expr • , rp]	
[U → expr • , rb]	

State 7	Goto
[S → lb M rb • , $]	

State 8	Goto
[S → lb U rp • , $]	

State 9	Goto
[S → lp U • rb , $]	12

State 10	Goto
[S → lp M • rp , $]	11

State 11	Goto
[S → lp M rp • , $]	

State 12	Goto
[S → lp U rb • , $]	

State 13	Goto
[Start → S $ • , $]	

State 14	Goto
[M → expr • , rb]	
[U → expr • , rp]	

Figure 6.40: LR(1) construction.

in response to reduce/reduce conflicts that arise in LALR(k) constructions.

Summary

This concludes our study of bottom-up parsers. We have investigated a number of LR table-building methods, from LR(0) to LR(1). The intermediate methods—SLR(1) and LALR(1)—are the most practical. In particular, LALR(1) provides excellent conflict resolution and generates very compact tables. Tools based on LALR(1) grammars are available for most languages. Such tools are indispensable for language modification and extension. Changes can be prototyped using an LALR(1) grammar for the language's syntax. When conflicts occur, the methods discussed in Section 6.4 help identify why the proposed modification may not work.

Because of their efficiency and power, LALR(1) grammars are available for most modern programming languages. Indeed, the syntax of modern programming languages is commonly designed with LALR(1) parsing in mind.

Exercises

1. Build the CFSM and the parse table for the grammar shown in Figure 6.2.

2. Using the knitting analogy of Section 6.2.2, show the sequence of LR shift and reduce actions for the grammar of Figure 6.2 on the following strings:

   (a) plus plus num num num $

   (b) plus num plus num num $

3. Figures 6.6 and 6.7 trace a bottom-up parse of an input string using the table shown in Figure 6.5. Trace the parse of the following strings.

   (a) q $

   (b) c $

   (c) a d c $

4. Build the CFSM for the following grammar:

1	Prog	→	Block $
2	Block	→	begin StmtList end
3	StmtList	→	StmtList semi Stmt
4		|	Stmt
5	Stmt	→	Block
6		|	Var assign Expr
7	Var	→	id
8		|	id lb Expr rb
9	Expr	→	Expr plus T
10		|	T
11	T	→	Var
12		|	lp Expr rp

5. Show the LR parse table for the CFSM constructed in Exercise 4.

6. Which of following grammars are LR(0)? Explain why.

(a)

1	S	→	StmtList $
2	StmtList	→	StmtList semi Stmt
3		|	Stmt
4	Stmt	→	s

(b)

1	S	→	StmtList $
2	StmtList	→	Stmt semi StmtList
3		|	Stmt
4	Stmt	→	s

(c)

1	S	→	StmtList $
2	StmtList	→	StmtList semi StmtList
3		|	Stmt
4	Stmt	→	s

(d)

1	S	→	StmtList $
2	StmtList	→	s StTail
3	StTail	→	semi StTail
4		|	λ

7. Show that the CFSM corresponding to a LL(1) grammar has the following property. Each state has exactly one kernel item if the grammar is λ-free.

8. Prove or disprove that all λ-free LL(1) grammars are LR(0).

9. Explain why the following grammar is unambiguous:

1	Start	→	Single a
2		|	Double b
3	Single	→	0 Single 1
4		|	0 1
5	Double	→	0 Double 1 1
6		|	0 1 1

10. Show the LR(0) construction for the following grammars:

(a)

1	Start	→	S $
2	S	→	id assign E semi
3	E	→	E plus P
4		|	P
5	P	→	id
6		|	lp E rp
7		|	id assign E

(b)

1	Start	→	S $
2	S	→	id assign A semi
3	A	→	id assign A
4		|	E
5	E	→	E plus P
6		|	P
7	P	→	id
8		|	lp A rp

(c)

1	Start	→	S $
2	S	→	id assign A semi
3	A	→	id assign A
4		|	E
5	E	→	E plus P
6		|	P
7		|	P plus
8	P	→	id
9		|	lp A rp

(d)

1	Start	→	S $
2	S	→	id assign A semi
3	A	→	Pre E
4	Pre	→	Pre id assign
5		|	λ
6	E	→	E plus P
7		|	P
8	P	→	id
9		|	lp A rp

```
         1  Start → S $
         2  S     → id assign A semi
         3  A     → Pre E
         4  Pre   → id assign Pre
(e)      5         | λ
         6  E     → E plus P
         7         | P
         8  A     → id
         9         | lp A rp
```

```
         1  Start → S $
         2  S     → id assign A semi
         3  A     → id assign A
         4         | E
         5  E     → E plus P
         6         | P
(f)      7  P     → id
         8         | lp A semi A rp
         9         | lp V comma V rp
        10         | lb A comma A rb
        11         | lb V semi V rb
        12  V     → id
```

```
         1  Start → S $
         2  S     → id assign A semi
         3  A     → id assign A
         4         | E
(g)      5  E     → E plus P
         6         | P
         7  P     → id
         8         | lp id semi id rp
         9         | lp A rp
```

11. Explain why the language defined by the grammar in Exercise 9 is **inherently nondeterministic**—there is no LALR(*k*) grammar for such languages.

12. Given the claim of Exercise 11, explain why the following statement is true or false:

    There is no LR(*k*) grammar for the language

    $$\{\,0^n1^n\mathsf{a}\,\}\cup\{\,0^n1^{2n}\mathsf{b}\,\}.$$

13. Discuss why is it not possible during LR(0) construction to obtain a shift/reduce conflict on a nonterminal.

14. Discuss why there cannot be an unambiguous CFGs for the language

$$\{ \mathsf{a}^i\mathsf{b}^j\mathsf{c}^k \mid i = j \text{ or } j = k; i, j, k \geq 1 \}$$

15. Complete the LR(0) construction for the grammar in Figure 6.19.

16. Show that LL(1) construction fails for an unambiguous grammar that is not LR(1).

17. Show that the grammar in Figure 6.20 is LR(0).

18. Complete the LR(0) construction for the grammar shown in Figure 6.25. Your state numbers should agree with those shown in the partial LR(0) construction.

19. Which of the grammars in Exercise 10 are LR(0)? Justify your answers.

20. Complete the SLR(1) construction for the grammar shown in Figure 6.26. Show the resulting parse table.

21. Extend the grammar given in Figure 6.21 to accommodate standard expressions involving addition, subtraction, multiplication, and division. Model the syntax and semantics for these operators according to Java or C.

22. Extend the grammar as directed in Exercise 21, but introduce an exponentiation operator that is *right-associating*. Let the operator (denoted by "$\star$") have the highest priority, so that the value of $3 + 4 \times 5 \star 2$ is 103.

23. Repeat Exercise 22, but add the ability to enclose expressions with parentheses to control how expressions are grouped together. Thus, the value of $((3 + 4) \times 5) \star 2$ is 1225.

24. Which of the grammars in Exercise 10 are SLR(1)? Justify your answers.

25. Generalize the algorithm given in Section 6.5.1 from SLR(1) to SLR(k).

26. Show that the following holds for any LALR(1) construction:

    (a) For any state s containing the item $\mathsf{A}\rightarrow\alpha \bullet \beta$,

    $$\mathit{ItemFollow}((s, \mathsf{A}\rightarrow\alpha \bullet \beta)) \subseteq \mathsf{Follow}(\mathsf{A})$$

    (b)

    $$\bigcup_{s} \bigcup_{\mathsf{A}\rightarrow\alpha_i \bullet \beta_i \in s} \mathit{ItemFollow}((s, \mathsf{A}\rightarrow\alpha_i \bullet \beta_i)) = \mathsf{Follow}(\mathsf{A})$$

27. Perform the LALR(1) construction for the following grammar:

```
 1  Start → S  $
 2  S     → x  C1  y1  C2  y2  C3  y3
 3        | A1
 4  A1    → b1  C1
 5        | a1
 6  A2    → b2  C2
 7        | a2
 8  A3    → b3  C3
 9        | a3
10  C1    → A2
11  C2    → A1
12        | A3
13  C3    → A2
```

28. Recall the EvalItemPropGraph algorithm given in Figure 6.28. Using the grammars in Figure 6.31 and Exercise 27 as a guide, show how to generate a LALR(1) grammar that requires n iterations for *ItemFollow* sets to converge in EvalItemPropGraph.

29. For the grammar shown in Figure 6.35, complete the LALR(1) construction from Figure 6.37.

30. Which of the grammars in Exercise 10 are LALR(1)? Justify your answers.

31. Show the LR(1) construction for the grammar in Exercise 4.

32. Define the **quasi-identical states** of an LR(1) parsing machine to be those states whose kernel productions are identical. Such states are distinguished only by the lookahead symbols associated with their productions. Given the LR(1) machine built for Exercise 31, complete the following:

    (a) List the quasi-identical states of the LR(1) machine.

    (b) Merge each set of quasi-identical states to obtain an LALR(1) machine.

33. Starting with the CFSM built in Exercise 4, compute the LALR(1) lookahead information. Compare the resulting LALR(1) machine with the machine obtained in Exercise 32.

34. Which of the grammars in Exercise 10 are LR(1)? Justify your answers.

35. Consider a grammar *G* and its LALR(1) construction. Suppose that a shift/reduce conflict occurs in *G*'s LALR(1) construction. Prove that *G*'s LR(1) construction also contains a shift/reduce conflict.

36. Describe an algorithm that computes LALR(1) and then splits states as needed in an attempt to address conflicts. Take note of the issue raised in Exercise 35.

37. Using a grammar for the C programming language, try to extend the syntax to allow *nested* function definitions. For example, you might allow function definitions to occur inside any compound statement.

    Report on any difficulties you encounter and discuss possible solutions. Justify the particular solution you adopt.

38. Using a grammar for the C programming language, try to extend the syntax so that a compound statement can appear to compute a value. In other words, allow a compound statement to appear wherever a simple constant or identifier could appear. Semantically, the value of a compound statement could be the value associated with its last statement.

    Report on any difficulties you encounter and discuss possible solutions. Justify the particular solution you adopt.

39. In Figure 6.3, Marker ② pushes a state on the parse stack. In the bottom-up parse shown in Figures 6.6 and 6.7, stack cells show both the state and the input symbol causing the state's shift onto the stack. Explain why the input symbol's presence in the stack cell is superfluous.

40. Recall the **dangling else** problem introduced in Chapter 5. A grammar for a simplified language that allows conditional statements follows:

```
1  Start → Stmt $
2  Stmt → if e then Stmt else Stmt
3        | if e then Stmt
4        | other
```

Explain why the grammar is or is not LALR(1).

41. Consider the following grammar:

```
1  Start      → Stmt $
2  Stmt       → Matched
3              | Unmatched
4  Matched    → if e then Matched else Matched
5              | other
6  Unmatched → if e then Matched else Unmatched
7              | if e then Unmatched
```

   (a) Explain why the grammar is or is not LALR(1).

   (b) Is the language of this grammar the same as the language of the grammar in Exercise 40? Why or why not?

42. Repeat Exercise 41, adding the production Unmatched→other to the grammar.

43. Consider the following grammar:

```
1  Start      → Stmt $
2  Stmt       → Matched
3              | Unmatched
4  Matched    → if e then Matched else Matched
5              | other
6  Unmatched → if e then Matched else Unmatched
7              | if e then Stmt
```

   (a) Explain why the grammar is or is not LALR(1).

   (b) Is the language of this grammar the same as the language of the grammar in Exercise 40? Why or why not?

44. Based on the material in Exercises 40, 41, and 43, construct an LALR(1) grammar for the language defined by the following grammar:

```
1  Start → Stmt $
2  Stmt  → if e then Stmt else Stmt
3        | if e then Stmt
4        | while e Stmt
5        | repeat Stmt until e
6        | other
```

45. Show that there exist non-LL(1) grammars that are

   (a) LR(0)

   (b) SLR(1)

   (c) LALR(1)

46. Normally, an LR parser traces a rightmost derivation (in reverse).

   (a) How could an LR parser be modified to produce a leftmost parse as LL(1) parsers do? Describe your answer in terms of the algorithm in Figure 6.3.

   (b) Would it help if we knew that the LR table was constructed for an LL grammar? Explain your reasoning.

47. For each of the following, construct an appropriate grammar:

   (a) The grammar is SLR(3) but not SLR(2).

   (b) The grammar is LALR(2) but not LALR(1).

   (c) The grammar is LR(2) but not LR(1).

   (d) The grammar is LALR(1) and SLR(2) but not SLR(1).

48. Construct a single grammar that has *all* of the following properties:

   - It is SLR(3) but not SLR(2).
   - It is LALR(2) but not LALR(1).
   - It is LR(1).

49. For every $k > 1$ show that there exist grammars that are SLR($k + 1$), LALR($k + 1$), and LR($k + 1$) but not SLR(k), LALR(k), or LR(k).

50. Consider the grammar generated by $1 \le i, j \le n,\ i \ne j$ using the following template:

$$\begin{aligned} \mathsf{S} &\rightarrow \mathsf{X}_i\ \mathsf{z}_i \\ \mathsf{X}_i &\rightarrow \mathsf{y}_j\ \mathsf{X}_i \\ &\ \mid\ \mathsf{y}_j \end{aligned}$$

The resulting grammar has $O(n^2)$ productions.

(a) Show that the CFSM for this grammar has $O(2^n)$ states.

(b) Explain why the grammar is, or is not, SLR(1).

51. The bottom-up parsing techniques given in this chapter are more powerful than top-down techniques given in Chapter 5.

Using the alphabet $\{a, b\}$, devise a language that is not LL(k) for any k but is LR(k) for some k. What property of LR(k) parsing allows such a grammar to be constructed?

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7

Syntax-Directed Translation

The parsers discussed in Chapters 5 and 6 can recognize syntactically valid inputs. However, compilers are typically required to perform some translation of the input source into a target representation, as discussed in Chapter 2. Some compilers are completely **syntax-directed**, translating programs in a single phase without taking any intermediate steps. Most compilers accomplish translation using multiple phases. Instead of repeatedly scanning the input program, compilers typically create an intermediate structure called the **abstract syntax tree** (AST) as a by-product of the parse. The AST then serves as a mechanism for conveying information between compiler phases.

In this chapter we study how to formulate grammars and production-triggered code sequences to enable syntax-directed translation or to create an AST for subsequent phases.

7.1 Overview

The work performed by a compiler while parsing is generally termed *syntax-directed translation*. The grammar on which the parser is based causes a specific sequence of derivation steps to be taken for a given input program. In constructing the derivation, a parser performs a sequence of *syntactic actions* as described in Chapters 5 and 6; such actions (e.g., *shift* and *reduce* for bottom-up parsing) are concerned only with the grammar's terminal and nonterminal symbols.

7.1.1 Semantic Actions and Values

To achieve syntax-directed translation, we insert code into the parser that executes in concert with the parser's syntactic actions.

Semantic actions Each production can have an associated code sequence that will execute when the production is applied. There is no imposed limit on what the code sequence can do, and the code is typically compiled with the parser. Such code can therefore print messages, stop the parsing activity, or manipulate the compiler's data structures.

Operations that execute in concert with productions are called **semantic actions**, because they usually address compilation concerns beyond the grammar's syntax that are related to the meaning of a program.

Semantic values When a semantic action is performed for the production $\mathsf{A} \rightarrow \mathcal{X}_1 \ldots \mathcal{X}_n$, the semantic actions associated with the production are given access to a set of **semantic values** related to the production, one for each symbol. In a bottom-up parse, the semantic values for $\mathcal{X}_1 \ldots \mathcal{X}_n$ are available when $\mathsf{A} \rightarrow \mathcal{X}_1 \ldots \mathcal{X}_n$ is applied, and the semantic actions determine a value for A. In a top-down parse, a value for A is available as the production is applied, and the symbols $\mathcal{X}_1 \ldots \mathcal{X}_n$ have values just after the production is applied.

For terminal symbols, their values originate from the scanner. For example, the syntactic token id has a specific value—the name of the associated identifier—when a production involving id is applied. The associated semantic actions can then reference the identifier's name, perhaps for the purpose of generating code to load or store the value associated with the identifier, or to enter the name in a symbol table.

For nonterminals, productions have already been applied to compute their semantic values. The semantic action associated with a production typically computes a value to be associated with A based on the values already assigned to $\mathcal{X}_1 \ldots \mathcal{X}_n$.

In automatically generated parsers, the parser driver (Figure 6.3 on page 185 for bottom-up parsing) is usually responsible for executing the semantic actions. To simplify calling conventions, the driver and the grammar's semantic actions are written in the same programming language. Semantic actions are also easily inserted into ad hoc parsers by specifying code sequences that execute in concert with the parser.

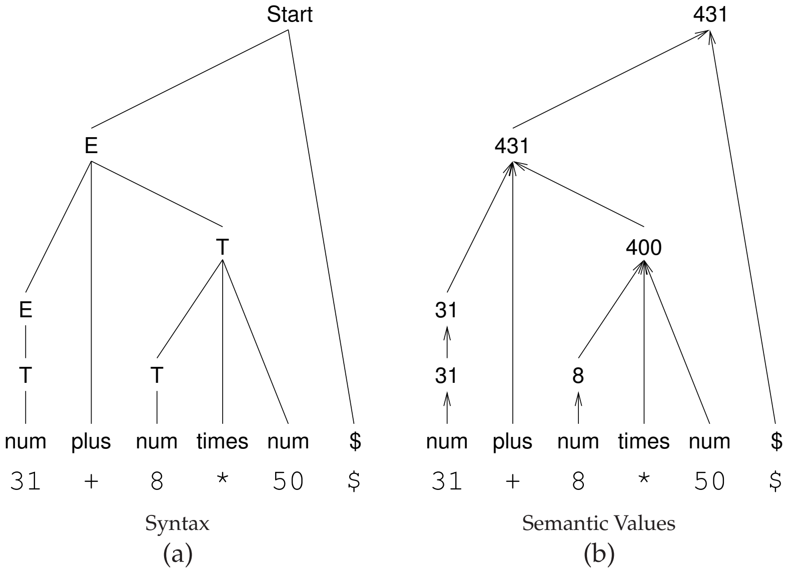
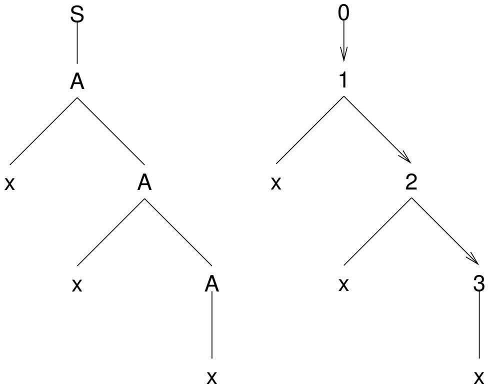


Figure 7.1: (a) Parse tree for the displayed expression;
(b) Synthesized attributes transmit values up the parse tree toward the root.

Formulating an appropriate set of semantic actions requires a firm understanding of how derivations are traced in a given parsing technique (bottom-up or top-down). Writing clear and elegant semantic actions often requires grammar restructuring to aid computation of semantic values at appropriate places. After the initial step of obtaining a grammar suitable for top-down or bottom-up parsing, it is not unusual to revise the grammar to facilitate semantic actions during this phase of compiler construction.

7.1.2 Synthesized and Inherited Attributes

In Section 7.2, we examine how to specify semantic actions for bottom-up parsing, which essentially create parse trees in a *postorder* fashion. For syntax-directed translation, this style nicely accommodates situations in which attributes primarily flow from the leaves of a derivation tree toward its root. If we imagine that each node of a parse tree can consume and produce information, then nodes consume information from their children and produce information for their parent in a bottom-up parse. An example using such **synthesized attributes** flow is shown in Figure 7.1. The parse tree in Fig-



Syntax
(a)

Semantic Values
(b)

Figure 7.2: (a) Parse tree for the displayed input string; (b) Inherited attributes pass from parent to child.

ure 7.1(a) is generated by the standard expression grammar from Figure 6.21 on page 205. As shown in Figure 7.1(b), a rule such as $E \rightarrow E$ plus T synthesizes a result at the parent that is the sum of the values transmitted by its first and third children. Section 7.2 considers synthesized attributes and bottom-up parsing in much greater detail.

On the other hand, consider the problem of counting the position of each x in a string. The parse tree in Figure 7.2(a) is derived from a simple right-linear grammar. Each A node computes its semantic value in Figure 7.2(b) by incrementing the value received from its parent. Values that flow in this manner are called **inherited attributes**. Section 7.3 considers the more general case of synthesized and inherited attributes in the context of top-down parsing.

Syntax-directed translation of most programming languages requires both synthesized and inherited attributes. While a given style of parsing favors attribute flow in one direction, value flow in the other direction is managed using other techniques. For example, symbol tables (introduced in Section 2.7.1 on page 47 with details in Chapter 8) effectively allow type information to propagate up the tree (from a variable's declaration) and down the tree (to a variable's use).

7.2 Bottom-Up Syntax-Directed Translation

We now consider how to incorporate semantic actions into bottom-up parsers. Such parsers are almost always generated automatically by tools (e.g., JavaCUP, `yacc`, Bison) that allow incorporation of code sequences that execute as reductions are performed. Such parsers also execute *shift* actions, but no provision is generally made for semantic actions when symbols are shifted.

Consider an LR parser that is about to perform a reduction using the production

$$\mathsf{A} \rightarrow \mathcal{X}_1 \ldots \mathcal{X}_n$$

As discussed in Chapter 6, the symbols $\mathcal{X}_1 \ldots \mathcal{X}_n$ are on top-of-stack prior to the reduction, with $\mathcal{X}_n$ topmost. The reduction pops n symbols from the stack and pushes the symbol A onto the stack. It is likely that previous reductions have associated semantic values with the symbols $\mathcal{X}_1 \ldots \mathcal{X}_n$ in the bottom-up parse. The semantic action associated with the reduction consists of an arbitrary fragment of code that can reference semantic values associated with $\mathcal{X}_1 \ldots \mathcal{X}_n$ and can associate a semantic value with the resulting A.

The notation for referencing a given semantic value varies among parser-generating tools. For example, `yacc` and Bison use the ordinal occurrence of the symbol in semantic action code: $\$i$ denotes the semantic value of $\mathcal{X}_i$ and \$0 denotes A's semantic value. Other tools (e.g., JavaCUP) use the notation $\mathcal{X}$:*val* to specify *val* as the semantic value associated with $\mathcal{X}$. In effect, the bottom-up parser operates two stacks:

- The *syntactic stack* (also called the *parse stack*) manipulates terminal and nonterminal symbols as described in Chapter 6
- The *semantic stack* manipulates semantic values associated with the grammar symbols.

The code for maintaining both stacks is generated automatically by the parser generator, based on the grammar and semantic-action code specified by the compiler writer.

7.2.1 Example

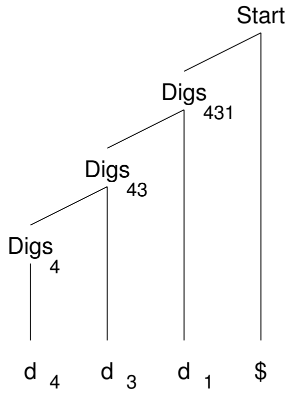
Consider the translation task of computing the value of a string of base-10 digits. For example, given the input `4 3 1 $`, the translation would produce the numerical value 431. This task—normally handled by the scanning phase of a compiler—is illustrated here as a for syntax-directed, bottom-up translation using the grammar shown in Figure 7.3(a). The grammar is augmented with semantic actions, shown beneath each production. Semantic values are denoted by subscripts on grammar symbols. For example, the variable *ans* is

1 $\text{Start} \rightarrow \text{Digs}_{ans}\ \$$
  call PRINT(*ans*)

2 $\text{Digs}_{up} \rightarrow \text{Digs}_{below}\ \text{d}_{next}$
  $up \leftarrow below \times 10 + next$

3 $\mid\ \text{d}_{first}$
  $up \leftarrow first$

(a)



(b)

Figure 7.3: (a) Grammar with semantic actions; (b) Parse tree and propagated semantic values for the input `4 3 1 $`.

specified as the semantic value associated with the Digs symbol in Rule 1. The code associated with a production is shown indented beneath the production. For example, the code beneath Rule 1 causes the final value passed up the parse tree (*ans*) to be printed.

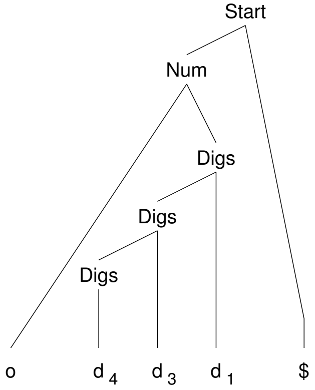
Because each nonterminal is the result of some production, the semantic values associated with nonterminals are computed by semantic action code segments. The values associated with terminal symbols must be established by the scanner. For example, Rule 3 in Figure 7.3 includes the symbol d_{first}. The syntactic element d represents a decimal digit, as discovered by the scanner; the semantic tag *first* represents the digit's value, which must also be provided by the scanner. Parser generators offer methods for declaring the *type* of the semantic symbols; for the sake of simplicity we omit type declarations in our examples. In the grammar of Figure 7.3(a), all semantic tags would be declared of type integer.

We now analyze how the semantic actions in Figure 7.3(a) compute the base-10 value of the digit string. To appreciate the interaction of Rules 2 and 3, we examine the order in which a bottom-up parse applies these rules. Figure 7.3(b) shows a parse tree for the input string 4 3 1 $. In Chapter 6 we learned that a bottom-up parse traces a rightmost derivation in reverse. The rules for Digs are therefore applied as follows:

Digs → d In a bottom-up parse, Rule 1 must be applied first, so d corresponds to the first input digit 4. The semantic action $up \leftarrow first$ causes the value of the first digit (4) to be assigned to the semantic value for Digs. Semantic

1 Start → Num \$
2 Num → o Digs
3 | Digs
4 Digs → Digs d
5 | d

(a)



(b)

Figure 7.4: (a) Grammar and (b) parse tree for the input `o 4 3 1 $`.

actions such as these are often called *copy rules* because they serve only to propagate values up the parse tree.

Digs → Digs d Each subsequent d is processed by Rule 2. The semantic action *up* ← *below* × 10 + *next* multiplies the value computed thus far (*below*) by 10 and then adds in the semantic value of the current d (*next*).

This example illustrates that left-recursive rules are amenable to semantic processing of input text from left to right in a bottom-up parser. Exercise 1 considers the problem of computing a digit string's value when the grammar rule is right recursive.

Figure 7.4(a) extends our language slightly, so that a string of digits is optionally prefaced with an o; a parse tree showing the new syntax is shown in Figure 7.4(b). Rule 3 generates strings of digits that should be interpreted base-10. Rule 2 generates an o followed by a string of digits that should be interpreted base-8 (octal).

While the grammar in Figure 7.4(a) suffices to parse the language described above, it suffers from the following drawbacks:

- The nonterminal Digs generates a string of decimal digits, even in cases where those digits should be interpreted base-8. Octal digits should be restricted to 0–7, but the grammar of Figure 7.4(a) is inconvenient to enforce that restriction. Rules 4 and 5 need to process base-10 as well as base-8 digits. Enforcing base-8 digits at Rule 2 would require rescanning the digits that reduced to Digs.

- Consider the parse tree shown in Figure 7.4(b). As was the case with our previous example, the first d symbol is processed first by Rule 5 and the remaining d symbols are processed by Rule 4. If the string is to be interpreted base-8, then the semantic action for Rule 4 should multiply the number passed up the parse tree by 8; otherwise, 10 should be used at the multiplier.

  Unfortunately, the grammar of Figure 7.4(a) will cause the o to be shifted on the stack. Because semantic actions are allowed only at reductions, no action is possible at that point. When the semantic actions for Rule 4 execute, it is unknown whether we are processing a base-10 or base-8 number.

Such situations arise often in the context of syntax-directed translation. The structure of the parse (as seen in Figure 7.4(b)) is not well suited to the translation task at hand. In terms of attribute propagation up the parse tree (synthesized attributes), the information required at a semantic action is not available from below.

We next discuss a number of approaches for remedying this problem and consider their advantages and disadvantages. Each approach involves some modification to the grammar, so a word of caution is in order before we begin: In general, there can be no algorithm that determines whether the languages denoted by two context-free grammars are the same. This means that when a grammar is modified, it cannot in general be proved that the modification did not change the language in some unacceptable way. Also, grammar modification can affect the suitability of the grammar for a given parsing technique. Thus, grammar modifications must be performed with great care:

- At the beginning of such a project, sample inputs should be written for submission to the parser. The samples should include inputs that should and should not be accepted by the parser, and the sample set should be as complete as possible.

- Grammar changes should be planned and carried out in small steps.

- After each step, **regression tests** should ensure that the parser based on the new grammar accepts and rejects the proper set of strings.

As dictated by common software engineering practices, bugs that develop because of faults in the parser or grammar should be resolved and then turned into new regression tests. This will ensure the bug does not resurface if the grammar or parsing actions are subsequently modified.

```
1 Start               → $\text{Num}_{ans}$ $
                          call PRINT($ans$)
2 $\text{Num}_{ans}$          → o $\text{OctDigs}_{octans}$
                          $ans \leftarrow octans$
3                     | $\text{DecDigs}_{decans}$
                          $ans \leftarrow decans$
4 $\text{DecDigs}_{up}$       → $\text{DecDigs}_{below}$ $\text{d}_{next}$
                          $up \leftarrow below \times 10 + next$
5                     | $\text{d}_{first}$
                          $up \leftarrow first$
6 $\text{OctDigs}_{up}$       → $\text{OctDigs}_{below}$ $\text{d}_{next}$
                          if $next \geq 8$                          ①
                          then ERROR("Non-octal digit")
                          $up \leftarrow below \times 8 + next$
7                     | $\text{d}_{first}$
                          if $first \geq 8$                         ②
                          then ERROR("Non-octal digit")
                          $up \leftarrow first$
```

Figure 7.5: Grammar with cloned productions.

7.2.2 Rule Cloning

Our first approach observes that a similar sequence of input symbols—a string of digits—should be treated differently depending on context. Following that observation, we can *clone* the productions in the grammar to derive similar syntax but with different semantic actions. We therefore construct two kinds of digit strings, one derived from OctDigs and the other derived from DecDigs, to obtain the grammar and semantic actions shown in Figure 7.5. Rules 4 and 5 interpret strings of digits base-10; Rules 6 and 7 generate the same syntactic strings but interpret their meaning base-8. Moreover, the separation of octal and decimal digit recognition allows checks in the semantic actions of Rules 6 and 7 that the octal digits are in the proper range.

With this example, we see that grammars are modified for reasons other than the parsing issues raised in Chapters 4, 5, and 6. Often, a translation task can become significantly easier if the grammar can be modified to accommodate convenient flow of semantic information.

Rule cloning is an improvement over our initial attempt at syntax-directed translation for this example. However, rule cloning cloning inflates a gram-

```
1  Start          → $\mathsf{Num}_{ans}$ $
                      call PRINT($ans$)
2  $\mathsf{Num}_{ans}$        → SignalOctal $\mathsf{Digs}_{octans}$
                      $ans \leftarrow octans$
3                 | SignalDecimal $\mathsf{Digs}_{decans}$
                      $ans \leftarrow decans$
4  SignalOctal    → o
                      $base \leftarrow 8$
5  SignalDecimal  → $\lambda$
                      $base \leftarrow 10$
6  $\mathsf{Digs}_{up}$        → $\mathsf{Digs}_{below}$ $\mathsf{d}_{next}$
                      $up \leftarrow below \times base + next$
7                 | $\mathsf{d}_{first}$
                      $up \leftarrow first$
```

Figure 7.6: Use of λ-rules to force semantic action.

mar with productions that are not necessary from a syntactic point of view. While those extra productions accommodate differentiated semantic actions at appropriate points in the parse, our next approach avoids such redundancy.

7.2.3 Forcing Semantic Actions

Bottom-up parsers normally are prepared to execute semantic actions only on reductions. If a semantic action is desired on the shift of some symbol $\mathcal{X}$, then a **unit production** of the form $\mathsf{A}\rightarrow\mathcal{X}$ can be introduced, with occurrences of $\mathcal{X}$ replaced by A in the grammar's productions. The semantic action can then be associated with the reduction of $\mathsf{A}\rightarrow\mathcal{X}$. Similarly, if a semantic action is desired *between* two symbols $\mathcal{X}_m$ and $\mathcal{X}_n$, then a production of the form $\mathsf{A}\rightarrow\lambda$ can be introduced into the grammar. The semantic action is associated with that rule, and all occurrences of $\mathcal{X}_m\ \mathcal{X}_n$ are replaced by $\mathcal{X}_m\ \mathsf{A}\ \mathcal{X}_n$.

We can apply these idea to the grammar of Figure 7.4 to obtain the grammar shown in Figure 7.6:

- The o is replaced by the nonterminal SignalOctal, which derives the o and whose semantic action sets the global variable $base \leftarrow 8$.
- Correspondingly, if the o is not present, the production SignalDecimal → λ sets $base \leftarrow 10$.

```
1 Start        → $\text{Num}_{ans}$ $
                   call PRINT($ans$)
2 $\text{Num}_{ans}$     → x SetBase $\text{Digs}_{baseans}$
                   $ans \leftarrow baseans$
3              | SetBaseTen $\text{Digs}_{decans}$
                   $ans \leftarrow decans$
4 SetBase      → $\text{d}_{val}$
                   $base \leftarrow val$
5 SetBaseTen   → $\lambda$
                   $base \leftarrow 10$
6 $\text{Digs}_{up}$      → $\text{Digs}_{below}$ $\text{d}_{next}$
                   if $next \geq base$                                   ③
                   then ERROR("Digit outside allowable range")
                   $up \leftarrow below \times base + next$
7              | $\text{d}_{first}$
                   if $first \geq base$                                  ④
                   then ERROR("Digit outside allowable range")
                   $up \leftarrow first$
```

Figure 7.7: Strings with an optionally specified base.

This grammar avoids copying the productions that generate a string of digits; instead, the grammar assigns and references a global variable (*base*) as a *side-effect* of processing an o (Rule 4) or λ (Rule 5).

We expand our example yet again, using the terminal x to indicate that the next digit is the base to which the subsequent digits should be interpreted. If no x is present, then the digits should be interpreted base-10, as before. Some examples of legal inputs and their interpretation are as follows:

Input	Meaning	Value (base-10)
`4 3 1 $`	431_{10}	431
`x 8 4 3 1 $`	431_{8}	281
`x 5 4 3 1 $`	431_{5}	116

A grammar for the new language is shown in Figure 7.7. Unit- and λ-productions are introduced to set the global variable *base* properly.

1 Start $\rightarrow$ Digs_{ans} \$
   call PRINT($ans.val$)

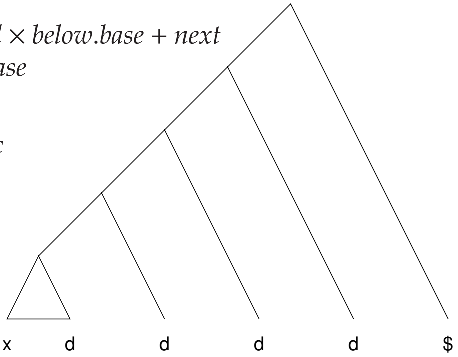
2 Digs_{up} $\rightarrow$ Digs_{below} d_{next}
   $up.val \leftarrow below.val \times below.base + next$
   $up.base \leftarrow below.base$

3 | $\text{SetBase}_{basespec}$
   $up.base \leftarrow basespec$
   $up.val \leftarrow 0$

4 Setbase_{n} $\rightarrow \lambda$
   $n \leftarrow 10$

5 | x d_{num}
   $n \leftarrow num$

(a)



(b)

Figure 7.8: (a) Grammar that avoids global variables; (b) Parse tree reorganized to facilitate bottom-up attribute propagation.

7.2.4 Aggressive Grammar Restructuring

Global variables are easily introduced into semantic actions, but there are good reasons to avoid using them:

- A grammar's rules are often invoked recursively during parsing, and the use of global variables can introduce unwanted interactions as semantic actions are applied (see Exercise 12).

- Global variables can make semantic actions difficult to write and maintain, as any semantic action could read or write a global variable. Moreover, proper initialization and reinitialization of global variables can be problematic.

- Global variables may require setting or resetting

A more robust solution attempts to restructure the parse tree so that information flows where it is needed for semantic actions. The following steps are suggested as a mechanism for obtaining a grammar more appropriate for bottom-up, syntax-directed translation:

1. Sketch the parse tree that would allow bottom-up synthesis and translation, without use of global variables.

2. Revise the grammar to achieve the desired parse tree.

3. Verify that the revised grammar is still suitable for parser construction. For example, if JavaCUP or `yacc` must process the grammar, then the grammar should remain LALR(1) after revision.

4. Verify that the grammar still generates the same language. This is usually accomplished using (grammar-specific) proof techniques or rigorous testing.

For the problem at hand, we can avoid a global *base* variable if we can process the base early, and then have it propagate up the parse tree along with the value of the processed input string. The semantic value synthesized up the tree becomes a *tuple* (i.e., a `struct` in C or a class in Java$^{\text{TM}}$) containing both the value of the digits thus far and the base used to compute that value.

Figure 7.8(b) sketches the parse tree we desire for the input `x 5 4 3 1 $`. The x and the first d (which specifies the base) are processed in the triangle; the base can then propagate up the tree and participate in the semantic actions. From this tree we rewrite the rules for Digs to obtain the grammar shown in Figure 7.8(a).

The grammar in Figure 7.8(a) reflects the newly structured parse tree. The semantic value for Digs consists of two components, *val* and *base*, that represent respectively the interpreted value of the input string thus far and the base for the interpretation. The semantic actions for Rule 2 serve to copy the base from its inception at Rule 3.

Experienced compiler writers make frequent use of rich semantic types and grammar restructuring to localize the effects of semantic actions. The resulting parsers are (mostly) free of global variables and easier to understand and modify.

Unfortunately, our grammar modification is deficient because it effected a subtle change in the language (see Exercise 2).

7.3 Top-Down Syntax-Directed Translation

In this section we discuss how semantic actions can be performed in top-down parsers. Such parsers are typically generated by hand, using the *recursive-descent* style discussed in Chapters 2 and 5. The result of such parser construction is simply a program; semantic actions can be written directly into the parser.

To illustrate this style of translation, we consider the processing of Lisp-like [McC60, FF86] expressions, defined by the grammar shown in Figure 7.9.

```
1  Start   → Value $
2  Value   → num
3          | lparen Expr rparen
4  Expr    → plus Value Value
5          | prod Values
6  Values  → Value Values
7          | λ
```

Figure 7.9: Grammar for Lisp-like expressions.

> For a review of this style of writing top-down parsers, see Section 2.5 on page 39 and Section 5.3 on page 149. The relevant grammar analyses and parser constructions are given as Exercise 2 on page 173.

Rule 1 generates an outermost expression whose value should be printed as the result of the syntax-directed translation. For example, the input

```
( plus 31 ( prod 10 2 20 ) ) $
```

should print `431`.

A Value is defined by Rule 2, which allows a simple numeric value via num, and Rule 3, which treats the result of a parenthesized expression as a value. Rules 4 and 5 generate the sum of two values and a product of zero or more values, respectively. The grammar lacks many features that would be expected in an expression-oriented language. A more complete expression grammar is considered in Exercises 4 and 5. The recursive rule for Values is right recursive to accommodate top-down parsing (Section 5.5 on page 154).

A recursive-descent parser for the grammar in Figure 7.9 is shown in Figure 7.10. As discussed in Chapter 5, the parser contains a procedure for each nonterminal in the grammar. To conserve space, the error checks normally present at the end of each **switch** statement are absent in Figure 7.10.

The parser in Figure 7.10 is also equipped with semantic actions that compute and print expression values. It is common in recursive descent parsing for semantic actions to make use of inherited and synthesized values. Inherited values are manifest as parameters passed into a method; synthesized values are returned by methods after deriving their parse subtrees. These ideas are combined in the parser of Figure 7.10 as follows:

- At Marker ⑤, the semantic value synthesized from a Value parse subtree is printed.

```
procedure START( )
    switch (...)
        case ts.PEEK( ) ∈ { num, lparen }
            ans ← VALUE( )
            call MATCH( $ )
            call PRINT( ans )                                  ⑤
end
function VALUE( ) returns int
    switch (...)
        case ts.PEEK( ) ∈ { num }
            call MATCH( num )
            ans ← num.VALUEOF( )
            return (ans)
        case ts.PEEK( ) ∈ { lparen }
            call MATCH( lparen )
            ans ← EXPR( )
            call MATCH( rparen )
            return (ans)
end
function EXPR( ) returns int
    switch (...)
        case ts.PEEK( ) ∈ { plus }
            call MATCH( plus )
            op1 ← VALUE( )                                     ⑥
            op2 ← VALUE( )                                     ⑦
            return (op1 + op2)                                 ⑧
        case ts.PEEK( ) ∈ { prod }
            call MATCH( prod )
            ans ← VALUES( 1 )                                  ⑨
            return (ans)
end
function VALUES( thusfar ) returns int
    case ts.PEEK( ) ∈ { num, lparen }
        next ← VALUE( )                                        ⑩
        ans ← VALUES( thusfar × next )                         ⑪
        return (ans)
    case ts.PEEK( ) ∈ { rparen }
        return (thusfar)                                       ⑫
end
```

Figure 7.10: Recursive-descent parser with semantic actions. The variable *ts* is the token stream produced by the scanner.

- Markers ⑥ and ⑦ capture the values synthesized from adjacent Value subtrees to form their sum. The result is synthesized by the **return** at Marker ⑧.

- The parameter $thusfar$ of Values represents the product of the factors parsed so far. Marker ⑨ causes Values to inherit the value of an empty partial product (i.e., 1).

  The product continues at Marker ⑪ by incorporating the next factor, which is synthesized at Marker ⑩.

  The product finishes at Marker ⑫, when the production Values $\rightarrow \lambda$ is applied. The partial product passed in as $thusfar$ is the complete product, and the **return** at Marker ⑫ initiates synthesis of that value up the Values call-chain.

7.4 Abstract Syntax Trees

While many of a compiler's tasks could be performed in a single phase via syntax-directed translation, modern software practices discourage implementing so much functionality to a single component such as the parser. Tasks such as semantic analysis, symbol table construction, program optimization, and code generation are each deserving of separate treatment in a compiler. Squeezing all of those tasks into a single compiler phase is an admirable feat of engineering, but the resulting compiler is difficult to understand, extend, and maintain.

We therefore consider the design and implementation of a data structure, known as the AST, that will serve as the central data structure for all post-parsing activities. The goal of syntax-directed translation is then simplified to the construction of an AST. The AST must be concise, but it must also be sufficiently flexible to accommodate the diversity of post-parsing phases. It is not uncommon to revisit the design of the AST during the life cycle of a compiler's development. As discussed in Section 7.7, object-oriented techniques such as the *visitor pattern* facilitate robust compiler design, separation of concerns, and phase interoperability.

7.4.1 Concrete and Abstract Trees

We begin by distinguishing *concrete* from *abstract* syntax trees, as first discussed in Section 2.6 on page 45. Parse trees such those shown in Figures 7.3 and 7.4 are *concrete*, in the sense that they show every detail of the parse. Consider the nonterminal Digs and the parse tree shown in Figure 7.3. The tree leans to the left because the rules for Digs are left recursive. If these rules were right recursive, then the parse subtree for Digs would change accordingly.

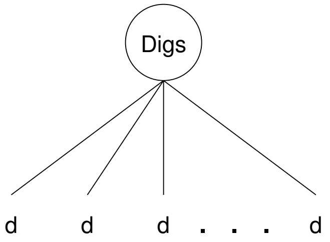


Figure 7.11: Abstract syntax tree for Digs.

Abstractly, the symbol Digs represents a list of d symbols. The order in which the particular d symbols occur is important to proper translation of the meaning of the list. The use of one particular style of recursion in the grammar's rules may be well suited to a given parsing method, but the *meaning* of the list should be the same either way (see Exercise 1).

The symbols derived from Digs can instead be represented abstractly by the AST shown in Figure 7.11. The Digs node serves as parent for *any* number of d nodes. The parsing infrastructure that generated the sequence of d symbols is lost. However, the order of the digits is important for synthesizing the value of the string of symbols. Thus, the order among the d symbols is retained. No grammar could generate the tree in Figure 7.11 as a concrete parse tree: each production in a grammar has a fixed number of symbols on its right-hand side.

7.4.2 An Efficient AST Data Structure

While there are many data structure choices to represent a tree, the design of an AST should take into account the following:

- The AST is typically constructed bottom-up: a list of siblings is generated, and that list is later adopted by a parent. The AST data structure should therefore support tree construction from the leaves toward its root.

- Lists of siblings are typically generated by recursive rules. The AST data structure should simplify adoption of siblings at either end of a list.

- Some AST nodes have a fixed number of children. For example, binary addition and multiplication require two children. However, some programming language constructs may require an arbitrarily large number of children. Such constructs include compound statements, which accommodate zero or more statements, and method parameter and argument lists. The data structure should efficiently support tree nodes with an arbitrary number of children.

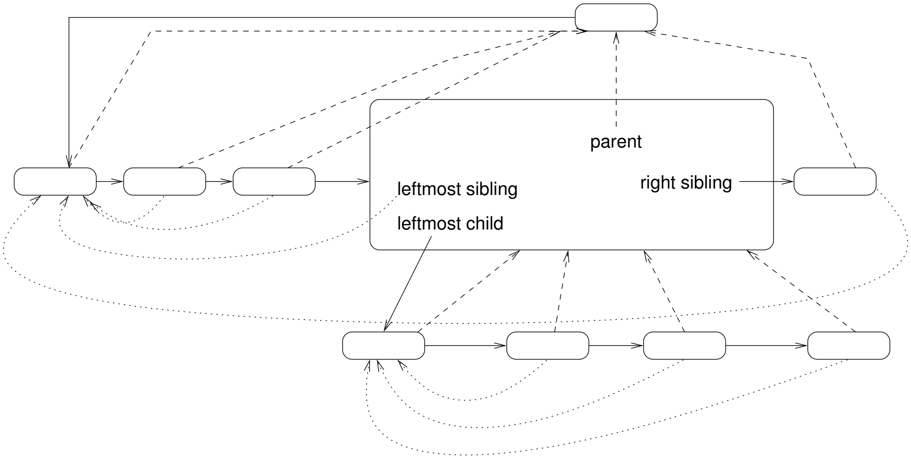


Figure 7.12: Internal format of an AST node. A dashed line connects a node with its parent; a dotted line connects a node with its leftmost sibling. Each node also has a solid connection to its leftmost child and right sibling.

Figure 7.12 shows an organization for an AST data structure based on the above discussion. Although each node can have an arbitrary number of children, each node is of constant size:

- Each node points to its next (right) sibling. These pointers form a singly-linked list of siblings.

  To facilitate access to the head of that list in constant time, each node points also to its leftmost sibling.

- Each node n points to its leftmost child, which forms the beginning of the list of n's children.

  Thus, a node with k children uses one pointer to reach the leftmost child. That child's siblings are then reached using right sibling pointers.

- Each node points to its parent.

7.4.3 Infrastructure for Creating ASTs

To facilitate construction of an AST, we fashion the following methods to create and manage AST nodes.

```
/*  Assert: $y \neq$ null                                        */
function makeSiblings( $y$ ) returns Node
    /*  Find the rightmost node in this list                     */
    $xsibs \leftarrow this$
    while $xsibs.rightSib \neq$ null do $xsibs \leftarrow xsibs.rightSib$
    /*  Join the lists                                           */
    $ysibs \leftarrow y.leftmostSib$
    $xsibs.rightSib \leftarrow ysibs$
    /*  Set pointers for the new siblings                        */
    $ysibs.leftmostSib \leftarrow xsibs.leftmostSib$
    $ysibs.parent \leftarrow xsibs.parent$
    while $ysibs.rightSib \neq$ null do
        $ysibs \leftarrow ysibs.rightSib$
        $ysibs.leftmostSib \leftarrow xsibs.leftmostSib$
        $ysibs.parent \leftarrow xsibs.parent$
    return ( $ysibs$ )
end

/*  Assert: $y \neq$ null                                        */
function adoptChildren( $y$ ) returns Node
    if $this.leftmostChild \neq$ null
    then $this.leftmostChild$.makeSiblings( $y$ )
    else
        $ysibs \leftarrow y.leftmostSib$
        $this.leftmostChild \leftarrow ysibs$
        while $ysibs \neq$ null do
            $ysibs.parent \leftarrow this$
            $ysibs \leftarrow ysibs.rightSib$
end
```

Figure 7.13: Methods for building an AST.

makeNode(t) is a *factory* method that creates a node whose contents and accessor methods depend on the type of t.

For example, makeNode(int n) instantiates a node that represents the constant integer n and that offers an accessor method that returns n.

makeNode(Symbol s) instantiates a node for a symbol s. Methods must be included to set and get the symbol table entry for s, from which its type, protection, and scope information can be retrieved.

makeNode(Operator o) instantiates a node for an operation, such as addition or subtraction. Details of the operation must be provided by accessor methods.

makeNode( ) instantiates a *null* node that explicitly represents the absence of structure. For consistency in processing an AST, it is better to have a null node than to have gaps in the AST or null pointers.

x.makeSiblings(y) causes y to become x's sibling, using the code shown in Figure 7.13. In the case where x has no right sibling and y is its own leftmost sibling, y becomes x's right sibling.

More generally, x is a (typically rightmost) node on a list of siblings *xsibs*; y is a node on a (typically singleton) list of siblings *ysibs*. The method appends the lists *xsibs* and *ysibs*. All siblings point to x's parent as their parent and to x's leftmost sibling as their leftmost sibling.

To facilitate chaining of nodes through recursive grammar rules, this method returns a reference to the rightmost sibling resulting from the invocation.

x.adoptChildren(y) adopts y and all of its siblings under the parent x, using the code shown in Figure 7.13.

makeFamily($op, kid_1, kid_2, \ldots, kid_n$) is included for convenience. This method generates a family with exactly n children under a parent node op. The code for the most common case ($n = 2$) is:

```
function makeFamily(op, kid1, kid2) returns Node
    return (makeNode(op).adoptChildren(kid1.makeSiblings(kid2)))
end
```

With our AST data structure and methods in place, we next consider issues related to the design of a particular AST based on a given grammar and language translation problem.

7.5 AST Design and Construction

Because the AST is centrally involved in most of a compiler's activities, its design can and should evolve with the functionality of the compiler. A properly designed AST simplifies both the work required within a single compiler phase as well as the manner in which the compiler phases communicate. There are important *forces* that influence the design of an AST:

- It should be possible to *unparse* (i.e., reconstitute) an AST into a form whose execution is sufficiently similar to the execution of the program represented by the AST.

  Thus, the AST nodes must hold sufficient information to recall the essential elements of the program fragment they represent.

- The implementation of an AST should be decoupled from the essential information represented within the AST.

  Accessors are therefore provided to hide a node's internal representation and to facilitate interoperability in the compiler's phases.

- Because the phases of a compiler view elements of an AST in fundamentally different ways, there is no single class hierarchy that can serve to describe AST nodes for all purposes.

  The class structure of AST nodes is therefore designed for expediency of constructing the AST. Use of the AST by a compiler's phases is facilitated by the various phase-specific *interfaces* implemented by an AST's nodes.

Given a source programming language L, the development of a grammar and the design of an appropriate AST structure typically proceed as follows:

1. An unambiguous grammar for L is devised. The grammar may contain productions that serve specifically to disambiguate the grammar. Recall a grammar is unambiguous if a top-down or bottom-up parser can be constructed for the grammar.

2. An AST for L is devised. The AST design typically discards grammar details concerned with disambiguation. Semantically useless symbols and punctuation such as , and ; are also omitted. The AST retains sufficient information to allow the compiler's phases to perform their work efficiently and cleanly.

3. Semantic actions are placed in the grammar to construct the AST. The design of the AST may require grammar modifications to simplify or localize construction. The semantic actions can rely on the methods described in Section 7.4.3 to create and manipulate the AST's nodes and edges.

4. Passes of the compiler are designed. Each phase may place new requirements on the AST in terms of its structure and contents; the grammar and the design of the AST may need to be revisited.

We illustrate the above methodology with the following example.

```
 1  Start  → Stmt $
 2  Stmt   → id assign E
 3         | if lparen E rparen Stmt else Stmt fi
 4         | if lparen E rparen Stmt fi
 5         | while lparen E rparen do Stmt od
 6         | begin Stmts end
 7  Stmts  → Stmts semi Stmt
 8         | Stmt
 9  E      → E plus T
10         | T
11  T      → id
12         | num
```

Figure 7.14: Grammar for a simple language.

7.5.1 Design

Figure 7.14 shows a grammar for a language that is relatively simple but contains features found in most programing languages. The language uses only integer data types, so declarations are unnecessary. We begin by considering each portion of the grammar in Figure 7.14 with the purpose of designing its AST structure.

Assignment statements Type analysis and code generation will require information about the target of the assignment and the value that will be stored at that target. The AST structure for an assignment statement can therefore be close to its concrete syntax. The assignment operator becomes the parent of the identifier and expression subtree, as shown in Figure 7.15(a).

if **statements** There are two forms of if statements in the grammar: Rule 3 generates an else clause and Rule 4 does not. We could design separate AST structures for each, but a more consistent approach views the second form as an instance of the first, with a *null* node inserted to represent the else clause.

Figure 7.15(c) show an AST structure suitable for both cases. The concrete syntax uses 6–8 symbols for an if statement, but most of those are punctuation required by the source language's syntax. Those symbols serve to disambiguate the language and make programs written in the language easier to read. However, only the essential elements are retained in Figure 7.15(c): the predicate that is tested by the statement and

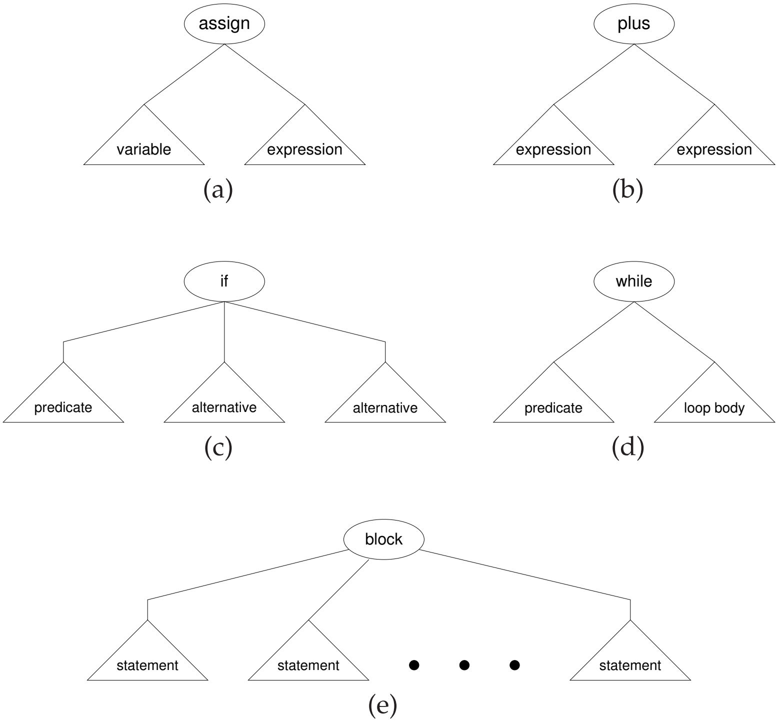


Figure 7.15: AST structures: A specific node is designated by an ellipse. Tree structure of arbitrary complexity is designated by a triangle.

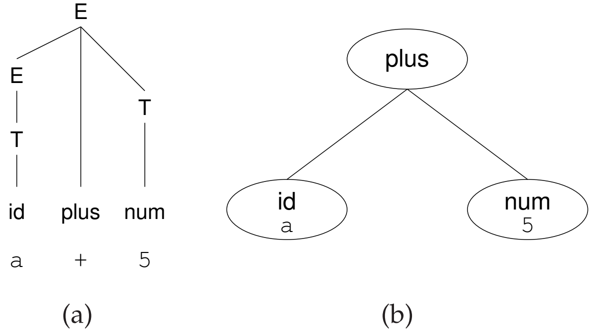


Figure 7.16: (a) Derivation of `a + 5` from E;
(b) Abstract representation of `a + 5`.

the resulting code that should be executed, if any, based on the outcome of that test.

while **statements** The AST structure shown in Figure 7.15(d) retains the two essential elements of a while statement: the predicate expression and the loop body.

Block of statements Rules 6, 7, and 8 work together to synthesize a block (sequence) of statements. As with the digits example of Figure 7.11, we need only record the order of the statements. The result of Rule 6 is therefore the structure shown in Figure 7.15(e).

plus **operations** The section of the grammar in Figure 7.14 that derives symbols from E has rules that serve only to disambiguate the grammar. The concrete syntax derived from E is shown in Figure 7.16(a). By eliminating structure unnecessary for translation, we arrive at the AST design shown in Figure 7.16(b). A nodes such as plus that represents binary (two-operand) operations becomes a parent with two children that supply the operands.

The arithmetic operations can thus be modeled after the assignment structure in Figure 7.15(a), with the assignment operator replaced by the given arithmetic operator. The result, shown in Figure 7.15(b), represents the sum of its two children.

7.5.2 Construction

Next, semantic actions must be added into the parser to construct the AST structures shown in Figure 7.15. When the parse is finished, the AST is returned as its artifact at Marker ⑬. Markers ⑭, ⑮, ⑯, and ⑰ synthesize the structures shown in Figure 7.15 for assign, if, and while statements. A block of statements is generated at Marker ⑱, assuming that Stmts does its job properly and returns a list of siblings—one for each statement in the block. The list is begun with the Stmt first reduced at Marker ⑳. Thereafter, each succeeding Stmt is added to the sibling list at Marker ⑲. The rules for E work similarly, with the first operand of the sum reduced at Marker ㉒. The plus structure is generated at Marker ㉑. Leaf nodes for variables and integer constants are generated at Markers ㉓ and ㉔.

Figure 7.18 shows a concrete syntax tree for a program written in the language defined by the grammar in Figure 7.14. The semantic actions of Figure 7.17 create the AST shown in Figure 7.19.

```
 1 Start          → Stmt_ast $
                     return (ast)                                              (13)
 2 Stmt_result    → id_var assign E_expr
                     result ← makeFamily(assign, var, expr)                    (14)
 3                | if lparen E_p rparen Stmt_s fi
                     result ← makeFamily(if, p, s, makeNode( ))                (15)
 4                | if lparen E_p rparen Stmt_s1 else Stmt_s2 fi
                     result ← makeFamily(if, p, s1, s2)                        (16)
 5                | while lparen E_p rparen do Stmt_s od
                     result ← makeFamily(while, p, s)                          (17)
 6                | begin Stmts_list end
                     result ← makeFamily(block, list)                          (18)
 7 Stmts_result   → Stmts_sofar semi Stmt_next
                     result ← sofar.makeSiblings(next)                         (19)
 8                | Stmt_first
                     result ← first                                            (20)
 9 E_result       → E_e1 plus T_e2
                     result ← makeFamily(plus, e1, e2)                         (21)
10                | T_e
                     result ← e                                                (22)
11 T_result       → id_var
                     result ← makeNode(var)                                    (23)
12                | num_val
                     result ← makeNode(val)                                    (24)
```

Figure 7.17: Semantic actions for grammar in Figure 7.14.

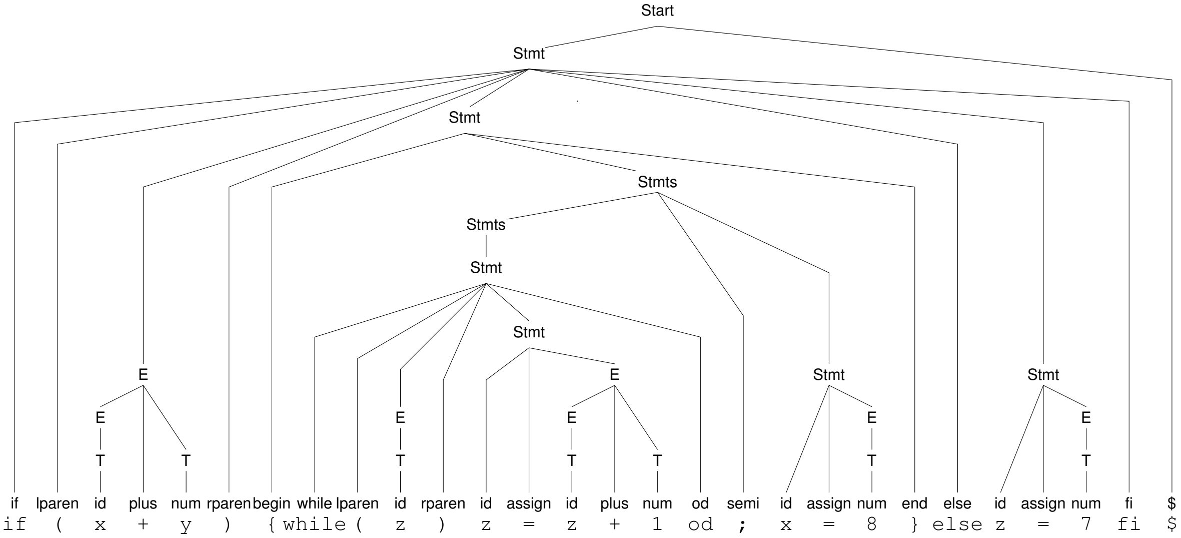

Start
Stmt
Stmt
Stmts
Stmts
Stmt
Stmt
E
E
T
E
T
E
T
E
T
E
T
Stmt
E
T
Stmt
E
T
if lparen id plus num rparen begin while lparen id rparen id assign id plus num od semi id assign num end else id assign num fi $
if (x + y) { while (z) z = z + 1 od ; x = 8 } else z = 7 fi $

Figure 7.18: Concrete syntax tree.

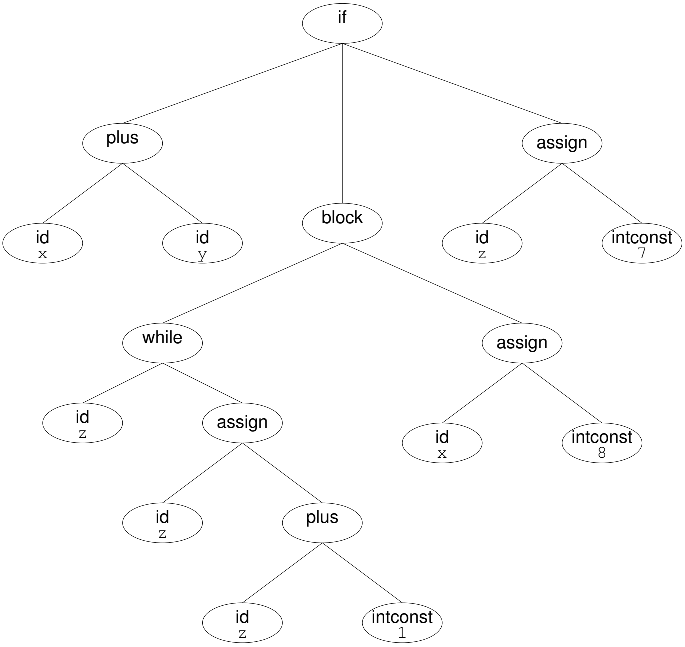


Figure 7.19: AST for the parse tree in Figure 7.18.

```
1 Start → Stmt $
2 Stmt  → L assign R
3 L     → id
4       | deref R
5 R     → L
6       | num
7       | addr L
```

Figure 7.20: Grammar for left and right values.

7.6 AST Structures for Left and Right Values

When an identifier is used in a programming language, it typically means one of the following: the *value* associated with the name, or the the *location* (address) at which a value is stored. Programming language definitions specify when an identifier means its value or its location—most often, the meaning depends on the context in which the identifier is used. For example, the assignment statement

$$x = y$$

references two identifiers, but their location on either side of the assignment operator significantly changes their meaning:

$x = \underline{y}$ The identifier y refers to the *value* of y. Such usage is commonly called a **right value** (R-value), from its appearance to the right of the assignment operator.

Some programming language elements have only right-value forms. A constant's value can be referenced, but the location of a constant is typically beyond reference, and languages prohibit changing a constant's value. An object's self reference (`this`) is typically available only in right-value form.

$\underline{x} = y$ The identifier x refers to the *location* of x, not its value. Such usage is called the **left value** (L-value) of x from its appearance to the left of the assignment operator.

Some languages provide a mechanism by which any R-value can be treated as an L-value using a *dereference* operator. For example, the expression $\star\, e$ in C allows the R-value of e to serve as an L-value. Other languages (e.g., Java) carefully limit L-values so as to reduce the possibility of changing storage unintentionally.

For the language defined in Figure 7.14, and in the ASTs generated from that grammar, the meaning of an identifier in terms of its R-value or L-value is

```
1 Start          → $\mathsf{Stmt}_{ast}$ $
                   return (ast)
2 $\mathsf{Stmt}_{result}$ → $\mathsf{L}_{target}$ assign $\mathsf{R}_{source}$
                   $result \leftarrow$ MAKEFAMILY(assign, $target$, $source$)
3 $\mathsf{L}_{result}$   → $\mathsf{id}_{name}$
                   $result \leftarrow$ MAKENODE($name$)
4                | deref $\mathsf{R}_{val}$
                   $result \leftarrow val$                                   ㉕
5 $\mathsf{R}_{result}$   → $\mathsf{L}_{val}$
                   $result \leftarrow$ MAKEFAMILY(deref, $val$)              ㉖
6                | $\mathsf{num}_{val}$
                   $result \leftarrow$ MAKENODE($val$)
7                | addr $\mathsf{L}_{val}$
                   $result \leftarrow val$                                   ㉗
```

Figure 7.21: Semantic actions to create ASTs for the grammar in Figure 7.20.

clear: an L-value appears only as the left child of an assignment operator; all other uses of identifiers refer to their R-values.

Differentiating left and right values can be more difficult for languages like C that contain explicit syntax for treating R-values as L-values and vica versa:

$\underline{\star\, p} = 0$ Without the $\star$, p would be treated as an L-value, and the assignment would set p to 0. With the $\star$, p is treated as an R-value, and the value stored at p becomes an L-value. Thus, the assignment stores 0 where p points.

$x = \underline{\&\, p}$ Without the &, p would be treated as an R-value, and the assignment would copy the value from p to x. With the &, p is treated as an L-value, and the resulting address is then treated as an R-value. Thus, the assignment sets x to the address of p.

Given the location of an identifier, we can always find the value stored there by indirection; but we cannot find an identifier's location from its value. For our AST representation of names, we will therefore always regard an identifier as representing that name's location. Explicit deref nodes will be placed in the AST to show exactly where indirection (dereferencing) occurs to obtain the value of a name from its location.

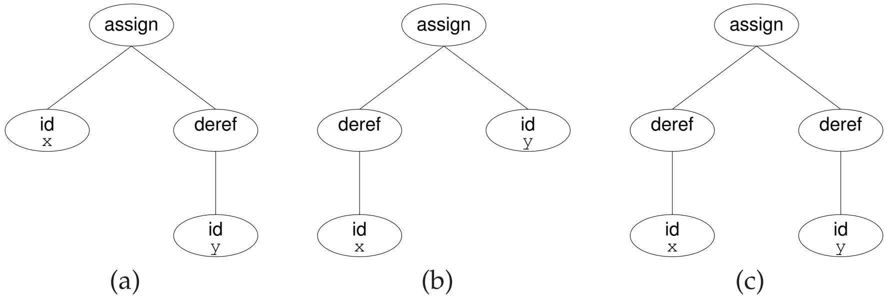


Figure 7.22: ASTs illustrating left and right values for the assignments:
(a) x = y
(b) x = & y
(c) ★ x = y

The grammar in Figure 7.20 models the syntax used in C for left and right values:

- Rules 3 and 4 define the syntax for L-values. Rule 3 allows an identifier to appear, in which case its interpretation is its location, as stated above. Rule 4 allows an R-value to appear left of the assignment operator, provided it is preceded by a ★ symbol. The presence of ★ is purely syntactic, as no dereferencing takes place on its account. For example, the statement ★ 431 = 0 sets the value at location 431 to 0.

- Rules 5, 6, and 7 define the syntax for R-values. Rule 6 allows numeric constants, whose interpretation is simply their value. Rule 7 allows the location of an L-value to be an R-value. The phrase $x + 1$ is one greater than the value of x; the phrase & $x + 1$ is the location just past where x is stored. If the language did not offer the & operator, there would be no syntax for expressing arithmetic on addresses (as is the case in Java).

  Although it seems simple, the rule deserving the most attention is Rule 5. Any references of an id right of the assignment operator must first be derived from L which can then be reduced to R by the production R→L. An L-value is transformed into an R-value by an actual dereference—by obtaining the value located at the L-value. The ★ and & operators require no work to implement. They serve to provide syntax that alters the usual meaning of identifiers on either side of the assignment operator.

The above observations provide a basis for synthesizing an AST for the grammar in Figure 7.20. Semantic actions are incorporated into the grammar as shown in Figure 7.21. Note that Markers ㉕ and ㉗ simply return the AST structure created below, with no extra operations inserted. However, Marker (26) inserts a node indicating an actual dereference (fetch through a pointer). Figure 7.22 shows the ASTs for various uses of the operators ⋆ and & .

7.7 Design Patterns for ASTs

With the AST established as the primary post-parsing artifact of compilation, we now consider the AST's role in the remaining phases. Chapter 2 presented an overview of compilation, demonstrating the use of an AST for semantic analysis and code generation in Sections 2.7 and 2.8. While the infrastructure for creating and managing ASTs could be based on the coverage in Sections 7.4 and 7.5 alone, it is worthwhile to consider how those tasks could be simplified through application of modern, object-oriented principles and techniques.

Design patterns [GHJV95] have emerged as a mechanism for capturing idiomatic gestures in software and reasoning about effective solutions to common design problems. While a complete treatment of design patterns is beyond this text, we discuss one design pattern that can be applied with great success in crafting a compiler. A guiding principle that greatly facilitates understanding when to apply a design pattern is that patterns are generally meant to *save time and effort* in developing and maintaining software. Patterns can make software easier to understand, thereby facilitating maintenance and modification. Patterns can also simplify software construction by helping developers recognize problems that already have reasonable solutions.

7.7.1 Node Class Hierarchy

Chapter 2 presented a small language and considered construction of its AST in Section 2.6 on page 45. Figure 2.9 on page 44 shows an example of such an AST, which includes nodes that declare variables, represent computations, and call for the printing of a variable's value.

The node-management issues presented in Section 7.4 are common to all nodes, so it is reasonable to have every node of Figure 2.9 extend a base class that can connect siblings, adopt children, and facilitate traversal of an AST. Beyond those basic functions, what class hierarchy should we impose on the node types of an AST? To answer that question, we must look at how the various compiler phases treat the AST nodes.

Consider the phases described in Sections 2.7 and 2.8. There is a close physical and logical resemblance between the plus and assign nodes: both have two children and both are involved in computation. However, their treatment varies substantially by phase:

- Type analysis in Figure 2.12 on page 49 attempts to reconcile the type of the subtrees beneath a plus node by finding the least-general type *t* that is suitable for the subtrees. The values used by the plus node are converted to type *t*, and the result of the addition is also of type *t*.

  Type analysis of an assign node in Figure 2.12 on page 49 attempts to coerce the right subtree to match the type of the left subtree, so that the value to be stored is appropriate for the variable receiving that value.

- The distinction between left and right values differs as well for code generation in Figure 2.14 on page 52. Within a plus tree, identifiers are assumed to represent their right values. For an assign node, identifiers within the right subtree are right values, but the target of the assignment is a left value.

On the other hand, some nodes are treated similarly by almost every phase:

- The plus and minus nodes of the ac language in Chapter 2 can be treated identically except for the arithmetic operation performed on their subtrees.

- Treatment of the if construct in most programming languages is very similar to the ternary operator (e.g., ? in C and Java), even though the specifics and nature of their syntax is very different.

It turns out there is no single node class hierarchy that is well suited to all phases of compilation. While one hierarchy may be well suited to semantic analysis, code generation or optimization phases becomes harder to write when saddled with a class hierarchy that favors some other phase.

As a result, the node class hierarchy is relatively flat. Node management is placed in a common superclass, say *AbstractNode*. Each type of node (if, plus, etc.) is then a simple extension of *AbstractNode* with enough construct-specific functionality to allow the phases to do their work. Superclasses can be introduced to simplify AST construction, by factoring common code between node types. However, the resulting class hierarchy should not necessarily be considered as a basis for designing the compiler's phases.

7.7.2 Visitor Pattern

The next issue to consider is the crafting of a compiler phase, in terms of the classes available to host such code, and the node organization established thus far. ASTs for Languages like Java contain ~50 node types, and compilers like the **GNU Compiler Collection** (GCC) have ~200 phases. To manage the relatively large space of potential phase and node interactions, modern software engineering principles dictate that the code for a phase should be

```
class Visitor
    /* Generic visit                                        */
    procedure visit( AbstractNode n )                       ㉘
        n.accept( this )                                    ㉙
    end
end
class TypeChecking extends Visitor                          ㉚
    procedure visit( IfNode i )
    end
    procedure visit( PlusNode p )
    end
    procedure visit( MinusNode m )
    end
end

class IfNode extends AbstractNode
    procedure accept( Visitor v )                           ㉛
        v.visit( this )
    end
    ...
end
class PlusNode extends AbstractNode
    procedure accept( Visitor v )                           ㉜
        v.visit( this )                                     ㉝
    end
    ...
end
class MinusNode extends AbstractNode
    procedure accept( Visitor v )                           ㉞
        v.visit( this )
    end
    ...
end
```

Figure 7.23: Visitor pattern

written in a single class, and not distributed among the various node types (see Exercise 20).

A phase is thus crafted by writing VISIT methods in the phase's class—one for each kind of node for which some action must be performed. An example of this style of code is shown in Figure 2.14 on page 52. A phase f then performs its work for a particular node n in response to the method call:

f.VISIT(*AbstractNode* n)

Most object-oriented languages use **single dispatch** to determine which VISIT method should be invoked in response to the above method call. The dispatch is based on the *actual type* of the receiver object f. Unfortunately, single dispatch finds a match for VISIT based on the *declared type* of its parameters at the call site. Thus, if a phase contained a method VISIT(*IfNode* n), that method would not be invoked on an actual *IfNode*, because the match is based on the declared type (*AbstractNode*) of the supplied parameter.

While other solutions are possible (see Exercises 20 and 21), invoking VISIT based on the compiler's phase f *and* a the supplied node n's *actual* type requires **double dispatch** (a limited form of **multiple dispatch**). The **visitor pattern** achieves a form of double dispatch for languages that offer only single dispatch. The visitor pattern enables phase- and node-specific code to be invoked cleanly, while aggregating the functionality of a phase in a single class. Figure 7.23 illustrates the application of the visitor pattern for our example. The code is organized as follows:

- Every phase extends the *Visitor* class, as shown at Marker ㉚, so that it inherits the VISIT(*AbstractNode* n) method.
- Every concrete node class includes the method shown at Markers ㉛, ㉜, and ㉞ that *accepts* a visitor and accomplishes double dispatch as described below.

  While the inclusion of the ACCEPT method in every node class seems redundant, it cannot be factored into a common superclass, because the type of **this** must be specific to the visited node.

As an example, consider the invocation of f.VISIT(*AbstractNode* n) when f is an instance of *TypeChecking* and n is an instance of *PlusNode*. Multiple dispatch is accomplished as follows:

- The inherited method VISIT(*AbstractNode* n) at Marker ㉘ is invoked, with **this** bound to the *TypeChecking* phase.
- Marker ㉙ invokes n.ACCEPT(**this**). Although n is declared of type *AbstractNode*, single dispatch will invoke the ACCEPT method that is most specialized to the actual type of n.

Thus, with n actually of type *PlusNode*, the method at Marker ㉜ is invoked. Within that method, the declared type of **this** is *PlusNode*—the type of the containing class.

- Finally, Marker ㉝ is executed, which invokes a method within the particular visitor (*TypeChecking*) that has signature visit(*PlusNode*).

Thus, the effect of f.visit(*AbstractNode* n) to invoke a method within f that is specialized by the actual type of n.

7.7.3 Reflective Visitor Pattern

Section 7.7.2 is a classic application of the visitor pattern to achieve double dispatch in a language offering only single dispatch. From a code-authoring perspective, the following disadvantages remain:

- Every concrete node class must include the method visit(*Visitor*) to achieve double dispatch based on the node's type. That method cannot be moved into a superclass because the parameter supplied to the visit method must match the node type that accepted the visitor.

- Every visitor must be prepared to visit any concrete node type, even if such nodes require no action on the part of a given visitor.

  To avoid redundancy and clutter, an *EmptyVisitor* class could be constructed that offers a visit method for every node type that simply returns without performing any actions. A useful visitor could extend the *EmptyVisitor* class and override those visit methods in which some actions must take place.

- Within a phase visitor, the visit methods' signatures are limited to node class types. A phase is not able to factor commonality of treatment into a single method except by delegation from methods that intercept a concrete node type. As discussed in Section 7.7.1, there is no single inheritance hierarchy that is well suited to every phase.

By using *reflection*, we can view node types on a per-visitor basis and avoid the need to specify visit methods for every node type. **Reflection** is a programming language's ability to inspect, reason about, manipulate, and act upon elements of the language, such as object types.

The code for a *reflective visitor* is shown in Figure 7.24. The method visit(*AbstractNode* n) does not call n.accept(**this**) to perform the second dispatch, as was the case at Marker ㉙. Instead, the dispatch method is invoked at Marker ㊱ to determine the best match for visiting the supplied node n. The reflective visitor works as follows:

```
class ReflectiveVisitor
    /⋆  Generic visit                                        ⋆/
    procedure visit( AbstractNode n )                        ㉟
        this.dispatch( n )                                   ㊱
    end
    procedure dispatch( Object o )
        /⋆  Find and invoke the visit( n ) method            ⋆/
        /⋆  whose declared parameter n is the closest match  ⋆/
        /⋆  for the actual type of o.                        ⋆/
    end
    procedure defaultVisit( AbstractNode n )                 ㊲
        foreach AbstractNode $c \in Children(n)$ do this.visit( c )
    end
end
class IfNode                                                 ㊳
    extends AbstractNode
    implements { NeedsBooleanPredicate }
end
class WhileNode                                              ㊴
    extends AbstractNode
    implements { NeedsBooleanPredicate }
end
class PlusNode
    extends AbstractNode
    implements { NeedsCompatibleTypes }
end
class TypeChecking extends ReflectiveVisitor                 ㊵
    procedure visit( NeedsBooleanPredicate nbp )             ㊶
        /⋆  Check the type of nbp.getPredicate( )            ⋆/
    end
    procedure visit( NeedCompatibleTypes nct )               ㊷
    end
    procedure visit( NeedsLeftChildType nlct )               ㊸
    end
end
```

Figure 7.24: Reflective Visitor

1. An extension of *ReflectiveVisitor* is instantiated, such as *TypeChecking* at Marker ㊵. We denote the instance of that visitor as *v*.

2. The invocation *v*.visit(*root*) initiates the *TypeChecking* visitor's processing of the AST at the *root* node of the AST.

3. With *root* processed as a generic *AbstractNode*, the invocation is matched by single dispatch with the method visit(*AbstractNode n*) at Marker ㉟.

4. At Marker ㊱, the dispatch method is called to accomplish the second dispatch, by determining reflectively which particular visit should be invoked, as follows.

   All methods in the actual visitor (*TypeChecking*) are examined, and the visit method that accepts a node type *most closely matched* to the supplied node's *actual* type is invoked. The nature of this matching process is described below.

   If no suitable match is found, then the method at Marker ㊲ is invoked as a default action, and its behavior passes the visitor along to the children of the supplied node.

   Note that the defaultVisit(*Object o*) at Marker ㊲ method can be redefined by a reflective visitor subclass, so this default behavior can be customized.

How do we find the visit method in a reflective visitor *v* that is most appropriate for handling node *n*?

- If node *n*'s type is *t*, then the method visit(*t*) is the exact match that would have been found by the nonreflective visitor in Figure 7.23. If *v* contains such a method, then it is the best choice to handle node *n*.

- If no exact match is found, then the search widens to find a visit method that can handle a *superclass* of *t*.

A class in C++ may lack a unique immediate superclass, because C++ allows **multiple inheritance**. Every class (except *Object*) has a unique superclass in Java. However, classes in Java can implement any number of *interfaces*. Thus, the search for visit(*w*), where *w* is a wider type than *t*, does not necessarily yield a unique result.

In practice, the reflective visitor is crafted so that the desired match is clearly present in the visitor and easily found by the widening process, as follows.

- The instantiable node type hierarchy is largely disregarded. If a node has actual type *t*, then it is unlikely that a visitor will offer a method visit(*t*).

- Instead, the behavior of the visitor is considered in terms of its common treatment of an AST's nodes.

  For example, consider the `while` and `if` statements. While the structure of those statements differs greatly, each involves a predicate whose type should be **Boolean** (**true** or **false**).

- A *TypeChecking* phase that checks predicates for proper type should treat the predicates of the `while` and `if` statements similarly. To achieve this effect, we arrange for the `if` and `while` nodes to inherit from a common class that will be handled by the *TypeChecking* visitor.

  The `if` and `while` node types implement the *NeedsBooleanPredicate* interface at Markers ㊳ and ㊴ in Figure 7.24.

  The *TypeChecking* visitor intercepts such nodes with the visit method shown at Marker ㊶.

The interfaces (or abstract classes) associated with a given node type allow clear and proper treatment of that node type for each visitor that must perform some action for such a node. Moreover, the visitor code itself becomes self documenting, in the sense that the visit methods capture the intent and scope of the visitor based on the abstract classes and the properties they represent. An implementation of the reflective visitor pattern, and the application of that pattern to some examples, can be found in the *Crafting a Compiler Supplement*.

Summary

Syntax-directed translation can accomplish translation directly via semantic actions that execute in concert with top-down or bottom-up parses. More commonly, an AST is constructed as a by-product of the parse; the AST serves as a record of the parse as well as a repository for information created and consumed by a compiler's phases. The design of an AST is routinely revised to simplify or facilitate compilation.

Exercises

1. Consider a right-recursive formulation for Digs of Figure 7.3, resulting in the following grammar.

$$\begin{array}{llll}
1 & \mathsf{Start} & \rightarrow & \mathsf{Digs}_{ans}\ \ \$ \\
 & & & \textbf{call}\ \text{PRINT}(ans) \\
2 & \mathsf{Digs}_{up} & \rightarrow & \mathsf{d}_{next}\ \ \mathsf{Digs}_{below} \\
 & & & up \leftarrow below \times 10 + next \\
3 & & \mid & \mathsf{d}_{first} \\
 & & & up \leftarrow first
\end{array}$$

   Are the semantic actions still correct? If so, explain why they are still valid; if not, provide a set of semantic actions that does the job properly.

2. The grammar in Figure 7.8 is almost faithful to the language originally defined in Figure 7.7. To appreciate the difference, compare how each grammar treats the input string `x 5 $`.

   (a) In what respects do the grammars generate different languages?

   (b) Modify the grammar in Figure 7.8 to maintain its style of semantic processing but to respect the language definition in Figure 7.7.

3. The language generated by the grammar in Figure 7.8 uses the terminal x to introduce the base. A more common convention is to separate the base from the string of digits by some terminal symbol. Instead of `x 8 4 3 1` to represent 431_8, a language following the common convention would use `8 x 4 3 1`.

   (a) Design an LALR(1) grammar for such a language and specify the semantic actions that compute the string's numeric value. In your solution, allow the absence of a base specification to default to base 10, as in Figure 7.8.

   (b) Discuss the tradeoffs of this language and your grammar as compared with the language and grammar of Figure 7.8.

4. Consider the addition of the the rule

$$\text{Expr} \rightarrow \text{sum Values}$$

to the grammar in Figure 7.9.

   (a) Does this change make the grammar ambiguous?

   (b) Is the grammar still LL(1) parseable?

   (c) Show how the semantic actions in Figure 7.10 must be changed to accommodate this new language construct; modify the grammar if necessary but avoid use of global variables.

5. Consider the addition of the rule

$$\text{Expr} \rightarrow \text{mean Values}$$

to the grammar in Figure 7.9. This rule defines an expression that computes the average of its values, defined by:

$$(\texttt{mean}\ v_1\ v_2\ \ldots\ v_n) = \frac{v_1 + v_2 + \ldots v_n}{n}$$

   (a) Does this new rule render the grammar ambiguous?

   (b) Is the grammar still LL(1) parseable?

   (c) Describe your approach for syntax-directed translation of this new construct.

   (d) Modify the grammar of Figure 7.10 accordingly and insert the appropriate semantic actions into Figure 7.10.

6. Although arithmetic expressions are typically evaluated from left to right, the semantic actions in VALUES cause a product computed from right to left. Modify the grammar and semantic actions of Figures 7.9 and 7.10 so that products are computed from left to right.

7. Verify that the grammar in Figure 7.14 is unambiguous using an LALR(1) parser generator.

8. Suppose that the terminals assign, deref, and addr correspond to the input symbols =, ⋆, and &, respectively. Using the grammar in Figure 7.20

   - show parse trees for the following strings;
   - show where indirections actually occur by circling the parse tree nodes that correspond to the rule R→L.

   (a) `x = y`

   (b) `x = ⋆ y`

   (c) `⋆ x = y`

   (d) `⋆ x = ⋆ y`

   (e) `⋆ ⋆ x = & y`

   (f) `⋆ 16 = 256`

9. Construct an LL(1) grammar for the language in Figure 7.14.

10. Consider extending the grammar in Figure 7.14 to include binary subtraction and unary negation, so that the expression

    ```
    minus y minus x times 3
    ```

    has the effect of negating `y` prior to subtraction of the product of `x` and `3`.

    (a) Following the steps outlined in Section 7.4, modify the grammar to include these new operators. Your grammar should grant negation strongest precedence, at a level equivalent to deref. Binary subtraction can appear at the same level as binary addition.

    (b) Modify the chapter's AST design to include subtraction and negation by including a minus operator node.

    (c) Install the appropriate semantic actions into the parser.

11. Modify the grammar and semantic actions of Figure 7.3 so that the rule for Digs is right recursive. Your semantic actions should create the identical AST.

12. The grammar below generates nested lists of numbers. The semantic actions are intended to count the number of elements just inside each parenthesized list. For each list found by Rule 2, Marker ㊹ prints out the number of elements found just inside the list.

    For example, the input

    ```
    ( ( 1 2 3 ) ( 1 2 3 4 5 6 ) )
    ```

    should print 3, 6, and 2.

    1 Start → List$_{avg}$ \$

    2 List$_{result}$ → lparen Operands$_{ops}$ rparen
    PRINT(*count*) ㊹

    3 | num$_{val}$

    4 Operands → Operands List
    $count \leftarrow count + 1$

    5 | List
    $count \leftarrow 1$

    (a) The grammar uses a global variable *count* to determine the number of elements in a list. What is wrong with that approach?

    (b) Change the semantic actions so that the appropriate values are synthesized by the rules to allow counting without a global variable.

13. Using a standard LALR(1) grammar for C or Java, find syntactic punctuation that can be eliminated without introducing LALR(1) conflicts. Explain why the (apparently unnecessary) punctuation is included in the language. As a hint, consider the parentheses that surround the predicate of an `if` statement.

14. The semantic actions in Figure 7.5 contains a test for non-octal digits at Markers ① and ②. Rewrite the grammar so that such testing is performed on behalf of exactly one production. *Hint*: Consider the use of a unit production as discussed in Section 7.2.3.

15. The semantic actions in Figure 7.7 contains a test for non-octal digits at Markers ③ and ④. Rewrite the grammar so that such testing is performed on behalf of exactly one production. *Hint*: Consider the use of a unit production as discussed in Section 7.2.3.

```
class IfNode extends AbstractNode
    procedure typeCheck( )
        /★  Type-checking code for an if                    ★/
    end
    procedure codeGen( )
        /★  Generate code for an if                         ★/
    end
    . . .
end

class PlusNode extends AbstractNode
    procedure typeCheck( )
        /★  Type-checking code for a plus                   ★/
    end
    procedure codeGen( )
        /★  Generate code for a plus                        ★/
    end
    . . .
end
. . .
```

Figure 7.25: Inferior design: phase code distributed among node types.

16. Figure 7.8(a) does not check that digits are within range of the specified base. Insert semantic actions to perform such checks, rewriting the grammar if necessary to support such checking as cleanly and concisely as possible.

17. The semantic values shown in Figure 7.2(b) serve to count the position of each `x` in a string.

    (a) What grammar is implied by the parse tree of Figure 7.2(a)?

    (b) Why is that grammar unsuitable for top-down parsing?

    (c) Transform the grammar so that it is suitable for top-down parsing.

    (d) Write a recursive-descent parser based on your grammar.

    (e) Add semantic actions into the parser that compute the semantic values as shown in Figure 7.2(b) using only *inherited* attribute flow.

    (f) Add more semantic actions to your parser that return (synthesize) the total number of `x` symbols found in the string.

```
foreach AbstractNode $n \in AST$ do
    switch ($n$.getType( ))
        case IfNode
            call $f$.visit( $\langle$IfNode $\Downarrow n\rangle$ )
        case PlusNode
            call $f$.visit( $\langle$PlusNode $\Downarrow n\rangle$ )
        case MinusNode
            call $f$.visit( $\langle$MinusNode $\Downarrow n\rangle$ )
```

Figure 7.26: An alternative for achieving double dispatch.

18. Compute the location of each x and the total number of x symbols, as in Exercise 17, using bottom-up parser with semantic actions. You may restructure the grammar as you see fit, but use only synthesized attribute flow.

19. Based on the discussion of Section 7.4 and using the pseudocode in Figure 7.13 as a guide, design a set of AST classes and methods to support AST construction in a real programing language.

20. In contrast to the approach discussed in Section 7.7.2, Figure 7.25 shows the partial results of a design in which each phase contributes code to each node type.

    (a) What are the advantages and disadvantages of the approach used in Figure 7.25?

    (b) How does the visitor pattern discussed in Section 7.7.2 address the disadvantages?

21. In contrast with the approaches discussed in Sections 7.7.2 and 7.7.3, consider the idea sketched in Figure 7.26.

    (a) What are the advantages and disadvantages of the approach used in Figure 7.26?

    (b) How does the visitor pattern discussed in Sections 7.7.2 and 7.7.3 address the disadvantages?

22. In addition to the *NeedsBooleanPredicate* type discussed in Section 7.7.3, Figure 7.24 references the following types: *NeedCompatibleTypes* and *NeedsLeftChildType*.

    (a) Which node types inherit from those types?

    (b) Describe the actions that should be performed by the visitor at Markers ㊷ and ㊸ on behalf of the *NeedCompatibleTypes* and *NeedsLeftChildType* types.

8

Symbol Tables and Declaration Processing

Chapter 7 considered the construction of an **abstract syntax tree** (AST) as an artifact of a top-down or bottom-up parse. On its own, a top-down or bottom-up parser cannot fully accomplish the compilation of modern programming languages. The AST serves to represent the source program and to coordinate information contributed by the various passes of a compiler. This chapter begins with a presentation of one such pass—the harvesting of symbols from an AST. Most programming languages allow the declaration, definition, and use of symbolic names to represent constants, variables, methods, types, and objects. The compiler checks that such names are used correctly, based on the programming language's definition.

The first half of this chapter describes the organization and implementation of a symbol table. This structure records the names and important attributes of a program's names. Examples of such attributes include a name's type, scope, and accessibility. We are interested in two aspects of a symbol table: its use and its organization. Section 8.1 defines a simple symbol table interface and shows how to use this interface to manage symbols for a block-structured language. Section 8.2 explains the effects of program scopes on symbol table management. Section 8.3 examines various implementations of a symbol table. Advanced topics, such as type definitions, inheritance, and overloading are considered in Section 8.4.

The second half of the chapter examines techniques for processing declarations and then goes on to show how the information derived from declarations is used to do type checking on assignments and expressions. Section 8.5 describes the representations used for the attributes that are associated with names by the symbol table. Section 8.6 details how AST structures representing simple declarations are processed to build a representation of the declarations in the symbol table. Techniques necessary for a handling class and method declarations are presented in Section 8.7. Finally, Section 8.8 illustrates how type checking is done, using the information stored in the symbol table as declarations are processed.

8.1 Constructing a Symbol Table

In this section, we consider how to construct a **symbol table** for a simple, block-structured language. Assuming an AST has been constructed as described in Chapter 7, we *walk* (make a *pass* over) the AST for two purposes:

- to process symbol declarations and
- to connect each symbol reference with its declaration.

Symbol references are connected with declarations through the symbol table. An AST node that mentions a symbol by name is enriched with a reference to the name's entry in the symbol table. If such a connection cannot be made, then the offending reference is improperly declared and an error message is issued. Otherwise, subsequent passes can use the symbol table reference to obtain information about the symbol, such as its type, storage requirements, and accessibility.

The block-structured program shown in Figure 8.1(a) contains two nested scopes. Although the program uses keywords such as `float` and `int`, no symbol table action is required for these symbols if they are recognized by the scanner as terminal symbols. Most programming language grammars demand such precision of the scanner to avoid ambiguity.

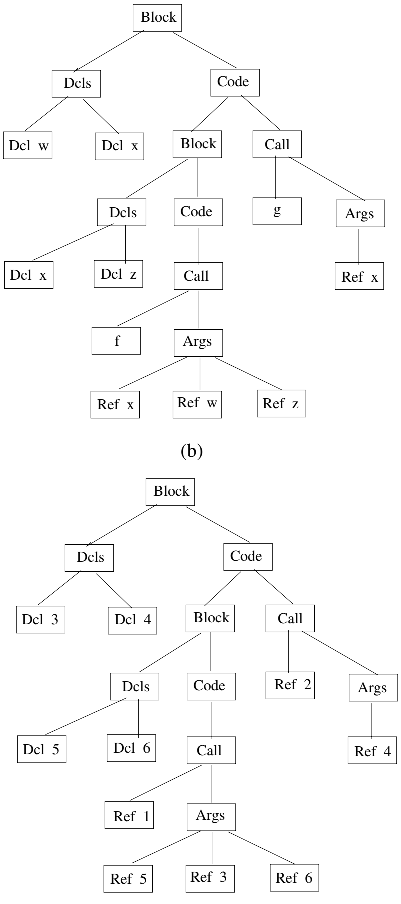
The program in Figure 8.1(a) begins by importing function definitions for `f` and `g`. The compiler finds these, determines that their return types are `void`, and then records the functions in the symbol table as its first two entries. The declarations for `w` and `x` in the outer scope are entered as symbols 3 and 4, respectively, in the symbol table illustrated in Figure 8.1(c). The inner scope's redeclaration of `x` and its declaration of `z` are entered as symbols 5 and 6, respectively. The AST in Figure 8.1(b) refers to symbols by name. In Figure 8.1(d), the names are replaced by symbol table references. In particular, the references to `x` shown only by name in Figure 8.1(b) are declaration-specific in Figure 8.1(d). In the symbol table, the references contain the original name as well as type information processed from the symbol declarations.

```
import f(float, float, float)
import g(int)
{
  int w,x
  {
    float x,z
    f(x,w,z)
  }
  g(x)
}
```

(a)

Symbol Number	Symbol Name	Attributes
1	`f`	`void func(float,float,float)`
2	`g`	`void func(int)`
3	`w`	`int`
4	`x`	`int`
5	`x`	`float`
6	`z`	`float`

(c)



(b)

(d)

Figure 8.1: Symbol table processing for a block-structured program

8.1.1 Static Scoping

Modern programming languages offer scopes to confine the activity of a name to a prescribed region of a program. A name may be declared no more than once in any given scope. For statically scoped, block-structured languages, references are typically resolved to the declaration in their closest containing scope. Additionally, most languages contain directives to promote a given declaration or reference to the program's **global scope**—a name space shared by all compilation units. Section 8.4.4 discusses these issues in greater detail.

Proper use of scopes results in programs whose behavior is easier to understand. Figure 8.1(a) shows a program with two nested scopes. Declared in the program's outer scope, `w` is available in both scopes. The activity of `z` is confined to the inner scope. The name `x` is available in both scopes, but the meaning of `x` changes when the inner scope redeclares `x`. In general, the **static scope** of an identifier includes its defining block as well as any contained blocks that do not themselves contain a declaration for the identifier.

Languages are typically designed to use punctuation or keywords to define static scopes. For example, in C and Java$^{\text{TM}}$, scopes are opened and closed by the appropriate braces, as shown in Figure 8.1(a). In these languages, a scope can declare types and variables. However, methods (function definitions) cannot appear in inner scopes. In, Ada and other languages with Algol-like syntax, the reserved keywords `begin` and `end` open and close scopes, respectively. Within each scope, types, variables, and procedures can be declared.

In some languages, references to names in outer scopes can incur overhead at runtime. As discussed in Chapter 11, an important consideration is whether a language allows the nested declaration of methods. C and Java prohibit this, while ML allows functions to be defined in any scope. For C and Java, a method's local variables can be *flattened* by renaming nested symbols and moving their declaration to the method's outermost scope. Exercise 14 considers this in greater detail.

8.1.2 A Symbol Table Interface

A symbol table is responsible for tracking which declaration is in effect when a reference to the symbol is encountered. In this section, we define a symbol table interface for processing symbols in a block-structured, statically scoped language. The methods in our interface are as follows:

openScope() opens a new scope in the symbol table. New symbols are entered in the resulting scope.

closeScope() closes the most recently opened scope in the symbol table. Symbol references subsequently revert to outer scopes.

```
procedure buildSymbolTable( )
    call processNode( ASTroot )
end

procedure processNode( node )
    switch (kind( node ))
        case Block
            call symtab.openScope( )                                   ①
        case Dcl
            call symtab.enterSymbol( node.name, node.type )
        case Ref
            sym ← symtab.retrieveSymbol( node.name )
            if sym = null
            then  call error( "Undeclared symbol : ", sym )
    foreach c ∈ node.getChildren( ) do  call processNode( c )
    if kind( node ) = Block
    then
        call symtab.closeScope( )                                      ②
end
```

Figure 8.2: Building the symbol table

enterSymbol(*name*, *type*) enters *name* in the symbol table's current scope. The parameter *type* conveys the data type and access attributes of *name*'s declaration.

retrieveSymbol(*name*) returns the symbol table's currently valid declaration for *name*. If no declaration for name is currently in effect, then a null pointer is returned.

declaredLocally(*name*) tests whether *name* is present in the symbol table's current (innermost) scope. If it is, true is returned. If *name* is in an outer scope, or is not in the symbol table at all, false is returned.

To illustrate the use of this interface, Figure 8.2 contains code to build the symbol table for the AST shown in Figure 8.1. The code is specialized to the type of the AST node. Actions may be performed both before and after a given node's children are visited. Prior to visiting the children of a *Block* node, code at Marker ① increases opens a new scope. After the subtree of the *Block* is processed, Marker ② abandons the scope. The code for *Ref* retrieves the symbol's current definition in the symbol table. If none exists, then an error message is issued.

8.2 Block-Structured Languages and Scopes

Most programming languages allow scopes to be nested statically, based on concepts introduced by Algol 60. Languages that allow nested name scopes are known as **block-structured languages**. While the mechanisms that open and close scopes can vary by language, we assume that the openScope and closeScope methods are the uniform mechanism for opening and closing scopes in the symbol table. In this section, we consider various language constructs that call for the opening and closing of scopes. We also consider the issue of allocating a symbol table for each scope as compared with using a single, global symbol table.

8.2.1 Handling Scopes

Every symbol reference in an AST occurs in the context of defined scopes. The scope defined by the *innermost* such context is known as the **current scope**. The scopes defined by the current scope and its surrounding scopes are known as the **open scopes** or **currently active scopes**. All other scopes are said to be **closed**. Based on these definitions, current, open, and closed scopes are not fixed attributes; instead, they are defined relative to a particular point in the program. The following are some common visibility rules that define the interpretation of a name in the presence of multiple scopes:

- At any point in the text of a program, the accessible names are those that are declared in the current scope and in all other open scopes.

- If a name is declared in more than one open scope, then a reference to the name is resolved to the *innermost* declaration—the one that most closely surrounds the reference.

- New declarations can be made only in the current scope.

Most languages offer mechanisms to install or resolve symbol names in the outermost, program-global scope. In C, names bearing the `extern` attribute are resolved globally. In Java, a class can reference any class's `public static` fields, but these fields do not populate a single, flat name space. Instead, each such field must be fully qualified by its containing class.

Programming languages have evolved to allow various useful levels of scoping. C and C++ offer a compilation-unit scope where names declared outside of all methods are available within the compilation unit's methods. Java offers a package-level scope in which classes can be organized into packages that can access all package-scoped methods and fields. In C, every function definition is available in the global scope, unless the definition has the `static` attribute. In C++ and Java, names declared within a class are available to all methods in the class. In Java and C++, a class's fields and methods bearing the `protected` attribute are available to the class's subclasses. The parameters and local variables of a method are available within the given method. Finally, names declared within a statement-block are available in all contained blocks, unless the name is redeclared in an inner scope.

8.2.2 One Symbol Table or Many?

As noted above, there are two common approaches to implementing block-structured symbol tables. A symbol table may be associated with each scope or all symbols may be entered in a single, global table. A single symbol table must accommodate multiple, active declarations of the same symbol. Despite this complication, searching for a symbol can be faster in a single symbol table. We next consider this issue in greater detail.

An Individual Table for Each Scope

If an individual table is created for each scope, some mechanism must be in place to ensure that a search produces the name defined by the nested-scope rules. Because name scopes are opened and closed in a *last-in, first-out* (*LIFO*) manner, a stack is an appropriate data structure for organizing such a search. Thus, a scope stack of symbol tables can be maintained, with one entry in the stack for each open scope. The innermost scope appears at the top of the stack. The next containing scope is second from the top, and so forth. When a new scope is opened, openScope creates and pushes a new symbol table on the stack. When a scope is closed, closeScope, the top symbol table is popped.

A disadvantage of this approach is that we may need to search for a name in a number of symbol tables before the symbol is found. The cost of this stack search varies from program to program, depending on the number of nonlocal name references and the depth of nesting of open scopes. In fact, it is widely known that most lookups of symbols in block-structured languages return symbols in the inner- or outer-most scopes. With a table per scope, intermediate scopes must be checked before an outermost declaration can be returned.

An example of this symbol table organization is shown in Figure 8.3.

One Symbol Table

In this organization, all names in a compilation unit's scopes are entered into the same table. If a name is declared in different scopes, then the scope name or depth helps identify the name uniquely in the table. With a single symbol table, retrieveSymbol need not chain through scope tables to locate a name.

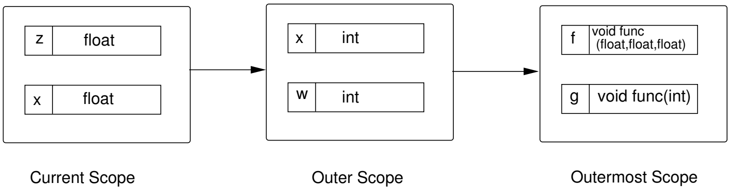


Figure 8.3: A stack of symbol tables, one per scope

Section 8.3.3 describes this kind of symbol table in greater detail. Such a symbol table is shown in Figure 8.8 on page 293.

8.3 Basic Implementation Techniques

Any implementation of the interface presented in Section 8.1 must correctly insert and find symbols. Depending on the number of names that must be accommodated and other performance considerations, a variety of implementations is possible. Section 8.3.1 examines some common approaches for organizing symbols in a symbol table. Section 8.3.2 considers how to represent the symbol names themselves. Based on this discussion, Section 8.3.3 proposes an efficient symbol table implementation.

8.3.1 Entering and Finding Names

We begin by considering various approaches for organizing the symbols in the symbol table. For each approach, we examine the time needed to insert symbols, retrieve symbols, and maintain scopes. These actions are not typically performed with equal frequency. A name can be declared no more than once in each scope, but names are typically referenced multiple times. It is therefore reasonable to expect that retrieveSymbol is called more frequently than the other methods in our symbol table interface. Thus, we pay particular attention to the cost of retrieving symbols.

Unordered List

This is the simplest possible storage mechanism. The only data structure required is an array, with insertions occurring at the next available location. For added flexibility, a linked list or resizable array avoids the limitations imposed by a fixed-size array. In this representation, enterSymbol inserts a name at the head of the unordered list. The scope name (or depth) is recorded with the

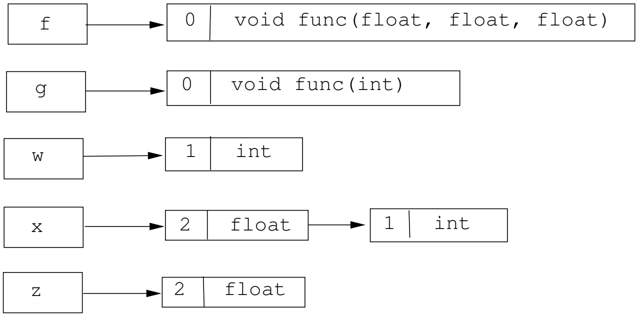


Figure 8.4: An ordered list of symbol stacks

name. This allows enterSymbol to detect if the same name is entered twice in the same scope, a situation disallowed by most programming languages. retrieveSymbol searches for a name from the head of the list toward its tail so that the closest, active declaration of the name is encountered first. All names for a given scope appear adjacently in the unordered list. Thus, openScope can annotate the list with a marker to show where the new scope begins. closeScope can then delete the currently active symbols at the head of the list. Although insertion is fast, retrieval of a name from the outermost scope can require scanning the entire unordered list. This approach is therefore impractically slow except for the smallest of symbol tables.

Ordered List

If a list of n distinct names is maintained alphabetically, binary search can find any name in $O(log\ n)$ time. In the unordered list, declarations from the same scope appear in sequence, an unlikely situation for the ordered list. How should we organize the ordered list to accommodate declarations of a name in multiple scopes? Exercise 5 investigates the potential performance of storing all names in a single, ordered list. Because retrieveSymbol accesses the currently active declaration for a name, a better data structure is an ordered list of stacks. Each stack represents one currently active name; the stacks are ordered by their representative names. retrieveSymbol locates the appropriate stack using binary search. The currently active declaration appears on top of the located stack. closeScope must pop those stacks containing declarations for the abandoned scope. To facilitate this, each symbol can be recorded along with its scope name or depth, as established by openScope. closeScope can then examine each stack in the list and pop those stacks whose top symbol is declared in the abandoned scope. When a stack becomes empty, it can be removed from the ordered list. Figure 8.4 shows such a symbol table for the example in Figure 8.1, at the point where method `f` is invoked.

A more efficient approach avoids touching each stack when a scope is abandoned. The idea is to maintain a separate linking of symbol table entries that are declared at the same scope level. Section 8.3.3 presents this organization in greater detail. The details of maintaining a symbol table using ordered lists are explored in Exercise 6. Although ordered lists offer fast retrieval, insertion into an ordered list is relatively expensive. Thus, ordered lists are advantageous when the space of symbols is known in advance, as in the case of reserved keywords.

Binary Search Trees

Binary search trees are designed to combine the efficiency of a linked data structure for insertion with the efficiency of binary search for retrieval. Given random inputs, it is expected that a name can be inserted or found in $O(log\ n)$ time, where n is the number of names in the tree. Unfortunately, average-case performance does not necessarily hold for symbol tables—programmers do not choose identifier names at random! Thus, a tree of n names could have depth n, causing name lookup to take $O(n)$ time. An advantage of binary search trees is their simple, widely known implementation. This simplicity and the common perception of reasonable average-case performance make binary search trees a popular technique for implementing symbol tables. As with the list structures, each name (node) in the binary search tree is actually a stack of currently active scopes that declare the name.

Balanced Trees

The worst-case scenario for a binary search tree can be avoided if a search tree can be maintained in *balanced* form. The time spent balancing the tree can be amortized over all operations so that a symbol can be inserted or found in $O(log\ n)$ time, where n is the number of names in the tree. Examples of such trees include red-black trees and splay trees. Exercises 9 and 10 further explore symbol table implementations based on balanced-tree structures.

Hash Tables

Hash tables are the most common mechanism for managing symbol tables, owing to their excellent performance. Given a sufficiently large table, a good hash function, and appropriate collision-handling techniques, insertion or retrieval can performed in constant time, regardless of the number of entries in the table. The implementation discussed in Section 8.3.3 uses a hash table, with collisions handled by chaining. Hash tables are widely implemented. Some languages (including Java) contain hash table implementations in their core

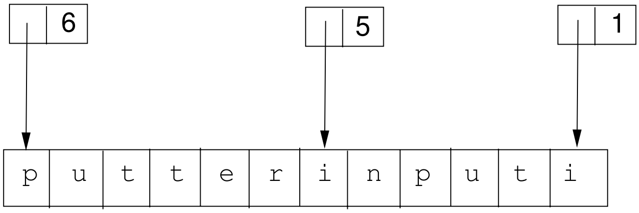


Figure 8.5: Name space for symbols *putter*, *input*, and *i*

library. The implementation details for hash tables are covered in most books on elementary data structures and are thoroughly discussed in [CLRS01].

8.3.2 The Name Space

At some point, a symbol table entry must represent the name of its symbol. Each name is essentially a string of characters. However, by taking the following properties into consideration, an efficient implementation of the name space can be obtained:

- The name of a symbol does not change during compilation. Thus, the strings associated with symbol table names are immutable—once allocated, they do not change.

- Although scopes come and go, the symbol names persist throughout compilation. Scope creation and deletion affects the set of currently available symbols, obtainable through retrieveSymbol. However, a scope's symbols are not completely forgotten when the scope is abandoned. Space must be reserved for the symbols at runtime, and the symbols may require initialization. Thus, the symbols' strings occupy storage that persists throughout compilation.

- There can be great variance in the *length* of identifier names. Short names—perhaps only a single character—are typically used for iteration variables and temporaries. Global names in a large software system tend to be descriptive and much longer in length. For example, the `X` windowing system contains names such as `VisibilityPartiallyObscured`.

- Unless an ordered list is maintained, comparisons of symbol names involve only equality and inequality.

The above points argue in favor of one logical name space, as shown in Figure 8.5, in which names are inserted but never deleted.

Name	Type	Var	Level	Hash	Depth

Figure 8.6: A symbol table entry

In Figure 8.5, each string is referenced by a pair of fields. One field specifies the string's origin in the string buffer, and the other field specifies the string's length. If the names are managed so that the buffer contains at most one occurrence of any name, then the equality of two strings can be tested by comparing the strings' references. If they differ in origin or length, then the strings cannot be the same. The Java class `String` contains the method `intern` that maps any string to a unique reference for the string. The strings in Figure 8.5 do not share any common characters. Exercise 11 considers **string spaces**, which store shared substrings more compactly. In some languages, the suffix of a name can suffice to locate the name. For example, a reference of `String` in a Java program defaults to `java.lang.String`. Exercise 12 considers the organization of name spaces to accommodate such access.

8.3.3 An Efficient Symbol Table Implementation

We have examined issues of symbol management and representation. Based on the discussion up to this point, we next present an efficient symbol table implementation. Figure 8.6 shows the layout of a symbol table entry containing the following fields:

Name is a reference to the symbol name space, organized as described in Section 8.3.2. The name is required to locate the symbol in a chain of symbols with the same hash table location.

Type is a reference to the type information associated with the symbol's declaration. Such information is processed as described in Section 8.6.

Hash threads symbols whose names hash to the same value. In practice, such symbols are doubly linked to facilitate symbol deletion.

Var is a reference to the next outer declaration of this same name. When the scope containing this declaration is abandoned, the referenced declaration becomes the currently active declaration for the name. Thus, this field essentially represents a stack of scope declarations for its symbol name.

Level threads symbols declared in the same scope. This field facilitates symbol deletion when a scope is abandoned.

Depth records the nesting depth of a symbol. It is useful in checking if a symbol at a given nesting level is already entered.

There are two *index* structures for the symbol table: a hash table and a **scope display**. The hash table allows efficient lookup and entry of names, as described in Section 8.3.1. The scope display maintains a list of symbols that are declared at the same level. In particular, the *i*th entry of the scope display references a list of symbols currently active at scope depth *i*. Such symbols are linked by their *level* field. Moreover, each active symbol's *var* field is essentially a stack of declarations for the associated variable.

Figure 8.7 shows the pseudocode for this symbol table implementation. Figure 8.8 shows the table that results from applying this implementation to the example in Figure 8.1, at the point where method `f` is invoked. Figure 8.8 assumes the following unlikely situation with respect to the hash function:

- `f` and `g` hash to the same location.
- `w` and `z` hash to the same location.
- All symbols are clustered in the same part of the table.

The code in Figure 8.7 relies on the following methods:

DELETE(*sym*) removes the symbol table entry *sym* from the collision chain found at *HashTable*.GET(*sym*.*name*). The symbol is not destroyed—it is simply removed from the collision chain. In particular, its *var* and *level* fields remain intact.

ADD(*sym*) adds the symbol *sym* to the collision chain at *HashTable*.GET(*sym*.*name*). Prior to the call to ADD, there is no entry in the table for *sym*.

When CLOSESCOPE is invoked to abandon the currently active scope, each symbol in the scope is visited by the loop at Marker ③. Each such symbol is removed from the hash table at Marker ④. If an outer scope definition exists for the symbol, then the definition is inserted into the hash table at Marker ⑤. Thus, the *var* field serves to maintain a stack of active scope declarations for each symbol. The *level* field allows CLOSESCOPE to operate in time proportional to the number of symbols affected by abandoning the current scope. Amortized over all symbols, this adds a constant overhead to the management of each declared symbol.

RETRIEVESYMBOL examines a collision chain to find the desired symbol. The loop at Marker ⑥ accesses all symbols that hash to the same table location, that is the chain that should contain the desired name. The code at Marker ⑦ follows the entries' *hash* fields until the chain is exhausted or the symbol is located. A properly managed hash table should have very short collision

```
procedure openScope( )
    $depth \gets depth + 1$
    $scopeDisplay[depth] \gets$ null
end

procedure closeScope( )
    foreach $sym \in scopeDisplay[depth]$ do                              ③
        $prevsym \gets sym.var$
        call delete($sym$)                                                ④
        if $prevsym \neq$ null                                            ⑤
        then call add($prevsym$)
    $depth \gets depth - 1$
end

function retrieveSymbol($name$) returns $Symbol$
    $sym \gets HashTable$.get($name$)
    while $sym \neq$ null do                                              ⑥
        if $sym.name = name$
        then return ($sym$)
        $sym \gets sym.hash$                                              ⑦
    return (null)                                                         ⑧
end

procedure enterSymbol($name, type$)
    $oldsym \gets$ retrieveSymbol($name$)
    if $oldsym \neq$ null and $oldsym.depth = depth$                      ⑨
    then call error($"Duplicate\ definition\ of", name$)
    $newsym \gets$ createNewSymbol($name, type$)                          ⑩
    /⋆ Add to scope display ⋆/
    $newsym.level \gets scopeDisplay[depth]$
    $newsym.depth \gets depth$
    $scopeDisplay[depth] \gets newsym$
    /⋆ Add to hash table ⋆/
    if $oldsym =$ null
    then call add($newsym$)
    else
        call delete($oldsym$)
        call add($newsym$)
    $newsym.var \gets oldsym$
end

function declaredLocally($name$) returns $Boolean$
    /⋆ See Exercise 7. ⋆/
end
```

Figure 8.7: Symbol table management

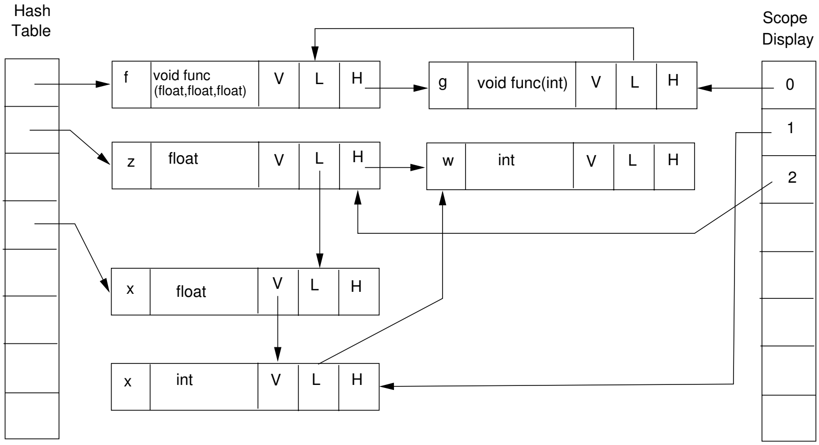


Figure 8.8: Detailed layout of the symbol table for Figure 8.1. The V, L, and H fields abbreviate the Var, Level, and Hash fields, respectively

chains. Thus, we expect that only a few iterations of the loop at Marker 6 should be necessary to locate a symbol or to detect that the symbol has not been properly declared.

ENTERSYMBOL first locates the currently active definition for *name*, should any exist in the table. Marker 9 checks that no declaration already exists in the current scope. A new symbol table entry is generated at Marker 10. The symbol is added to those in the current scope by linking it into the scope display. The remaining code inserts the new symbol into the table. If an active scope contains a definition of the symbol name, then that name is removed from the table and referenced by the *var* field of the new symbol.

Recalling the discussion in Section 8.2.2, an alternative approach segregates symbols by scope. A stack of symbol tables results (one symbol table per scope), as shown in Figure 8.3. The code to manage such a structure is left as an exercise. (See Exercise 4.)

8.4 Advanced Features

We next examine how to extend the simple symbol table framework to accommodate advanced features of modern programming languages. Extensions to our simple framework fall generally in the following categories:

- Name augmentation (overloading)

- Name hiding and promotion
- Modification of search rules

In each case, it is appropriate to rethink the symbol table design to arrive at an efficient, correct implementation of the desired features. In the following sections, we discuss the essential issues associated with each feature. However, we leave the details of the implementation as exercises.

8.4.1 Records and Typenames

Most languages allow aggregate data structures to be defined using the `struct` and `record` type constructors. Because such structures can be nested, access to a field may involve navigating through many containers before the field can be reached. In C, Ada, and Pascal, such fields are accessed by completely specifying the containers and the field. Thus, the reference `a.b.c.d` accesses field `b` of structure `a`, field `c` of structure designated by `a.b`, and finally field `d` of structure `a.b.c`. COBOL and PL/I allow intermediate containers to be omitted if the reference can be unambiguously resolved. In such languages, `a.b.c.d` might be abbreviated as `a.c` or `c.d`. This idea has not met with general acceptance, partly because programs that use such abbreviations are difficult to read. It is also possible that `a.d` is a mistake, but the compiler silently accepts the reference by filling in missing containers.

Structures can be nested arbitrarily deeply. Thus, structures are typically implemented using a tree. Each structure is represented as a node; its children represent the structure's subfield. Alternatively, a structure can be represented by a symbol table whose entries are the record's subfields. Exercise 15 considers the implementation of structures in symbol tables.

C offers the `typedef` construct, which establishes a name as an *alias* for a type. As with record types, it is convenient to establish an entry in the symbol table for the typedef. In fact, most C compilers use scanners that must distinguish between ordinary names and typenames. This is typically accomplished through a back door call to the symbol table to lookup each identifier. If the symbol table shows that the name is an active typename, then the scanner returns a typename token. Otherwise, an ordinary identifier token is returned.

8.4.2 Overloading and Type Hierarchies

The notion of an identifier has thus far been restricted to a string containing the identifier's name. Situations can arise where the name alone is insufficient to locate a desired symbol. Object-oriented languages such as C++ and Java allow method overloading. A method can be defined multiple times, provided that each definition has a unique **type signature**. The type signature of a method includes the number and types of its parameters and its return type. With overloading, a program can contain the method `print(int)` as well as `print(String)`.

When the type signatures are included, the compiler comes to view a method definition not only in terms of its name but also in terms of its type signature. The symbol table must be capable of entering and retrieving the appropriate symbol for a method. In one approach to method overloading, the type signature of a method is encoded along with its name. For example, the method M that accepts an integer and returns void is encoded as $M(int) : void$. It then becomes necessary to include a method's type signature each time a method name is retrieved from the symbol table. Alternatively, the symbol table could simply record a method along with a list of its overloaded definitions. In the AST, method invocations point to the entire list of method definitions. Subsequently, semantic processing scans the list to make certain that a valid definition of the method occurs for each invocation. (See Section 9.2 on page 376.)

Some languages, such as C++ and Ada, allow operator symbols to be overloaded. For example, the meaning of + would change if its arguments are strings instead of numbers. The symbol table for such languages must be capable of determining the definition of an operator symbol in every scope.

Ada allows literals to be overloaded. For example, the symbol `diamond` could participate simultaneously in two different enumeration types: as a playing card suit and as a gem.

Pascal and Fortran employ a small degree of overloading in that the same symbol can represent the invocation of a method as well as the value of the method's result. For such languages, the symbol table contains two entries. One represents the method while the other represents a name for the value returned by the method. It is clear in context whether the name means the value returned by the method or the method itself. As demonstrated in Figure 8.1, semantic processing makes an explicit connection between a name and its symbol.

C also has overloading to a certain degree. A program can use the same name as a local variable, a `struct` name, and a label. Although it is unwise to write such confusing programs, C allows this because it is clear in each context which definition of a name is intended. (See Exercise 16.)

Languages such as Java and C++ offer type extension through subclassing. The symbol table could contain a method `resize(Shape)`, while the program invokes the method `resize(Rectangle)`. If `Rectangle` is a subclass of `Shape`, then the invocation should resolve to the method `resize(Shape)`. However, if the program contains a `resize` method for both `Rectangle` and `Shape` types, then resolution should choose the method whose formal parameters most closely match the types of the supplied parameters. (See Section 9.2 for more details.)

8.4.3 Implicit Declarations

In some languages, the appearance of a name in a certain context serves to declare the name as well. As a common example, consider the use of labels in C. A label is introduced as an identifier followed by a colon. Unlike Pascal, the program need not declare the use of such labels in advance. In Fortran, the type of an identifier for which no declaration is offered can be inferred from the identifier's first letter. In Ada, a index is implicitly declared to be of the same type as the range specifier. Moreover, a new scope is opened for the loop so that the loop index cannot clash with an existing variable. (See Exercise 17.)

Implicit declarations are almost always introduced in a programming language for the convenience of those who use the language rather than those who implement it. Taking this point further, implicit declarations may ease the task of writing programs at the expense of those who must later read them. In any event, the compiler is responsible for supporting such features.

8.4.4 Export and Import Directives

Export rules allow a programmer to specify that some local scope names are to become visible outside that scope. This selective visibility is in contrast to the usual block-structured scope rule, which causes local scope names to be *invisible* outside the scope. Export rules are typically associated with modularization features such as Ada packages, C++ classes, C compilation units, and Java classes. These language features help a programmer organize a program's files by their functionality.

In Java, the `public` attribute causes the associated field or method to be known outside its class. To prevent name clashes, each class can situate itself in a package hierarchy through the `package` directive. Thus, the `String` class provided in the Java core library is actually part of the `java.lang` package. In contrast, all methods in C are known outside of their compilation units, *unless* the `static` attribute is bestowed. The `static` methods are available only within their compilation units.

With an export rule, each compilation unit advertises its offerings. In a large software system, the space of available global names can become polluted and disorganized. To manage this, compilation units are typically required to specify which names they wish to import. In C and C++, the use of a header file includes declarations of methods and structures that can be used by the compilation unit. In Java, the `import` directive specifies the classes and packages that a compilation unit might access. Ada's `use` directive serves essentially the same purpose.

To process the export and import directives, the compiler typically examines the import directives to obtain a list of potentially external references. These references are then examined by the compiler to determine their validity, to the extent that is possible at compile time. In Java, the `import` directives serve to initialize the symbol table so that references to abbreviated classes (`String` for `java.lang.String`) can be resolved.

8.4.5 Altered Search Rules

Pascal's `with` statement is a good example of a feature that alters the way in which symbols are found in the symbol table. If a Pascal program contains the phrase `with R do S`, then within the statements in `S`, the compiler must first try to resolve an identifier reference as a field of the record `R`. If no valid reference is found in record `R`, then the symbol table is searched as usual. This feature allows the programmer to avoid frequently restating `R` inside `S`, which is advantageous if `R` is actually a complex name. Moreover, in such a case, the compiler can usually generate faster code, since it is likely that there are multiple references to fields of the record `R` within `S`.

Forward references also affect a symbol table's search rules. Consider a set of recursive data structures or methods. In the program, the set of definitions must be presented in some linear order. It is inevitable that a portion of the program will reference a definition that has not yet been processed. Forward references suspend the compiler's skepticism concerning undeclared symbols. A forward reference is essentially a promise that a complete definition will eventually be provided.

Some languages require that forward references be announced. In C, it is considered good style to declare an as-yet-undefined function so that its types are known at all call points. In fact, some compilers *require* such declarations. On the other hand, a C structure may contain a field that is a pointer to itself. For example, each element in a linked list contains a pointer to another element. It is customary to process such forward references in two passes. The first pass makes note of type references that should be checked in the second pass.

Symbol Table Summary

Although the interface for a symbol table is quite simple, the details underlying a symbol table's implementation play a significant role in the performance of the symbol table. Most modern programming languages are statically scoped. The symbol table organization presented in this chapter efficiently represents scope-declared symbols in a block-structured language. Each language places its own requirements on how symbols can be declared and used. Most languages include rules for symbol promotion to a global scope. Issues such as inheritance, overloading, and aggregate data types must be considered when designing a symbol table.

8.5 Declaration Processing Fundamentals

This section presents the approach we use to represent the information that must be associated with identifiers in the symbol table and begins our discussion of the techniques used to process declarations and do type checking on an abstract syntax tree (AST) representation of a program.

8.5.1 Attributes in the Symbol Table

In the discussion in Section 8.1.2, symbol tables were presented as a means for associating identifiers with some attribute information. We did not specify what kind of information was included in the attributes associated with an identifier or how it was represented. These topics are considered in this section.

The attributes of an identifier generally include anything the compiler knows about it. Because a compiler's main source of information about identifiers is declarations, attributes can be thought of as internal representations of declarations. Compilers do generate some attribute information internally, typically from the context in which the declaration of an identifier appears. Some languages define use of an identifier as an implicit declaration, in which case all of the attribute information must be constructed by the compiler when a first use is encountered. Identifiers are used in many different ways in a modern programming language, including as variables, constants, types, procedures, classes, and fields. Every identifier, therefore, will not have the same set of attributes associated with it. Rather, it will have a set of attributes corresponding to its usage and thus to its declaration.

We need a data structure to store the variety of information necessary to represent the attributes associated with the many different uses of names that occur in a program. This capability can be achieved by using a `struct` that contains a tag indicating the kind of attributes being stored and a `union` with one alternative corresponding to each possible value of the tag. Using an object-based approach, we could define an abstract class named *Attributes* and an appropriate subclass to represent the information that must be stored to describe each kind of declaration.

In the pseudocode that follows, we will use the `struct` approach, since that will allow us to keep the code a bit simpler. The tag will be named *kind* and names for members will be introduced as needed. Translation to an object-based approach should be obvious, with each distinct tag value indicating the need for a corresponding subclass of *Attributes*.

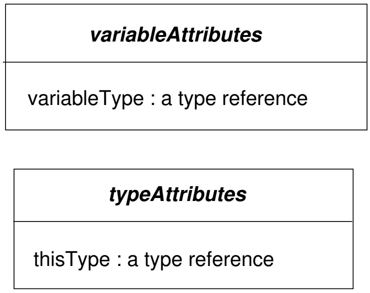


Figure 8.9: Attribute Descriptor Structures

To represent a variable declaration, we will need to store the type of the variable. Figure 8.9 illustrates the necessary *Attributes* structure. The *variableType* field will get a reference to a type descriptor as its value, but the reference may not be available yet. A second version is shown in Figure 8.9 that represents an identifier used as the name of a type rather than as a variable. A different tag value, *typeAttributes*, must be supplied to indicate this different use, although the information stored about it, a type reference, is the same as that for a variable name.

Arbitrarily complex structures, determined by the complexity of the information to be stored, may be used for attributes. Even the simple examples used here include references to other structures that represent type information. As we outline the declaration processing for other language features, we will define similar attribute structures for each.

8.5.2 Type Descriptor Structures

Among the attributes of almost any identifier is a type, which is represented by a type reference, as indicated in the previous examples. We will interpret a type reference as a reference to a struct of type *TypeDescriptor*. Representing types presents a compiler writer with much the same problem as representing attributes: there are many different types whose descriptions require different information, so our solution will be similar, with *typeKind* as the name of the tag member of a *TypeDescriptor* struct.

Figure 8.10 shows several variants of *TypeDescriptor*; the first assumes that *integer* is a built-in type in the language being compiled. The first example type descriptor is unusual in that it includes no information other than the fact that it represents the built-in *integer* type. The other examples show that we may include any information required to describe the type, including

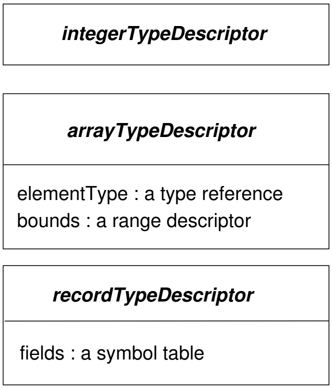


Figure 8.10: Type Descriptor Structures

references to other type descriptors or anything else necessary, even a symbol table. Such a representation is crucial in handling virtually all modern programming languages, which allow types to be constructed using powerful composition rules. Using this technique rather than some kind of fixed tabular representation also makes the compiler much more flexible in what it can allow a programmer to declare. For example, using this approach, there is no reason to have an upper bound on the number of dimensions allowed for an array or the number of fields allowed in a structure. Such limitations in early languages like Fortran stem purely from implementation considerations. We generally favor techniques that enable a compiler to avoid rejecting a legal program because of the size of the program or some part of it. Dynamic linked structures such as the type descriptor structure are the basis of such techniques.

8.5.3 Type Checking Using an Abstract Syntax Tree

We will use the **visitor pattern** described in Chapter 7 to implement a semantic processing pass over an AST. The primary activities of this pass will be constructing a symbol table structure that represents all of the declarations in the tree and preforming type checks as necessary throughout the tree. As pointed out when this visitor approach was introduced, it allows us to group all of the actions for this pass within a single class, *SemanticsVisitor*, a subclass of *Visitor*. Since the declaration processing involves specialized actions for building the symbol table structure that is the basis of the type checking to be done by *SemanticsVisitor*, the declaration processing actions will be implemented by a more specialized visitor, called *TopDeclVisitor*.

The actions to be performed by*SemanticsVisitor* and *TopDeclVisitor* for each node type will be specified in the sections that follow in the form of a VISIT method which takes an instance of the node type as a parameter. The exact same actions could be implemented by methods defined directly within the classes defining the abstract syntax tree node types. The disadvantage of this latter approach is that the code implementing the semantics pass would be scattered across many class definitions. These same actions can also be used even if the implementation language for a compiler does not support objects. A recursive traversal equivalent to that implemented by the visitor pattern can be performed by a single routine containing a large switch or case statement with an alternative for each kind of node in an abstract syntax tree. Once execution of the tree traversal defined by *SemanticsVisitor* is finished, the analysis phase of the compiler is complete. Subsequent chapters will describe additional phases of the compiler that ultimately synthesize the target code it produces.

A *reflective* mechanism for achieving such **double dispatch** is presented in Section 7.7.3 on page 268. A review of that material may be helpful before proceeding with this chapter.

Figure 8.11 provides an outline of the declarations processing visitors that will be described in Section 8.6, which covers variable and type declarations, and Section 8.7, which handles classes and methods. A given visitor is typically tasked with performing a relatively narrow set of activities. For the purposes of processing declarations, it is helpful to organize the visitors as follows:

SemanticsVisitor is the top-level visitor for processing declarations and doing semantic checking on an AST's nodes. There must be a VISIT method defined by *SemanticsVisitor* or one of its specializations for every kind of AST node. In addition to the declaration processing visitor to follow, type checking visitors are discusses in Section 8.8 and Chapter 9.

TopDeclVisitor is a specialized visitor invoked by *SemanticsVisitor* for processing declarations. It is responsible for building the symbol table structures corresponding to variable, type, class and method declarations. In the case of method declarations, it also initiates processing of each method's contents.

TypeVisitor is a specialized visitor used to handle an identifier that represents a type or a syntactic form that defines a type (such as an array).

For each AST node of interest, we present algorithm-style code in the form of a VISIT method that illustrates a strategy for processing the construct in the context of a declaration-handling pass over the AST .

```
class NodeVisitor
    procedure visitChildren( $n$ )
        foreach $c \in n$.getChildren( ) do
            call $c$.accept( this )                                      ⑪
    end
end

class SemanticsVisitor extends NodeVisitor
    /*  visit methods for other node types are defined in Section 8.8   */
end

class TopDeclVisitor extends SemanticsVisitor
    procedure visit( VariableListDeclaring $vld$ )                       ⑫
        /*  Section 8.6.1 on page 303                                  */
    end
    procedure visit( TypeDeclaring $td$ )                                ⑬
        /*  Section 8.6.3 on page 305                                  */
    end
    procedure visit( ClassDeclaring $cd$ )                               ⑭
        /*  Section 8.7.1 on page 317                                  */
    end
    procedure visit( MethodDeclaring $md$ )                              ⑮
        /*  Section 8.7.2 on page 321                                  */
    end
end

class TypeVisitor extends TopDeclVisitor
    procedure visit( Identifier $id$ )                                   ⑯
        /*  Section 8.6.2 on page 304                                  */
    end
    procedure visit( ArrayDefining $arraydef$ )                          ⑰
        /*  Section 8.6.5 on page 311                                  */
    end
    procedure visit( StructDefining $structdef$ )                        ⑱
        /*  Section 8.6.6 on page 312                                  */
    end
    procedure visit( EnumDefining $enumdef$ )                            ⑲
        /*  Section 8.6.7 on page 313                                  */
    end
end
```

Figure 8.11: Structure of the declarations visitors, with references to sections addressing specific constructs.

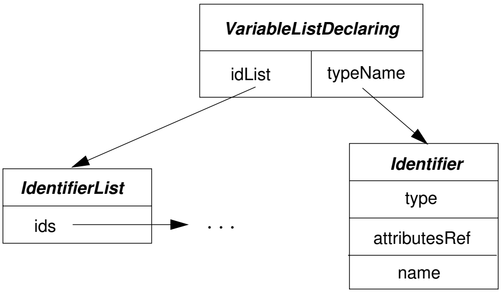


Figure 8.12: Abstract Syntax Tree for Variable Declarations

8.6 Variable and Type Declarations

We begin by examining the techniques necessary to handle declarations of variables and scalar types, and then move on to consideration of structured types.

8.6.1 Simple Variable Declarations

Our study of declaration processing begins with a simplified version of the variable declarations found in any programming language. This simple form of declaration includes a type name and a list of identifiers, indicating that all of the identifiers are to be declared as variables with the named type. The abstract syntax tree built to represent a variable declaration with the type defined by a type name is shown in Figure 8.12. Regardless of the exact syntax used in a particular programming language, this AST can be used to represent such declarations. However, it only represents a restricted version of variable declarations, which we are using to begin our discussion of declaration processing. A more general version of this visitor can be found in Section 8.6.4.

The visitor actions for simple variable declarations in Figure 8.13 illustrate use of a specialized visitor to process parts of the abstract syntax tree. An *Identifier* can be used in many contexts in a syntax tree. Since we want this particular identifier to be interpreted as a type name, a new instance of *TypeVisitor* is created at Marker ⑳ and invoked at Marker ㉑ to look up the type name in the symbol table and verify that it does indeed refer to a type.

The loop at Marker ㉒ processes the list of variable names, first checking each for a previous declaration (Marker ㉓). The block of code that begins at Marker ㉔ creates an *Attributes* structure for an identifier and then enters the identifier in the symbol table with the appropriate *Attributes*.

```
/★  Visitor code for Marker ⑫ on page 302                           ★/
procedure visit( VariableListDeclaring vld )
    typeVisitor ← new TypeVisitor( )                                   ⑳
    call vld.typeName.accept( typeVisitor )                            ㉑
    foreach id ∈ vld.idList do                                         ㉒
        if currentSymbolTable.declaredLocally( id.name )               ㉓
        then
            call error( "This variable is already declared : ", id.name )
            id.type ← errorType
            id.attributesRef ← null
        else
            id.type ← vld.typeName.type                                ㉔
            attr.kind ← variableAttributes
            attr.variableType ← id.type
            id.attributesRef ← attr
            call currentSymbolTable.enterSymbol( id.name, attr )
end
```

Figure 8.13: visit method in TopDeclVisitor for VariableListDeclaring.

8.6.2 Handling Type Names

The visit method in Figure 8.14 defines the actions performed by the *TypeVisitor* when it visits an *Identifier*. It begins by looking for the identifier in the symbol table using retrieveSymbol at Marker ㉕. If the attributes returned indicate that *id* does name a type, then a reference to the type descriptor for that type is assigned at Marker ㉖ to *type*, which stores the result of this retrieval process. If the *Identifier* does not name a type, an error message is produced at Marker ㉗. *type* is set to refer to a special type descriptor, *errorType*, indicating the error. Use of *errorType* to streamline error reporting will be discussed in

```
/★  Visitor code for Marker ⑯ on page 302                           ★/
procedure visit( Identifier id )
    attr ← currentSymbolTable.retrieveSymbol( id.name )                ㉕
    if attr ≠ null and attr.kind = typeAttributes
    then
        id.type ← attr.thisType                                        ㉖
        id.attributesRef ← attr
    else
        call error( "This identifier is not a type name : ", id.name )  ㉗
        id.type ← errorType
        id.attributesRef ← null
end
```

Figure 8.14: visit method in TypeVisitor for Identifier.

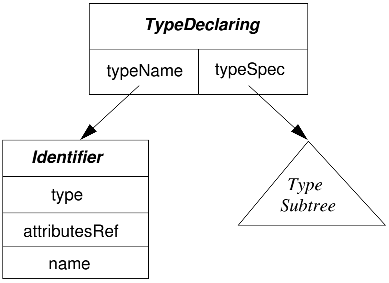


Figure 8.15: Abstract Syntax Tree for Type Declarations

detail in the following section.

8.6.3 Type Declarations

We have just seen how type name references are processed when used in variable declarations. We now consider how type declarations are processed to create the structures accessed by type references. A type declaration in any language includes a name and a description of the type to be associated with it. The abstract syntax tree shown in Figure 8.15 can be used to represent such a declaration regardless of its particular syntax.

As seen in Figure 8.16, the actions performed by the VISIT method that declares a type are similar for those used for declaring a variable. The type identifier must be entered into the current symbol table and an *Attributes* descriptor associated with it. In this case, the *Attributes* descriptor must indicate that the identifier names a type and must contain a reference to a *TypeDescriptor* for the type it names.

Figure 8.15 and the VISIT method leave unanswered the obvious question of what kind of subtree is pointed to by *typeSpec*. This part of the abstract syntax tree must define a type that is to be represented by *typeName*. Since type names are given to types defined by a programmer, *typeSpec* can point to a subtree that represents any form of type constructor allowed by the language being compiled. In the next two sections, we will be describing how to process two of the most common examples of such constructors: definitions of struct and array types.

By using *TypeVisitor* (rather than *TopDeclVisitor*) to process the subtree referenced by *typeSpec*, we require that the semantic processing for a type definition result in the construction of a *TypeDescriptor*. As we have just seen, the semantic processing for *TypeDeclaring* associates a reference to that *TypeDescriptor* with a type name. Notice that using this approach means that

```
/⋆ Visitor code for Marker ⑬ on page 302                              ⋆/
procedure VISIT( TypeDeclaring td )
    typeVisitor ← new TypeVisitor( )                                       ㉘
    call td.typeSpec.ACCEPT( typeVisitor )                                 ㉙
    name ← td.typeName.name                                                ㉚
    if currentSymbolTable.DECLAREDLOCALLY( name )                          ㉛
    then
        call ERROR( "This identifier is already declared : ", name )
        td.typeName.type ← errorType
        td.typeName.attributesRef ← null
    else
        attr ← new Attributes( typeAttributes )                            ㉜
        attr.thisType ← td.typeSpec.type
        call currentSymbolTable.ENTERSYMBOL( name, attr )
        td.typeName.type ← td.typeSpec.type
        td.typeName.attributesRef ← attr
end
```

Figure 8.16: VISIT method in TopDeclVisitor for TypeDeclaring.

it does not matter if a type definition is used rather than a type name in a variable declaration. In the previous section, we assumed that the type for a variable declaration was given by a type name. However, since we processed that name using *TypeVisitor*, it is actually irrelevant to the variable declaration method whether the type is described by a type name or by a type definition. In either case, processing the type subtree with *TypeVisitor* will result in a reference to a *TypeDescriptor* being produced. The only difference is that in one case (a type name), the reference is obtained from the symbol table, while in the other (a type definition), the reference is to a descriptor created by the *TypeVisitor*.

Handling Type Declaration and Type Reference Errors

In the pseudocode for the *TopDeclVisitor* VISIT method for an identifier, we introduced the idea of a **static semantic check**. That is, we define a situation in which an error can be recognized in a program that is syntactically correct. The error in that case was a simple one: a name was used as a type name, but it did not actually name a type. In such a situation, we want our compiler to generate an error message that explains why the program is erroneous. Ideally, we want to generate only one error message per error (though compilers too often fall far short of this ideal!). The simplest way to accomplish this goal is to immediately stop the compilation, though it is also at least approachable for a compiler that tries to detect as many errors as possible in a source program.

Whenever semantic processing finds that a semantic error has occurred, it will return a reference to a special *TypeDescriptor* that we will refer to as *errorType*. That particular value will be a signal to the code that initiated semantic processing that an error has been detected and that an error message has already been generated. In this calling context, an explicit check may be made for *errorType* or it may be treated like any other *TypeDescriptor*. In the specific instance of the variable declaration pseudocode, the possibility of an *errorType* return can be ignored. It works perfectly well to declare variables with *errorType* as their type. In fact, doing so prevents later, unnecessary error messages. That is, if a variable is left undeclared because of a type error in its declaration, each time it is used, the compiler will generate an undeclared variable error message. In later sections, we will see other uses of *errorType* to avoid the generation of extraneous error messages.

Type Compatibility

One question remains to be answered: Just what does it mean for types to be the same or for a constraint (as used in Ada) to be compatible with a type? Ada and Pascal have a strict definition of type equivalence that says that every type definition defines a new, distinct type that is incompatible with all other types. This definition means that the declarations

```
A, B : ARRAY (1..10) OF Integer;
C, D : ARRAY (1..10) OF Integer;
```

are equivalent to

```
type Type1 is ARRAY (1..10) OF Integer;
A, B : Type1;
type Type2 is ARRAY (1..10) OF Integer;
C, D : Type2;
```

`A` and `B` are of the same type and `C` and `D` are of the same type. However, the two types are defined by distinct type definitions and thus are incompatible. As a result, assignment of the value of `C` to `A` would be illegal. This rule is easily enforced by a compiler. Since every type definition generates a distinct type descriptor, the test for type equivalence requires only a comparison of pointers.

Other languages, most notably C, C++ and Algol 68, use different rules to define type equivalence. The most common alternative is to use **structural type equivalence**. As the name suggests, two types are equivalent under this rule if they have the same definitional structure. Thus `Type1` and `Type2` from the previous example would be considered equivalent. At first glance, this rule appears to be a much more useful choice because it seems more convenient for programmers using the language. However, counterbalancing this convenience is the fact that the structural type equivalence rule makes it impossible for a programmer to get full benefit from the concept of type checking. That is, even if a programmer wants the compiler to distinguish between `Type1` and `Type2` because they represent different concepts in the program despite their identical implementations, the compiler is unable to do so.

Structural equivalence is also much harder to implement. Rather than being determined by a single pointer comparison, a parallel traversal of two type descriptor structures is required. The code to do such a traversal requires special cases for each of the type descriptor alternatives. Another approach does the comparison work as a type definition is processed by the semantic checking pass through the tree. The type being defined is compared against previously defined types so that equivalent types are represented by the same data structure, even though they are defined separately. This technique allows the type equivalence test to be implemented by a pointer comparison, but it requires an indexing mechanism that makes it possible to tell during declaration processing whether each newly defined type is equivalent to any previously defined type.

Further, the recursion possible within pointer type definitions poses subtle difficulties to the implementation of a structural type equivalence test. Consider the problem of writing a routine that can determine whether the following two Ada types are structurally equivalent (`access` means ”points to”):

```
type A is access B;
type B is access A;
```

Even though such a definition is meaningless semantically, it is syntactically legal (presuming there is an incomplete type definition to introduce the name `B` before the definition of `A`). Thus a compiler for a language with structural type equivalence rules must be able to make the appropriate determination—that `A` and `B` are equivalent. If parallel traversals are used to implement the equivalence test, then the traversal routines must ”remember” which type descriptors they have visited during the comparison process in order to avoid an infinite loop. Suffice it to say that comparing pointers to type descriptors is much simpler!

8.6.4 Variable Declarations Revisited

Declarations of variables and, more generally, data members of classes can be more complex than the simple form shown in Section 8.6.1. Types need not be specified by a name, but can be constructed by a variety of syntactic forms. To cover the cases, the AST for declarations shown in Figure 8.17

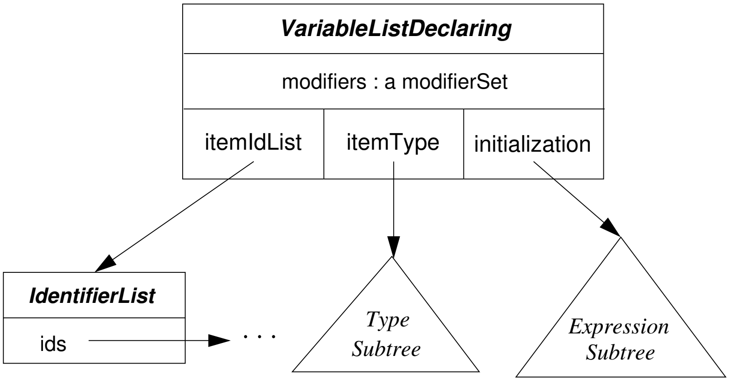


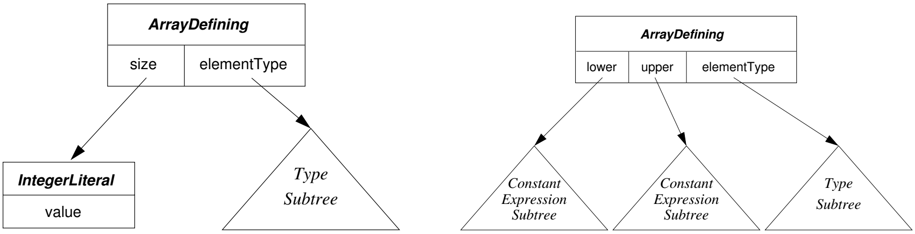
Figure 8.17: AST for Generalized Variable Declarations

includes a reference to a type subtree similar to the one used in Figure 8.15 of Section 8.6.3. The visit method for these generalized variable declarations in Figure 8.18 begins at Marker ㉝ just like the one Section 8.6.1, since it does not matter whether the type of the variables being declared is specified by a name or a type definition. An optional initialization subtree is also shown as part of the *VariableListDeclaring* AST node in Figure 8.17. If initialization is present (Marker ㉞), it is analyzed and checked for assignment compatibility with the declared type (Marker ㉟). Initialization is required if a constant is being declared, and appropriate checking is included in the visitor actions (Marker ㊱).

Depending on the language being compiled, the declaration syntax may include one or more modifiers, such as *const*, *static*, *public*, etc. Because many combinations of such modifiers may be possible, we have designed our AST to simply represent all of the modifiers present as a set rather than have different AST nodes representing each of the modifier keywords. Corresponding nodes would be present in the parse tree, but they are eliminated and instead are represented by the modifier set during AST creation. At Marker ㊲, we assume an extension to the *Attributes* descriptor for variables defined in Figure 8.9 to store this modifier set. Any visit method that needs information about the attributes of a variable can check this set. The body of the declaration loop ends after *id.name* is entered into the symbol table and the *type* and *attributeRef* fields in its AST node are set. The method ends once the entire list of variable names has been processed.

```
/★ Generalized visitor code for Marker (12)                                   ★/
procedure VISIT( VariableListDeclaring vld )
    typeVisitor ← new TypeVisitor( )                                         (33)
    call vld.itemType.ACCEPT( typeVisitor )
    declType ← vld.itemType.type
    if $vld.initialization \neq$ null                                        (34)
    then
        checkingVisitor ← new SemanticsVisitor( )
        call vld.initialization.ACCEPT( checkingVisitor )
        if not ASSIGNABLE( vld.initialization.type, declType )               (35)
        then
            call ERROR( "Initialization expression not assignable to variable type at", vld )
    else
        if $const \in vld.modifiers$                                         (36)
        then
            call ERROR( "Initialization expression missing in constant declaration at", vld )
    foreach $id \in vld.itemIdList$ do
        if currentSymbolTable.DECLAREDLOCALLY( id.name )
        then
            call ERROR( "Variable name cannot be redeclared : ", id.name )
            id.type ← errorType
            id.attributesRef ← null
        else
            attr.kind ← variableAttributes
            attr.variableType ← declType
            attr.modifiers ← declType                                        (37)
            call currentSymbolTable.ENTERSYMBOL( id.name, attr )
            id.type ← declType
            id.attributesRef ← attr
end
```

Figure 8.18: Code for TopDeclVisitor's VariableListDeclaring



Size of array specified
(a)

Upper and lower bounds included
(b)

Figure 8.19: Abstract Syntax Trees for Array Definitions

```
/★ Visitor code for Marker ⑰ on page 302                              ★/
procedure visit( ArrayDefining arraydef )
    call visitChildren( arraydef )
    arraydef.type ← new TypeDescriptor( arrayType )                       ㊳
    arraydef.type.elementType ← arraydef.elementType.type
    arraydef.type.arraysize ← arraydef.size.value
end
```

Figure 8.20: visit method in TypeVisitor for ArrayDefining.

8.6.5 Static Array Types

The most common form of array type constructor found in programming languages enables a programmer to define an array type by specifying the type of its elements and the number of elements it contains. The element type can be described by a type name or a general type definition included as part of the array definition. Since visiting either form of type specification yields a reference to a type descriptor, the visitor for the *ArrayDefining* node need not distinguish between these two cases.

The number of elements in the array is defined either by a single integer literal (in the case where the lower bound is a value defined by the language) or a pair of literals that specify lower and upper bounds. The AST in Figure 8.19(a) illustrates the case where only a single integer is allowed in the syntax for an array definition. This syntax is used in languages where the lower bound of an array is defined by the language as a fixed value (either 0 or 1) and the integer specifies the number of elements in the array. The visit method in this section is written for processing this form of AST. The tree in Figure 8.19(b) illustrates the case where a language allows more-flexible array definitions in which both the lower and upper bounds are specified and these bounds can be defined by expressions involving named constants as well as literals. Thus expression trees appear in the AST for the two bounds. The visit method for the tree in Figure 8.19(b) is considered in Exercise 18.

The visit method for an *ArrayDefining* node appears in Figure 8.20. It builds a *TypeDescriptor* that describes the array type. The required specialization of *TypeDescriptor* was presented in Figure 8.10 of Section 8.5.1. The pseudocode for the visit method begins by invoking visitChildren to process the subtrees that describe its size and element type. Note that if the language allows the size to be described by a constant expression, then the expression's value can be computed by visiting the expression subtree with a specialized visitor class. A new *ArrayTypeDescriptor* is created at Marker ㊳ and the values it must contain are obtained from the *elementType* and *size* subtrees on the following lines.

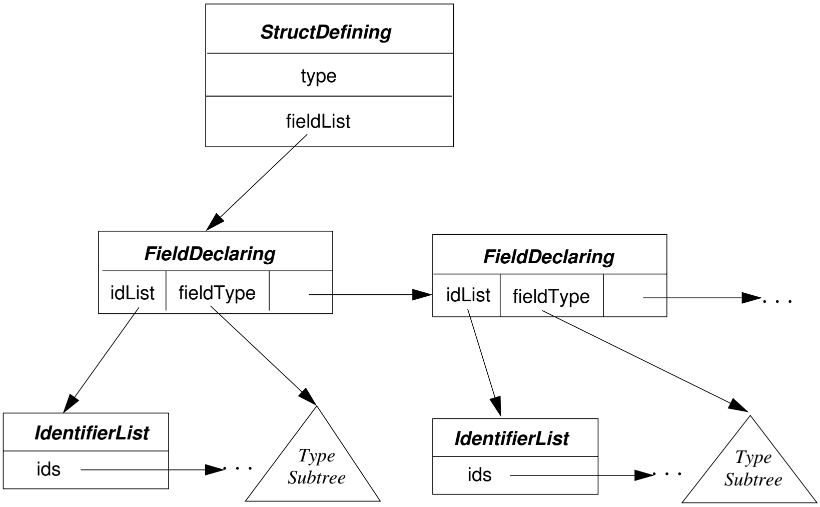


Figure 8.21: Abstract Syntax Tree for a Struct Definition

8.6.6 Struct and Record Types

Programming languages typically include a type constructor for a heterogeneous collection of named data items, commonly known as records or structs. The data items are named individually, using a syntax similar to variable declarations. Because the names and types of all of the fields of a record or struct must be individually specified, both the AST and semantic processing for this kind of type constructor is much more complicated than that for arrays.

The AST in Figure 8.21 illustrates the representation necessary for a struct definition. The *StructDefining* node provides access to a list of *FieldDeclaring* nodes. Each of the *FieldDeclaring* nodes looks much like *VariableListDeclaring* node from Section 8.6.4 and will be processed similarly. A struct defines a new name scope in which all of the fields will be declared, but this scope can only be accessed by naming an instance of the struct type. (See Section 8.8.4.) Thus the symbol table for the scope defined by the struct will not be pushed on the stack of currently open scopes. Rather, it will become part of the *TypeDescriptor* for the struct type. The specialization of *TypeDescriptor* needed for structs was presented in Figure 8.10 of Section 8.5.1.

The VISIT method for a *StructDefining* node is found in Figure 8.22. It begins at Marker (39) by building a *TypeDescriptor* for a struct type. It then creates a new symbol table to hold all of the field declarations.

Beginning at Marker (40) two nested loops process the list of *FieldDeclaring* nodes and the individual field declarations they contain. At Marker (41) in the

```
/*   Visitor code for Marker ⑱ on page 302                          */
procedure VISIT( StructDefining structdef )
    typeRef ← new TypeDescriptor( structType )                        ㊴
    typeRef.fields ← new SymbolTable( )
    foreach decl ∈ structdef.fieldList do                             ㊵
        call decl.fieldType.ACCEPT( this )                            ㊶
        foreach id ∈ vld.idList do
            if typeRef.fields.DECLAREDLOCALLY( id.name )              ㊷
            then
                call ERROR( "Name cannot be redeclared : ", id.name )
                id.type ← errorType
                id.attributesRef ← null
            else
                attr.kind ← fieldAttributes                           ㊸
                attr.fieldType ← decl.fieldType.type
                call typeRef.fields.ENTERSYMBOL( id.name, attr )
                id.type ← decl.fieldType.type
                id.attributesRef ← attr
    structdef.type ← typeRef                                          ㊹
end
```

Figure 8.22: VISIT method in TypeVisitor for StructDefining.

outer loop, the *TypeVisitor* is propagated to the AST representing the type specification for one set of fields. Within the inner loop, each field identifier is checked for previous declaration within the struct (at Marker ㊷). Presuming that it passes this test, beginning at Marker ㊸, a *FieldAttribures* representation is created for the name, which is then entered into the symbol table for the struct. The VISIT method finishes at Marker ㊹ by leaving a reference to the completed *TypeDescriptor* for the struct type within the *StructDefining* node that represents it in the AST.

8.6.7 Enumeration Types

An enumeration type is defined by a list of distinct identifiers. Each identifier is a constant of the enumeration type. These constants are ordered by their position in the type definition and are represented internally by integer values. Typically, the value used to represent the first identifier is zero, and the value for each subsequent identifier is one more than that for its predecessor in the list (although Ada does allow a programmer to specify the values used to represent enumeration literals). If runtime error checking is enabled, then the value zero might be used to represent "uninitialized," with valid enumeration values begining at one. An abstract syntax tree for an enumeration type definition is shown in Figure 8.24.

```
/*  Visitor code for Marker 19 on page 302                                   */
procedure VISIT( EnumDefining enumdef )
    typeRef ← new TypeDescriptor( enumType )                                 45
    nextval ← 0
    foreach id ∈ enumdef.constantNames do                                    46
        if currentSymbolTable.DECLAREDLOCALLY( id.name )                     47
        then
            call ERROR( "Name cannot be redeclared : ", id.name )
            id.type ← errorType
            id.attributesRef ← null
        else
            attr ← new Attributes( enumAttributes )                          48
            attr.enumType ← typeRef
            attr.myValue ← nextval
            nextval ← nextval + 1
            call currentSymbolTable.ENTERSYMBOL( id.name, attr )             49
            id.type ← typeRef
            id.attributesRef ← attr
            call typeRef.APPENDTOCONSTLIST( id.name, attr )
    enumdef.type ← typeRef                                                   50
end
```

Figure 8.23: VISIT method in TypeVisitor for EnumDefining.

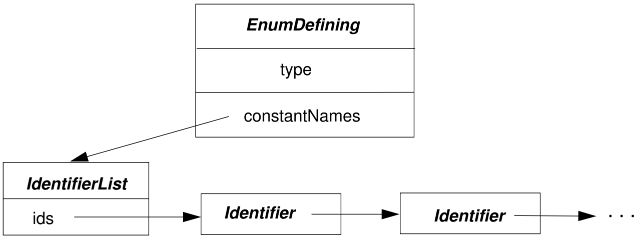


Figure 8.24: Abstract Syntax Tree for an Enumeration Type

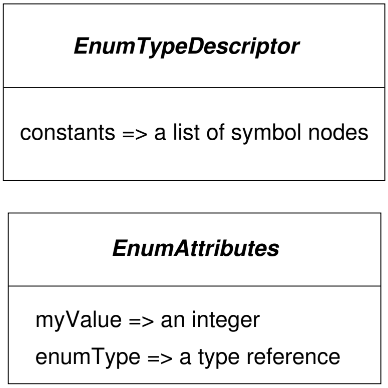

EnumTypeDescriptor
constants => a list of symbol nodes
EnumAttributes
myValue => an integer
enumType => a type reference

Figure 8.25: Type and Attribute Descriptor Objects for Enumerations

The visit method for processing an *EnumDefining* AST node, as with those for processing records and arrays, will build a *TypeDescriptor* that describes the enumeration type. In addition, each of the identifiers used to define the enumeration type is entered into the current symbol table. Its attributes will indicate that it is an enumeration constant, and include the value used to represent it and a reference to the *TypeDescriptor* for the enumeration type itself. The type will be represented by a list of the symbols and *Attributes* records for its constants. The required specializations of *Attributes* and *TypeDescriptor* are illustrated in Figure 8.25.

The visit method in Figure 8.23 begins at Marker ㊺ by getting a new *EnumTypeDescriptor* and initializing a local variable *nextval* that will be used to define the values of the constants of the enumeration type. The loop that begins at Marker ㊻ processes each of the constants of the enumeration type in turn. As with all of the other kinds of names declared in this chapter, a check is done at Marker ㊼ to be sure that the constant name is not already declared in this scope. If the name can be added to the current scope, then the block of code at Marker ㊽ creates an *Attributes* record for it, and sets its value from *nextval* and its type as the enumeration type currently being defined. The body of the constant declaring loop finishes at Marker ㊾ by entering the constant name in the symbol table and also adding its name to the list of constants in the *TypeDescriptor* that describes the enumeration type. After executing this loop for all of the constants in the type definition, the visit method completes its work at Marker ㊿ by including a reference to the now completed *EnumTypeDescriptor* in the *EnumTypeNode* in the AST for the enumeration type. Figure 8.26 shows the *EnumTypeDescriptor* that would be constructed to represent the enumeration type defined by `(red, yellow, blue, green)`.

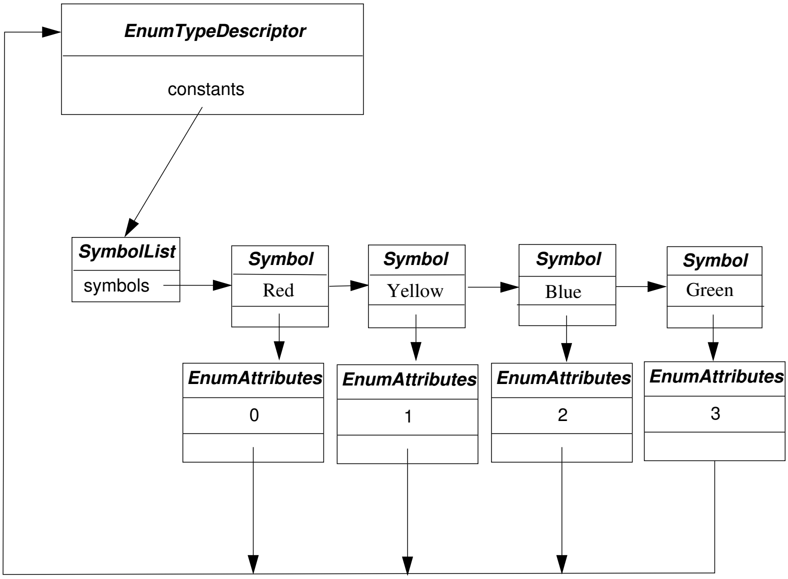


Figure 8.26: Representation of an Enumeration Type

8.7 Class and Method Declarations

Declarations of classes in Java, C++, or any other language generally require processing that is much like the struct definitions in Section 8.6.6. Because classes have a number of additional capabilities, the details will necessarily be more complex. As with structs, classes encapsulate a collection of declarations. Thus a symbol table will be created for each new class to provide a unique name space for the declarations in the class. A class defines a type, just as a struct does. However, a class declaration includes a name for that type, in contrast to the struct definition, which may be used as the type for a variable without there being any name for the struct type. Thus the VISIT method for a class declaration will not only construct a *TypeDescriptor* for the class, but also create an entry in the current symbol table for the class name.

Methods are an important part of class declarations, since they generally define the external interface used to access an instance of the class. Methods introduce one new concept that we have not used in our discussion of declaration processing, a **signature** , defined by the types of the parameters and the return type of the method. Information about the signature of a method must be constructed as the method declaration is processed and stored in the symbol table as part of the information associated with the method name.

One new mechanism is necessary to implement the semantic checking required by classes and methods. Certain validity checks described in Chapter 9 will require reference to the class or method currently being compiled. Our AST representation does not readily provide access to this information, since the node representing the current class or method may be arbitrarily far up the tree and, in addition, our tree nodes as described thus far do not include upward links. Rather than making a radical change in our representation, we introduce several methods that are defined as visible within all of our visit methods. The best way to implement these methods depends on the particular language being used, so no concrete implementation is specified here. The methods are:

```
procedure setCurrentClass( ClassAttributes c )
function getCurrentClass( ) returns ClassAttributes
procedure setCurrentMethod( MethodAttributes m )
function getCurrentMethod( ) returns MethodAttributes
procedure setCurrentConstructor( MethodAttributes m )
function getCurrentConstructor( ) returns MethodAttributes
```

The visit methods in this section will make use of the set methods as processing begins for declarations of each kind of construct. Various visit methods in Chapter 9 will use the get methods to access the information they need about the context in which they are operating. Note that these methods distinguish between the context created by a constructor declaration and a method declaration, though both are represented by a *MethodAttributes* structure.

8.7.1 Processing Class Declarations

A class declaration may specify a parent class to which the defined class is related by inheritance. In some languages, such as Java, a class declaration may begin with modifiers such as *abstract* and *final*, which affect properties or uses of the class. Java class declarations may also name a set of *interfaces* implemented by the class. The AST node in Figure 8.27 includes subtrees for all of these features except *interfaces* implemented. That feature will be considered separately.

As has typically been the case for our declarations-processing visitors, the visit method for a *ClassDeclaring* node works with the AST subtrees accessible from the node as needed in order to implement a class declaration. The code in Figure 8.29 begins at Marker 51 with the creation of a *TypeDescriptor* for the class and the creation of a symbol table to hold the names declared within the class. These steps are followed by creation of an *Attributes* structure for the class and entry of the name of the class in the current symbol table. Figure 8.28 illustrates the information that is associated with the class name in its symbol

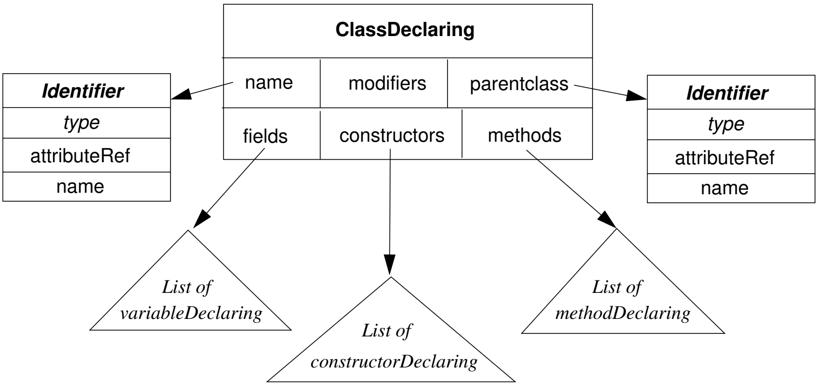


Figure 8.27: Abstract Syntax Tree for a Class Declaration

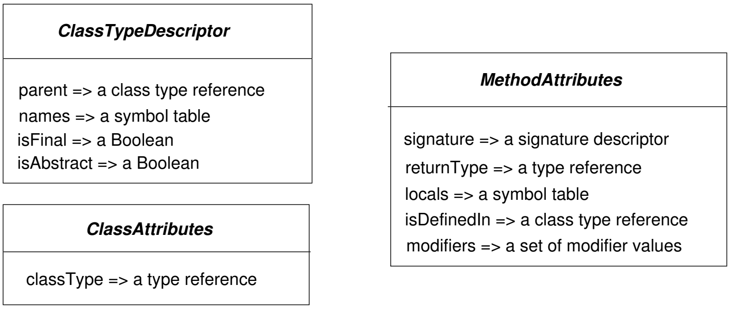

ClassTypeDescriptor
parent => a class type reference
names => a symbol table
isFinal => a Boolean
isAbstract => a Boolean
ClassAttributes
classType => a type reference
MethodAttributes
signature => a signature descriptor
returnType => a type reference
locals => a symbol table
isDefinedIn => a class type reference
modifiers => a set of modifier values

Figure 8.28: Attributes and Type Descriptor for Class Declarations

table entry. The value of *currentClass* is set so that it references the *Attributes* descriptor of this class.

Next, at Marker 52, we check to see if a parent class name was supplied in the declaration. If not, an AST node that refers to the class *Object* is attached to this node. Otherwise, an instance of *TypeVisitor* is created to process the node referenced by *parentClass*. If an error occurs during the visit to *parentClass*, or if *parentClass.name* is not a class name, then the *TypeDescriptor* for the current class declaration will be replaced by *errorType* and the rest of the class declaration is not processed. A misnamed parent class would likely cause more errors to be reported if the field and method declarations were processed, so we choose to skip over these components of the declaration in order to achieve our goal of generating only one error message per error in the source code.

If *parentClass.name* does designate a class, then processing continues at

```
/★ Visitor code for Marker 14 on page 302                                  ★/
procedure VISIT( ClassDeclaring cd )
    typeRef ← new TypeDescriptor( ClassType )                                   51
    typeRef.names ← new SymbolTable( )
    attr ← new Attributes( ClassAttributes )
    attr.classType ← typeRef
    call currentSymbolTable.ENTERSYMBOL( name.name, attr )
    call SETCURRENTCLASS( attr )
    if cd.parentclass = null                                                    52
    then  cd.parentclass ← GETREFTOOBJECT( )
    else
        typeVisitor ← new TypeVisitor( )
        call cd.parentclass.ACCEPT( typeVisitor )
    if cd.parentclass.type = errorType
    then  attr.classtype ← errorType
    else
        if cd.parentclass.type.kind ≠ classType
        then
            attr.classtype ← errorType
            call ERROR( parentClass.name, "does not name a class" )
        else
            typeRef.parent ← cd.parentClass.attributeRef                        53
            typeRef.isFinal ← MEMBEROF( cd.modifiers, final )
            typeRef.isAbstractl ← MEMBEROF( cd.modifiers, abstract )
            call typeRef.names.INCORPORATE( cd.parentclass.type.names )         54
            call OPENSCOPE( typeRef.names )
            call cd.fields.ACCEPT( this )                                       55
            call cd.constructors.ACCEPT( this )
            call cd.methods.ACCEPT( this )
            call CLOSESCOPE( )
    call SETCURRENTCLASS( null )
end
```

Figure 8.29: VISIT method in TopDeclVisitor forClassDeclaring

Marker 53, where values are designated for the rest of the fields of the type descriptor for the class. Since only a limited number of modifiers are possible in a class declaration, fields have been included in the type descriptor to record the presence or absence of each possible modifier. Going on to the code at Marker 54, a new symbol table feature is required in order to handle name resolution within the body of a class. All of the names declared in the parent class (and its ancestor classes) must be directly visible as the body is being processed. We assume there is a symbol table method named incorporate available to implement this extended lookup rule. It is used to include the names visible in the parent class in the symbol table defined for the current class (*TypeRef.names*). After this step, that symbol table is set as the currently open name scope. This step creates the appropriate symbol table environment for processing the *fields*, *constructors*, and *methods* subtrees. The three calls beginning at Marker 55 propagate the current *TopDeclVisitor* to each of these subtrees. In the last steps of this method, the symbol table for this class is popped off the symbol table stack so that the symbol environment returns to what it was before the execution of this visitor, with the addition of the name of this class to that environment, and the value of *currentClass* is returned to **null**, since we are no longer within a class declaration. If nested classes are allowed by the language being compiled, we would instead restore a saved value in this last statement.

The visit method as written above deals with modifiers *final* and *abstract*, as are included in Java. These modifiers put restrictions on how the class name can be used, and thus imply additional checks in appropriate visit methods. In fact, fully implementing the meaning of *final* requires the addition of a check to this *ClassDeclaring* visitor. Just after the check is done for *parentClass* naming a class, we must also add a check that the class it names has not been declared to be *final*. On the other hand, *abstract* will require a check elsewhere that the name of an *abstract* class is not used in a constructor.

The symbol table requirements of classes make the use of a separate symbol table structure for each scope, as illustrated in Figure 8.3, a more appropriate technique for object-oriented languages than the alternative single symbol table structure approach detailed in Section 8.3.3. In addition, the inclusion of the incorporate method in the interface requires us to consider yet another requirement on our data structure. With the addition of this feature, the lookup process for a name must proceed first up the list of symbol tables for ancestor classes before traversing through the stack of scopes that enclose the current classes (which may include another class, if nested classes are allowed by the language). Multiple inheritance complicates the picture still further. Exercise 23 considers the impact of this combination of features. One more change to the symbol table interface is required to handle class (and method) declarations. A new version of openScope is used just after the call to incorporate at Marker 54. It takes a symbol table as a parameter, in this

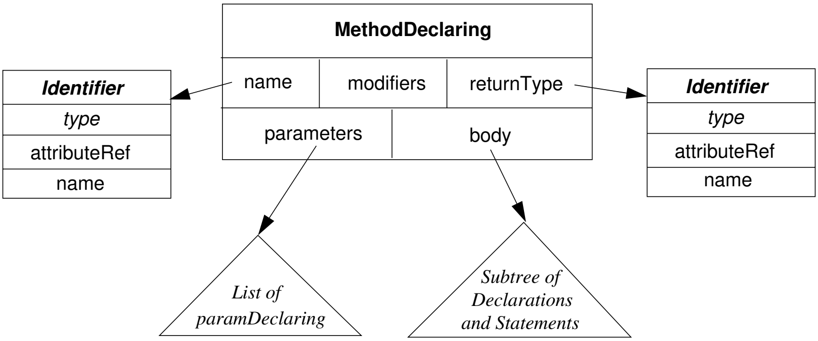


Figure 8.30: Abstract Syntax Tree for a Method Declaration

case, the symbol table for the current class, and makes that symbol table the new open scope. It can be distinguished from the previously defined version of OPENSCOPE by standard overloading resolution rules.

The final feature of classes that we will consider in this section is the list of interface names that may be part of a class declaration in Java. Each interface has a set of declarations associated with it, again represented by a symbol table. After all of the declarations of a class have been processed, its declarations must be checked against each of the interfaces specified, in order to ensure that all of the interfaces are indeed fully implemented by the class. The list of interfaces must become one of the fields of the class type so that appropriate type checking can be done when an instance of the class is used in a context that requires an object conforming to an interface.

8.7.2 Processing Method Declarations

The AST node in Figure 8.30 illustrates the information available to the VISIT method for a method declaration. As with other declarations, the visit to a *MethodDeclaring* node will create a new entry in the current symbol table for the method name. Since the body defines a new scope, as is the case for a class, a new nested symbol table will be created by this VISIT method.

The *MethodDeclaring* visitor is found in Figure 8.31. It does not make a call to VISITCHILDREN, as was the case for the VISIT method for a *ClassDeclaring* node. Rather, the various subtrees referenced by the *MethodDeclaring* node are processed by visitors in a more customized way. This process begins at Marker (56), where an instance of *TypeVisitor* is created for the *returnType*. The result of this visit, found in the *type* field of the *Identifier* node referenced by *returnType*, is set as the return type of the method. Note that no check is done for *errorType*, since we want to process the rest of this declaration even

```
/⋆  Visitor code for Marker (15) on page 302                          ⋆/
procedure visit( MethodDeclaring md )
    typeVisitor ← new TypeVisitor( )                                   (56)
    call md.returnType.accept( typeVisitor )
    attr ← new Attributes( MethodAttributes )
    attr.returnType ← md.returnType.type
    attr.modifiers ← md.modifiers
    attr.isDefinedIn ← getCurrentClass( )
    attr.locals ← new SymbolTable( )
    call currentSymbolTable.enterSymbol( name.name, attr )
    md.name.attributeRef ← attr
    call openScope( attr.locals )
    oldCurrentMethod ← getCurrentMethod( )
    call setCurrentMethod( attr )
    call md.parameters.accept( this )                                  (57)
    attr.signature ← parameters.signature.addReturn( attr.returntype )
    call md.body.accept( this )                                        (58)
    call setCurrentMethod( oldCurrentMethod )
    call closeScope( )
end
```

Figure 8.31: visit method in TopDeclVisitor for MethodDeclaring.

if the name given for the return type did not designate a valid type. Next, an *Attributes* descriptor for the method is created, with the return type, a newly created symbol table, and the value of *currentClass* being used to provide values for three of its fields. (See Figure 8.28 for the details of an *Attributes* descriptor for a method.) This symbol table is set as the current symbol scope and the *Attributes* descriptor for the method is set as the *currentMethod* before processing of the rest of the subtrees of the *MethodDeclaring* node begins. Note that the old value of *currentMethod* is saved so that it can be restored at the end of this method. If nested methods are allowed by the language, this step will make sure that the surrounding method is referenced again after this method declaration is finished. If nested methods are not allowed, then the restore step should return the *currentMethod* reference to a null value.

The *modifiers* field of the AST node was simply copied to the *Attributes* descriptor for the method rather than interpreting the values of the modifier set, as we did in the visit method for a *ClassDeclaring* node. Because of the many modifiers that can be specified for a method, it is simpler just to carry along the set of modifiers that were part of the declaration and let other visitors look for values in this set when needed.

At Marker (57), the current *TopDeclVisitor* is propagated to the list of parameter declarations. The visit method for the *ParameterDeclaring* node referenced by *parameters* will encounter a sequence of formal parameter names and their types that define the interface to the method. This visitor enters all of the names in the symbol table, similar to variables, and constructs a descriptor for the list of parameter types, which is subsequently incorporated in the *Attributes* descriptor for the method. Details of this method are left as an exercise. (See Exercise 22.) Since the final subtree representing the body of the method consists of a combination of declarations and executable statements, we have to consider what will happen as we propagate the current *TopDeclVisitor* to this subtree. When a declaration is encountered, the appropriate VISIT method declared within *TopDeclVisitor* will be invoked. When a statement is encountered during traversal of the list, no corresponding method will be found in *TopDeclVisitor*. Since the parent class of *TopDeclVisitor* is *SemanticsVisitor,* VISIT methods declared within that class for statement AST nodes will be invoked when corresponding statement nodes are visited. VISIT methods for statement nodes are described in Section 8.8 and Chapter 9. Finally, the value of *currentMethod* is returned to **null**, since we are no longer within a method declaration, and the VISIT method returns the symbol table stack to its original state before completing its actions for the *MethodDeclaring* node.

Java includes one additional significant feature as part of method declarations. A method may include a list of exceptions that might be thrown by execution of the method. A complete implementation of Java method declarations requires a reference to this list of exceptions as part of the *MethodDeclaring* AST node and a VISIT method to process the exception list. In addition the *Attributes* descriptor for the method must be extended to include a *declaredThrowsList* for use by the VISIT methods that handle throws, as defined in Section 9.1.7 on page 369.

Constructors are much like methods, except that no return type is specified, since the type they return is the type of the class in which they are declared. Certain restrictions apply to constructors, depending on the language being compiled. A VISIT method for a *ConstructorDeclaring* node will look much like the method defined here for a *MethodDeclaring* node.

8.8 An Introduction to Type Checking

Sections 8.6 and 8.7 presented the methods defined in *TopDeclVisitor* that process declarations in order to collect information in the symbol table. In this section, we define the actions performed by *SemanticsVisitor* that use the information in symbol table to check types and other semantic requirements as it visits the AST nodes for the executable parts of a program.

We begin our discussion of semantic checking by defining the type checking requirements for names and expressions in the context of an assignment statement. After a general discussion of the interpretation of names in this context, Section 8.8.1 presents the VISIT methods used for semantic analysis of

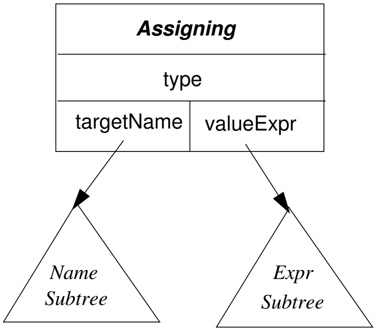


Figure 8.32: Abstract Syntax Tree for an Assignment

simple identifiers and literals. The method for assignment statements is then detailed in Section 8.8.2. Section 8.8.3 deals with expressions involving unary and binary operators and thereafter in Section 8.8.4 the techniques needed to compile simple record and array references are presented.

The AST form of an assignment statement is shown in Figure 8.32. A *targetName* subtree may be simply an identifier or something more complex, such as an indexed array or a field in a struct or class. Similarly, a *valueExpr* subtree may be a simple identifier or literal, or a complex computation involving operator evaluation and method calls. No matter how complex the subtrees are, our approach to semantic analysis will be uniform; visitors will traverse both subtrees checking for semantic errors and determining types.

A simple name can obviously be represented by the same *Identifier* node that we used to represent type names and in many other contexts as we processed declarations. The simplest form of an expression is also just a name (as seen in the assignment statement `a = b`) or a literal constant (as in `a = 5`).

However, semantic analysis of an assignment statement is not as simple as it first seems. Look at the trivial assignment `a = b`. In this assignment the two identifiers have rather different meanings. `a` stands for the *address* of `a`, the location that will receive the assigned value. `b` stands for the *value* at the address associated with `b`. We say that the `a` is providing an **left value** (L-value) because this is the meaning of an identifier on the left-hand side of an assignment statement (i.e., an address). Similarly, we say that `b` denotes a **right value** (R-value) because this is how we interpret an identifier on the right-hand side of an assignment (i.e., a value or the contents of an address). This differentiation between L-values and R-values explains why a named constant may be the source value of an assignment but not its target.

In terms of our semantic analysis, we shall use *SemanticsVisitor* to ana-

```
class NodeVisitor
    procedure visitChildren( n )
        foreach c ∈ n.getChildren( ) do call c.accept( this )
    end
end

class SemanticsVisitor extends NodeVisitor
    procedure visit( Identifier id )                                      59
        /★   Section 8.8.1 on page 327                              ★/
    end

    procedure visit( IntLiteral intlit )                                  60
        intlit.type ← integerType
    end
    procedure visit( Assigning assign )                                   61
        /★   Section 8.8.2 on page 328                              ★/
    end

    procedure visit( BinaryExpr bexpr )                                   62
        call visitChildren( bexpr )
        bexpr.type ← binaryResultType( bexpr.operator, bexpr.leftType.type, bexpr.rightType.type )
    end

    procedure visit( UnaryExpr uexpr )                                    63
        call visitChildren( uexpr )
        uexpr.type ← unaryResultType( uexpr.operator, uexpr.subExpr.type )
    end

    procedure visit( ArrayReferencing ar )
        /★   Figure 8.37 on page 331                                ★/
    end

    procedure visit( StructReferencing sr )
        /★   Figure 8.38 on page 331                                ★/
    end
end
```

Figure 8.33: Type checking visitors (Part 1)

```
class LHSSemanticsVisitor extends SemanticsVisitor
    procedure visit( Identifier id )                                       (64)
        visitor ← new SemanticsVisitor( )
        call id.accept( visitor )
        if not isAssignable( id.attributeRef )
        then
            call error( id.name, "is not assignable." )
            id.type ← errorType
            id.attributesRef ← null
    end

    procedure visit( ArrayReferencing ar )                                 (65)
        call ar.arrayName.accept( this )
        visitor ← new SemanticsVisitor( )
        call ar.accept( visitor )
    end

    procedure visit( StructReferencing sr )                                (66)
        visitor ← new SemanticsVisitor( )
        call sr.accept( visitor )
        if sr.type ≠ errorType
        then
            call sr.objectName.accept( this )
            st ← sr.objectName.type.fields
            attributeRef ← st.retrieveSymbol( fieldName.name )
            if not isAssignable( id.attributeRef )
            then call error( fieldName.name, "is not an assignable field" )
    end
end
```

Figure 8.34: Type checking visitors (Part 2)

lyze constructs expected to produce an R-value. This includes simple identifiers, literals, operators, and function calls. To analyze constructs expected to produce an L-value, we shall introduce a new specialized visitor class, *LHSSemanticsVisitor*. This visitor class will be defined for fewer AST nodes, since only a few constructs produce a memory location (a variable, an arry element or field reference). In most cases a VISIT method in *LHSSemanticsVisitor* will do the same analysis as a corresponding VISIT method in *SemanticsVisitor* *plus* a "bit more." The extra checking it will do is to verify that an assignable name (an L-value) is produced. Thus, in checking an *Identifier* as an L-value, we must check that it is properly declared and produces a valid type *plus* that it names a variable (or it can somehow be the target of an assignment).

Figures 8.33 and 8.34 outline the semantic processing visitors that handle each of the constructs discussed in this section. We only need VISIT methods to be defined in *LHSSemanticsVisitor* for a few constructs because the parser that builds the AST forbids many ill-formed trees. Thus we do not need a VISIT method for *IntLiteral* because an assignment like `1=a` is syntactically illegal. If there is any doubt about whether an illegal construct might appear as an L-value, a VISIT method for the corresponding AST node can be included in *LHSSemanticsVisitor* that warns that the construct may not be the target of an assignment.

8.8.1 Simple Identifiers and Literals

```
/⋆  Visitor code for Marker 59 on page 325                                ⋆/
procedure VISIT( Identifier id )
    id.type ← errorType
    id.attributeRef ← null
    attributeRef ← currentSymbolTable.RETRIEVESYMBOL( id.name )              67
    if attributeRef = null
    then  call ERROR( id.name, "has not been declared" )
    else
        if ISDATAOBJECT( attributeRef )                                       68
        then
            id.attributeRef ← attributeRef
            id.type ← id.attributeRef.variableType
        else   call ERROR( id.name, "does not name a data object" )
end
```

The VISIT method for *Identifer* uses the symbol table to discover the "meaning" of the identifier. It begins by setting the *type* field to *errorType* and the *attributesRef* field to null. These are default values, in case the *Identifer* is incorrectly defined or used. A call to RETREIVESYMBOL at Marker 67 then checks that the named identifer has been properly declared. It will return **null** as a result if the identifier is not found. Because the identifier is being used in a context where it must name a data object (which has a value), a call to ISDATAOBJECT (Marker 68) verifies that the identifier meets this requirement. (For example, an identifier declared as the name of a type would pass the first check but fail the second one.) Presuming the identifier does indeed name a data object, the type and attributes information of that data object are saved in the AST node. If either test fails, then the default *errorType* is retained.

The visitor for *Identifier* in *LHSSemanticVisitor* (Marker 64) first uses a *SemanticVisitor* instance to run the VISIT method defined in this section. In addition, it uses a method named ISASSIGNABLE to make sure that `id` names a data object that can be the target of assignment. For example, ISASSIGNABLE will return `false` if `id` is a constant. It also includes a check for `id.attributeRef` being **null** because `id` is not defined or does not name a data object and will return `false` in that case, too.

The VISIT method for a literal node is trivial, since the type is immediately available. The pseudocode is also included in Figure 8.33, at Marker (60). There is no corresponding VISIT method in *LHSSemanticVisitor* because we expect the parser to forbid interger literals in contexts where an L-value is expected. The case where literals may incorrectly appear as L-values is explored in Exercise 26.

We develop the type checking visitors for more complex names, such as record field and array element references, in Section 8.8.4. The processing that is done for an assignment is designed to completely ignore the complexity of the subtree that specifies the target of the assignment, just as it need not consider the complexity of the computation that specifies the source value.

8.8.2 Assignment Statements

```
/⋆   Visitor code for Marker (61) on page 325                          ⋆/
procedure VISIT( Assigning assign )
    lhsVisitor ← new LHSSemanticsVisitor( )                            (69)
    call assign.targetName.ACCEPT( lhsVisitor )
    call assign.valueExpr.ACCEPT( this )                               (70)
    if ASSIGNABLE( assign.valueExpr.type, assign.targetName.type )     (71)
    then  assign.type ← assign.targetName.type
    else
        call ERROR( "Right hand side expression not assignable to left hand side name at", assign )
        assign.type ← errorType
end
```

The major task of the VISIT method for an *Assigning* node involves obtaining the types of the components of the assignment statement and checking whether they are compatible according to the rules of the language. We create and use an *LHSSemanticsVisitor* at Marker (69) to get the target's type and verify that it may be the target of an assignment. We use the normal *SemanticsVisitor* at Marker (70) to check the type of the right-hand side. The test at Marker (71) determines whether the type of the value on the right-hand side of the assignment can be assigned to the variable on the left. The type field of the *Assigning* node itself is then set appropriately, with *errorType* being used if the assignability test fails.

8.8.3 Checking Expressions

At the beginning of our discussion of type checking, we saw two simple examples of the general concept of an expression, namely, a single identifier and a literal constant. More complex expressions are constructed using unary

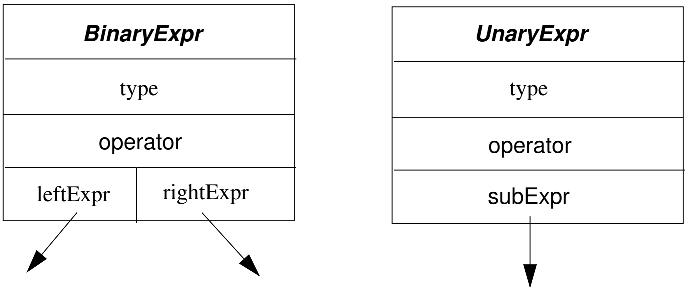


Figure 8.35: Abstract Syntax Tree Representations for Unary and Binary Expressions

and binary operators, as represented by at ASTs shown in Figure 8.35. The nodes referenced by *leftExpr*, *rightExpr*, and *subExpr* in the figure can reference any kind of expression subtree. Whether simple or complex, they are analyzed by the appropriate semantic visitor.

visit methods for binary and unary operators are defined in Figure 8.33 at Marker 62 and Marker 63. Each of the methods starts by propagating the visitor traversal to the operand expressions, after which only the types they produce are of interest.

The type checking done by these visit methods depends on the operator and the types of the operands. The pseudocode for the visitors uses the functions binaryResultType and unaryResultType to examine the operator and operands to determine whether they describe a legal operation according to the definition of the language being compiled. If the specified operation is meaningful, then the type of the result of the operation is returned; otherwise, the result of the call must be *errorType*. Consider a few examples. Adding integers is allowed in all programming languages, with the result being an integer. Addition of an integer and a float will produce a float in most languages, but in a language that does not allow implicit type conversions, such an expression is erroneous. Typically, comparision of two arithmetic values yields a Boolean result.

8.8.4 Checking Complex Names

We now examine the type checking that must be done when references to elements of arrays or fields of structs are analyzed. The AST representations of array and struct references are illustrated in Figure 8.36. These trees are actually simplified a bit, since *objectName* in a *StructReferencing* node and *arrayName* in an *ArrayReferencing* node need not always be *Identifier* nodes.

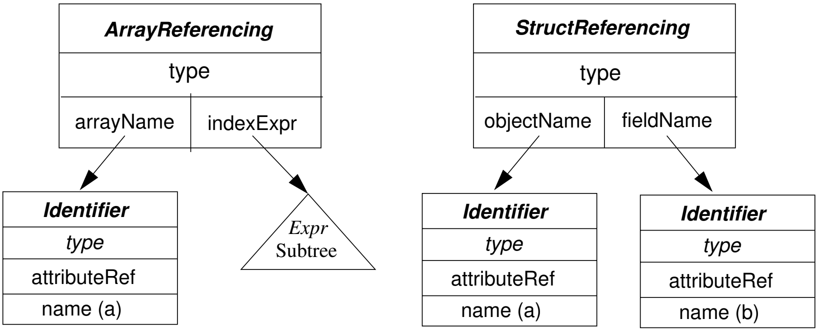


Figure 8.36: Abstract Syntax Trees for Array and Struct References

More generally, they reference subtrees that correspond to a data object, with an *Identifier* node a simple, but common, case.

The visit method for an *ArrayReferencing* AST node is found in Figure 8.37. The method begins with a call to visitChildren. As noted above, *arrayName* can reference an arbitrarily complex subtree, just as the *indexExpr* subtree can be any expression. For our type checking purposes, we are only interested in the *type* field in the root node of each of these subtrees, which is set by the visits initiated by visitChildren.

The test at Marker 73 checks whether the *arrayName* subtree does designate an array. The check at Marker 74 checks the type of the *indexExpr* subtree. This check is designed for languages that require arrays to be indexed by integers (most do). A more complex check must be added if the language being implemented allows enumerated types as array indices.

The *LHSSemanticsVisitor* visit method for arrays is shown at Marker 65. It visits its *arrayName* subtree to verify that the array is assignable. (Many languages allow arrays to be declared `const` or `final`.) It then calls the corresponding visit method in *SemanticsVisitor* to do the rest of the semantic analysis.

The pseudocode for the visit method for a *StructReferencing* AST node is found in Figure 8.38. The method does not begin with the usual call to visitChildren. Rather, we only traverse the AST referenced by the *objectName* field. The reason for this departure from the usual practice is seen at Marker 79, where the *fieldName* is interpreted in the symbol table context provided by the *objectName* subtree. Prior to doing so, the method checks whether this subtree evaluation has resulted in an error (Marker 77) and then ensures that it does indeed name a structure (Marker 78). Following our convention for error reporting, no error message is necessary in the first instance, while failure of the local check at Marker 78 does produce a message appropriate for the

```
procedure VISIT( ArrayReferencing ar )
    call VISITCHILDREN(ar)
    if ar.arrayName.type = errorType                                        (72)
    then  ar.type ← errorType
    else
        if ar.arrayName.kind ≠ arrayTypeDescriptor                          (73)
        then
            call ERROR(ar.arrayName, "is not an array")
            ar.type ← errorType
        else
        ar.type ← ar.arrayName.type.elementType
    if ar.indexExpr.type ≠ errorType and ar.indexExpr.type ≠ integer        (74)
    then
        call ERROR("Index expression is not an integer : ", ar.indexExpr)   (75)
end
```

Figure 8.37: Type checking array references

```
procedure VISIT( StructReferencing sr )
    call sr.objectName.ACCEPT( this )                                       (76)
    if sr.objectName.type = errorType                                       (77)
    then  sr.type ← errorType
    else
        if sr.objectName.type ≠ structTypeDescriptor                        (78)
        then
            call ERROR(sr.objectName, "does not name a struct.")
            sr.type ← errorType
        else
            st ← sr.objectName.type.fields                                  (79)
            attributeRef ← st.RETRIEVESYMBOL( fieldName.name )
            if attributeRef = null
            then
                call ERROR( fieldName.name, "is not a field of", sr.objectName. )
                sr.type ← errorType
            else   sr.type ← attributeRef.fieldType
end
```

Figure 8.38: Type checking structure references

error recognized. Finally, beginning at Marker 79, the meaning of *fieldName* is retrieved from the structure's symbol table and its type is returned as the type of this reference. As would be expected, an appropriate error message is produced if the name is not found.

The *LHSSemanticsVisitor* visit method for structs is shown at Marker 66. It first calls the normal semantic visitor to verify that a valid struct and field are present. If so, it then does two additional checks. First, it visits the *objectName* subtree to verify that the struct is assignable. It then looks up the *fieldName*'s attributes and checks that the field is assignable (to cover the case in which an individual field may be marked as `const` or `final`).

Complex Name Example

Figure 8.39 illustrates the AST that would be used to represent the name `s.a[i+1].f`. This name includes a structure name, a field name, which names an array of structures, and the name of a field of one of the array elements. Thus Figure 8.39 includes two *StructReferencing* nodes (nodes 1 and 4), and one *ArrayReferencing* node (node 2). When the visit method for node 1 in *SemanticsVisitor* is invoked to process this name, the type checking process will proceed through the following steps:

1. Through the call to the accept method at Marker 76 in Figure 8.38, the type checking traversal immediately moves to node 2.

2. The visit method for an *ArrayReferencing* node in Figure 8.37 begins with a call to visitChildren, which first invokes the accept method for node 4. (Node 5 will be visited later as a result of this call to visitChildren.)

3. Once again, the *StructReferencing* visit method uses the call to the accept method at Marker 76 to propagate the traversal on to the node referenced by its *objectName* field, node 6 in this case.

4. The meaning of name `s` is retrieved from the symbol table. Presuming that `s` names a data object, its type will be set as the value of the *type* field on node 6. Control will then return to the visit method for node 4.

5. The code at Marker 78 in Figure 8.38 checks that the type just retrieved for `s` is a structure. Presuming that it is, the identifier `a` from the *name* field in node 7 will be looked up in the symbol table associated with struct `s`. The type retrieved for `a` (an array) will be set as the value of the *type* field of node 4. Control will then return to the visit method for node 2.

   Note that the type checking traversal was not propagated to node 7. If that had been done, then `a` would have been looked up in the symbol table for the current scope rather than the one associated with structure `s`.

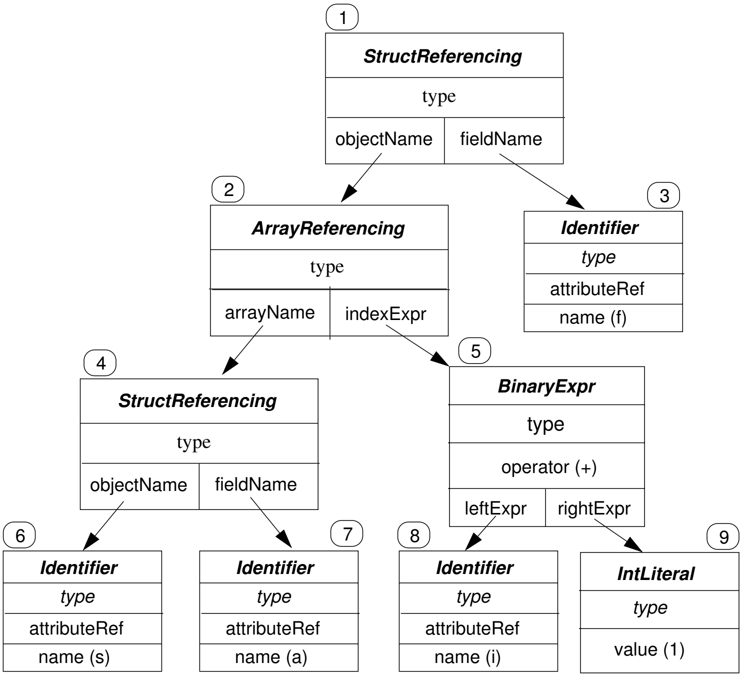


Figure 8.39: AST for Array and Struct Example

6. The second iteration within the execution of VISITCHILDREN for node 2 invokes the ACCEPT method for node 5.

7. The VISIT method at Marker (62) in Figure 8.33 is executed for node 5.

8. This method begins with a call to VISITCHILDREN, which first invokes the ACCEPT method for node 8.

9. The meaning of name `i` is retrieved from the symbol table. Our assumption in this example is that it is an integer, so *integer* is set as the value of the *type* field on node 8. Control returns to the visitor for node 5.

10. VISITCHILDREN then invokes the ACCEPT method for node 9. The VISIT method at Marker (60) in Figure 8.33 immediately assigns *integer* as the value of the *type* field in node 9 and then returns.

11. The call to BINARYRESULTTYPE in the VISIT method for the *BinaryExpr* node will result in *integer* being assigned as the value of the *type* field in node 5. Control then returns to the visitor for node 2.

12. After checking that the type of node 4 is an array and verifying that the type of the index expression represented by node 5 is indeed *integer* (Marker 75 in Figure 8.37), this visitor sets the *type* field of node 2 to the *elementType* field of type of node 4. Control then returns to the visitor for node 1.

13. The code at Marker 78 in Figure 8.38 checks that the type just set for node 2 designates a structure. Presuming that it does, the identifier `f` from the *name* field in node 3 is looked up in the symbol table associated with the struct type of node 2. The type retrieved for `f` will be set as the value of the *type* field of node 1 and control will then return to the visitor that invoked the traversal on node 1.

While this may seem like a rather complex process, it is important to understand that it enables the code for each visit method to deal with local information only. (Reaching down for field names is a special case, where the *fieldName* subtree can only be an *Identifier* node.) Names composed in arbitrarily complex ways can always be handled with the simple steps defined for each kind of AST node. In particular, the type checking done within each node never depends on the structure of the AST below or surrounding the node being processed, only on local information and the *type* fields of subtrees.

8.9 Summary

Although the interface for a symbol table is quite simple, the details underlying a symbol table's implementation play a significant role in the performance of the symbol table. Most modern programming languages are statically scoped. The symbol table organization presented in this chapter efficiently represents scope-relative symbols in a block-structured language. Each language places its own requirements on how symbols can be declared and used. Most languages include rules for symbol promotion to a global scope. Issues such as inheritance, overloading, and aggregate data types should be considered when designing a symbol table. A symbol table is used to associate names with a variety of information, generally referred to as atttributes; types are a common, distinguished kind of attribute. Types and other attributes are represented by complex, customized data structures designed based on language characteristics to store the appropriate descriptive information.

This chapter made extensive use of the visitor pattern first introduced in Chapter 7 to process an abstract syntax tree. visit methods associated with nodes of the tree representing declarations build the symbol table and the structures associated with names declared in the program being compiled. This same mechanism also accomplishes type checking during declaration processing and when applied to the AST nodes representing the executable parts of the program.

In the design of the VISIT methods, two goals were paramount. The first was that the operation of each method must be based on only the AST node with which it is associated and information supplied by processing the subtrees it references, while avoiding any dependency on the structure of any surrounding parts of the AST. The second principle was that checking must produce clear error messages so that a programmer can easily identify the program error that produced any particular message. The notion of using an `errorType` to deal with type errors was presented. Use of this technique in a consistent way throughout the VISIT methods is an effective way to minimize the number of error messages generated by the compiler, thus highlighting problems more precisely for the programmer.

Exercises

1. The two data structures most commonly used to implement symbol tables in production compilers are binary search trees and hash tables. What are the advantages and disadvantages of using each of these data structures for symbol tables?

2. Consider a program in which the variable is declared as a method's parameter and as one of the method's local variables. A programming language includes parameter hiding if the local variable's declaration can mask the parameter's declaration. Otherwise, the situation described in this exercise results in a multiply defined symbol. With regard to the symbol table interface presented in Section 8.1.2, explain the implicit scope-changing actions that must be taken if the language calls for

   (a) parameter hiding

   (b) no parameter hiding

3. Describe two alternative approaches to handling multiple scopes in a symbol table, and list the actions required to open and close a scope for each alternative. Trace the sequence of actions that would be performed for each alternative during compilation of the program in Figure 8.1.

4. Figure 8.7 provides code to create a single symbol table for all scopes. Recalling the discussion of Section 8.2.2, an alternative approach segregates symbols by scope, as shown in Figure 8.3. Modify the code of Figure 8.7 to manage a stack of symbol tables, one for each active scope. Retain the symbol table interface as defined in Section 8.1.2.

5. Recalling the discussion of Section 8.3.1, suppose that all active names are contained in a single, ordered list. An identifier would appear *k* times in the list if there are currently *k* active scopes that declare the identifier.

   (a) How would you implement the methods defined in Section 8.1.2 so that RETRIEVESYMBOL finds the appropriate declaration of a given identifier?

   (b) Explain why the lookup time does or does not remain *O*(*log n*) for a list of *n* entries.

6. Design and implement a symbol table using the ordered list data structure suggested in Section 8.3.1. Discuss the time required to perform each of the methods defined in Section 8.1.2.

7. Extend the symbol table implementation of Figure 8.7 to include an implementation of declaredLocally(*name*). Recall that declaredLocally tests whether *name* is present in the symbol table's current (innermost) scope. If it is, true is returned. If *name* is in an outer scope, or is not in the symbol table at all, false is returned.

8. Program a symbol table manager using the interface described in Section 8.1.2 and the implementation described in Section Section 8.3.3.

9. Program a symbol table manager using the interface described in Section 8.1.2. Maintain the symbol table using red-black trees. Describe the performance characteristics of this approach.

10. Program a symbol table manager using the interface described in Section 8.1.2. Maintain the symbol table using splay trees. Describe the performance characteristics of this approach.

11. Each string in Figure 8.5 occupies its own space in the string buffer. Suppose the strings were added in the following order: *i*, *input*, and *putter*. If the strings could share common characters, then these three strings could be represented using only 8 character slots. Design a symbol table string-space manager that allows strings to overlap, retaining the representation of each string as an offset and length pair, as described in Section 8.3.2.

12. Some languages allow special access to names that are actually suffixes of complex names. In Java, the classes in the package *java.lang* are available, so that the *java.lang.Integer* class can be referenced simply as *Integer*. This feature is also available for any explicitly imported classes. Similarly, Pascal's with statement allows field names to be abbreviated in the applicable blocks. Design a name space that supports the efficient retrieval of such abbreviated names, under the specified conditions. Be sure to document any changes you wish to make to the symbol table interface given in Section 8.1.2.

13. As discussed in Section 8.4.1, some languages allow a series of field references to be abbreviated, providing the abbreviation can uniquely locate the desired field. Certainly, the last field of such a reference must appear in the abbreviation. Moreover, the first field is necessary to distinguish the reference from other instances of the same type. We therefore assume that the first and last fields must appear in an abbreviation.

    As an example, the reference `a.b.c.d` could be abbreviated `a.d` if records `a` and `b` do not contain their own `d` field.

    Design an algorithm to allow such abbreviations, taking into consideration that the first and last fields of the reference cannot be omitted. Integrate your solution into the implementation of retrieveSymbol in Section 8.3.3.

14. Consider the following C program:

```
int func() {
   int x, y;
   x = 10;
   {
      int x;
      x = 20;
      y = x;
   }
   return(x * y);
}
```

    The identifier `x` is declared in the method's outer scope. Another declaration occurs within the nested scope that assigns `y`. In C, the nested declaration of *x* could be regarded as a declaration of some other name, provided that the inner scope's reference to is appropriately renamed `x`. Design a set of AST visitor methods that

    - Renames and moves nested variable declarations to the method's outermost scope
    - Appropriately renames symbol references to preserve the meaning of the program

15. Using the symbol table interface given in Section 8.1.2, describe how to implement structures (`structs` in C) under each of the following assumptions:

    - All structures and fields are entered in a single symbol table.
    - A structure is represented by its own symbol table, whose contents are the structure's subfields.

16. As mentioned in Section 8.4.2, C allows the same identifier to appear as a *struct* name, a label, and an ordinary variable name. Thus, the following is a valid C program:

    ```
    main() {
        struct xxx {
            int a,b;
        } c;
        int xxx;

        xxx:
            c.a = 1;
    }
    ```

    In C, the structure name never appears without the terminal *struct* preceding it. The label can only be used as the target of a goto statement. Explain how to use the symbol table interface given in Section 8.1.2 to allow all three varieties of *xxx* to coexist in the same scope.

17. Describe how you would use the symbol table interface given in Section 8.1.2 to localize the declaration and effects of a loop iteration variable. As an example, consider the variable in the Java-like statement

    ```
    for (int i=1; i<10; ++i)  { . . . }
    ```

    In this exercise, we seek the following:

    - The declaration of *i* cannot possibly conflict with any other declaration of *i*.
    - The effects on this *i* are confined to the body of the loop. That is, the scope of *i* includes the expressions of the for statement as well as its body, represented above as ... . The value of the loop's iteration variable is undefined when the loop exits.

18. Write a VISIT method for handling the array type definitions represented by the AST in Figure 8.19(b), with upper and lower bounds included in array specification. Note that some type checking on the bounds expressions will be required in this method.

19. Consider the following program fragment:

```
typedef
    struct {
        int x,y;
    }  *Pair;

Pair *(pairs[23]);
```

The `typedef` establishes `Pair` as a typename, defined as a pointer to a record of two integers. The declaration for `pairs` uses the typename, but adds one more level of indirection to an array of 23 `Pair`s. Describe how to implement `typedef` in C using the techniques presented in Section 8.6. Your design for `typedef` must be accommodate further type construction using `typedef`s.

20. Draw a diagram of the symbol table entries, attribute descriptors, and type descriptors that would result from processing the declarations in Exercise 19.

21. Extend the visit method in Section 8.7.1 to handle the feature in Java that allows a list of interfaces implemented as part of a class declaration (as described at the end of that section). Interface declarations themselves are similar to class declarations. Write a visit method for processing interface declarations.

22. Write the visit method that is applied to a list of *paramDeclaring* nodes, as described in Section 8.7.2.

23. Section 8.7.2 presented the need for an incorporate method as part of a symbol table interface in order to add the names in a parent class and its ancestors to the symbol table of a class. Propose two possible implementations of retrieveSymbol in a symbol table that includes incorporate in its interface and explain their relative advantages and disadvantages. How will your implementations and analysis have to change to handle multiple inheritance?

24. Draw a diagram of the symbol table entries, attribute descriptors and type descriptors that would result from processing the declaration of *LHSSemanticVisitor* in Figure 8.34.

25. Presuming that Java is the language being compiled, outline the implementation of isAsignable(*dataObect*), as used in the visit methods of *LHSSemanticVisitor* in Figure 8.34.

26. Recall from Section 8.8 that special checking must be done for names that are used on the left-hand side of an assignment. The visitor class *LHSSemanticVisitor* was introduced for this purpose. Section 8.8.1 noted that since a parser will not allow a literal to appear as the target of an assignment, no visit method for handling literals was included in *LHSSemanticVisitor*.

    - For a language of your choice, identify any other contexts in a program where an L-value is required.
    - Do the syntactic rules of the language guarantee that a literal can not appear in these contexts?
    - Write appropriate *LHSSemanticVisitor* visit methods for the AST nodes corresponding to any literals allowed by the language.

27. For the language of your choice, outline the implementation of the method assignable(*valueType*, *targetType*), as introduced in the visit method for an *Assigning* AST node in Section 8.8.2.

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9

Semantic Analysis

9.1 Semantic Analysis for Control Structures

Control structures are an essential component of all programming languages. With control structures a programmer can combine individual statements and expressions to form an unlimited number of specialized program constructs.

Some languages constructs, such as the `if`, `switch`, and `case` statements, provide for conditional execution of selected statements. Others, such as `while`, `do`, and `for` loops, provide for iteration (or repetition) of the body of a looping construct. Still other statements, such as the `break`, `continue`, `throw`, `return`, and `goto` statements force program execution to depart from normal sequential execution of statements.

The exact form of control structures differs in various programming languages. For example, in C, C++, C♯, and Java$^{\text{TM}}$, `if` statements do not use a `then` keyword; in Pascal, ML, Ada, and many other languages, `then` is required.

Minor syntactic differences are unimportant when semantic analysis is performed by a compiler. We can ignore syntactic differences by analyzing an abstract syntax tree (Section 7.4 on page 250). Recall that an abstract syntax tree (AST) represents the essential structure of a construct while hiding minor variations in source level representation. Using an AST, we can discuss the semantic analysis of fundamental constructs such as conditional and looping statements without any concern about exactly how they are represented at the source level.

A common way to organize the semantic analysis of a program is to create a number of **semantic analysis methods**, one for each kind of AST node. To understand the semantic analysis of a particular kind of construct (an expression, or assignment, or call) you need merely examine the corresponding AST node's semantic analysis method.

Using the visitor approach we developed in Chapters 7 and 8, we can factor all our semantic analysis into a number of specialized visitors, each implementing a portion of the overall semantic analysis task. In this chapter we shall focus on three aspects of semantic analysis: *type correctness*, *reachability and termination*, and *exceptions*.

Type correctness is the essence of semantic analysis. We visit an AST node and its children to verify that the types of all components conform to programming language rules. Thus, in an if statement, the control expression must be semantically valid and return a Boolean value. Because other structures may have similar (or even identical) type rules, the visitor classes that implement type correctness may share methods, leading to simpler and more reliable analysis.

Reachability and termination analysis, as discussed below (Section 9.1.1), determines whether a construct terminates normally (many do not!) and whether a construct can be reached during execution. Java and C♯ require this semantic analysis. Even languages such as C and C++, for which this analysis is optional, benefit from enhanced error analysis.

Almost all modern programming languages include some form of exception handling. In analyzing expressions and statements, we must be aware of the fact that constructs may throw an exception rather than terminate normally. Java requires accounting for all checked exceptions. That is, if a checked exception is thrown (see Section 9.1.7), then it *must* either be caught in a catch block or listed in a method's throw list.

To enforce this rule, we will accumulate the checked exceptions that can be generated by a given construct. Each AST node that contains an expression or statement will have a *throwsSet* field. This field will reference a set of exception types. The *throwsSet* will be propagated as ASTs are analyzed. It will be used when catch blocks, methods, and constructors are analyzed.

We will organize semantic analysis using the following visitors:

SemanticsVisitor is used to check that the type rules imposed on language constructs are satisfied. This includes checking that control expressions are Boolean-valued, that parameters have correct types in calls, that expected types are returned from calls, and so forth. This analysis, often called **static semantics**, is a necessary part of all compilers.

ReachabilityVisitor is a specialized visitor used to analyze control structures for reachability and proper termination. These visitors set two flags, *isReachable* and *terminatesNormally*, used for error analysis and optional code optimization.

ThrowsVisitor is a specialized visitor used to collect information about throws that may ″escape″ from a given construct. These visitors compute the *throwsSet* field, which records exceptions that may be thrown.

9.1.1 Reachability and Termination Analysis

An important issue in analyzing Java control structures is **reachability**. Java requires that unreachable statements be detected during semantic analysis with suitable error messages generated. For example, in the statement sequence:

```
...; return; a=a+1; ...
```

the assignment statement must be marked as unreachable during semantic analysis.

Reachability analysis is *conservative*. In general, determining whether a given statement can be reached is very difficult. In fact, it is impossible! Computer science theorists have proven that it is undecidable whether a given statement is ever executed, even when we know in advance all of the data that a program will access (reachability is a variant of the famous **halting problem** first discussed in [Tur36]).

Because our analyses will be conservative, we will not detect all occurrences of unreachable statements. However, the statements we recognize as unreachable will definitely be erroneous, so our analysis will certainly be useful. In fact, even in languages like C and C++, which do not require reachability analysis, we can still produce useful warnings about unreachable statements that may well be erroneous.

To detect unreachable statements during semantic analysis, we will add two Boolean-valued fields to the ASTs that represent statements and statement lists. The first, *isReachable*, marks whether a statement or statement list is considered reachable. We will issue an error message for any non-null statement or statement list for which *isReachable* is false.

The second field, *terminatesNormally*, marks whether a given statement or statement list is expected to terminate normally. A statement that terminates normally will continue execution ″normally″ with the next statement that follows. Some statements (such as a break, continue, or return) may force execution to proceed to a statement other than the normal successor statement. These statements are marked with *terminatesNormally* set to false. Similarly, a loop may never terminate its iteration (e.g., `for(;;) {a=a+1;}`). Loops that do not terminate (using a conservative analysis) also have their *terminatesNormally* flag set to false.

The *isReachable* and *terminatesNormally* fields are set according to the following rules:

- If *isReachable* is true for a statement list, then it is also true for the first statement in the list.
- If *terminatesNormally* is false for the last statement in a statement list, then it is also false for the whole statement list.
- The statement list that comprises the body of a method, constructor, or static initializer is always considered reachable (its *isReachable* value is true).
- A local variable declaration or an expression statement (assignment, method call, heap allocation, variable increment, or decrement) always has *terminatesNormally* set to true (even if the statement has *isReachable* set to false). (This is done so that all of the statements following an unreachable statement do not generate error messages.)
- A null statement or statement list never generates an error message if its *isReachable* field is false. Rather, the *isReachable* value is propagated to the statement's (or statement list's) successor.
- If a statement has a predecessor (it is not the first statement in a list), then its *isReachable* value is equal to its predecessor's *terminatesNormally* value. That is, a statement is reachable if and only if its predecessor terminates normally.

As an example, consider the following method body:

```
void example() {
   int v; v++; return; ; v=10; v=20; }
```

This method body is considered reachable, and thus, so is the declaration of v. This declaration and the increment of variable v complete normally, but the return does not (see Section 9.1.5). The null statement following the return is unreachable, and propagates this fact to the assignment of 10, which generates an error message. This assignment terminates normally, so its successor is considered reachable.

In the following sections we will study the semantic analysis of the control structures of Java. Included in this analysis will be whether or not the statement in question terminates normally. We will set the *terminatesNormally* value for

```
class NodeVisitor
    procedure visitChildren( $n$ )
        foreach $c \in n$.getChildren( ) do call $c$.accept( this )
    end
end

class SemanticsVisitor extends NodeVisitor
    procedure checkBoolean( $c$ )                                        ①
        if $c.type \neq Boolean$ and $c.type \neq errorType$
        then call error( "Require Boolean type at", $c$ )
    end

    procedure visit( IfTesting $ifn$ )                                   ②
        call visitChildren( $ifn$ )
        call checkBoolean( $ifn.condition$ )
    end

    procedure visit( WhileLooping $wn$ )                                 ③
        call visitChildren( $wn$ )
        call checkBoolean( $wn.condition$ )
    end

    procedure visit( DoWhileLooping $dwn$ )                              ④
        call visitChildren( $dwn$ )
        call checkBoolean( $dwn.condition$ )
    end

    procedure visit( ForLooping $fn$ )                                   ⑤
        call openScope( )
        call visitChildren( $fn$ )
        if $fn.condition \neq$ null
        then call checkBoolean( $fn.condition$ )
        call closeScope( )
    end

    procedure visit( LabeledStmt $ls$ )
        /★    Figure 9.11 on page 357                              ★/
    end
    procedure visit( Continuing $cn$ )
        /★    Figure 9.12 on page 358                              ★/
    end
    procedure visit( Breaking $bn$ )
        /★    Figure 9.15 on page 360                              ★/
    end
    procedure visit( Returning $rn$ )
        /★    Figure 9.18 on page 362                              ★/
    end
end
```

Figure 9.1: Semantic Analysis Visitors (Part 1)

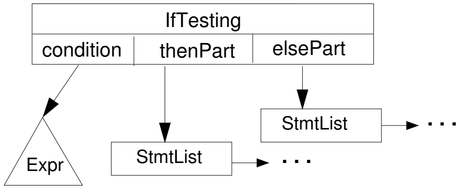


Figure 9.2: Abstract Syntax Tree for an If Statement

each kind of control statement, and from these successor statements we will set their *isReachable* field.

Interestingly, although we expect most statements and statement lists to terminate normally, in functions (methods that return a non-void value) we *require* the method body to terminate abnormally. Because a function must return a value, it cannot return by "falling though" to the end of the function. It must execute a return of some value or throw an exception. These both terminate abnormally, so the method body must also terminate abnormally. After the body of a function is analyzed, the *terminatesNormally* value of the statement list comprising the body is checked; if it is not false, an error message ("Function body must exit with a return or throw statement") is generated.

9.1.2 If Statements

The AST corresponding to an if statement is shown in Figure 9.2. An *IfTesting* node has three subtrees corresponding to the condition controlling the if, the then statements, and the else statements. The semantic rules for an if statement are simple—the condition must be a valid Boolean-valued expression, and the then and else statements must be semantically valid. It may happen that the condition itself contains a semantic error (e.g., an undeclared identifier or an invalid expression). In this case, semantic analysis returns the special type *errorType*, which indicates that further analysis of the condition is unnecessary (since we have already marked the condition as erroneous). Any type other than Boolean or *errorType* causes an error message to be produced. This analysis pattern is common enough that we create a method CHECKBOOLEAN (Figure 9.1, Marker ①) to implement it. Now the semantic analysis visitor for if statements becomes trivial—a call to VISITCHILDREN to verify type correctness of the condition, then and else parts followed by a call to CHECKBOOLEAN to verify that the condition is Boolean-valued. (Figure 9.1, Marker ②). Recall that if an if statement has no else part, the *elsePart* AST is null, which is trivially correct.

For purposes of reachability analysis, we assume that the condition controlling the if can evaluate to either true or false. (The special case in which

```
class ReachabilityVisitor extends NodeVisitor
    procedure visit( IfTesting ifn )                                  ⑥
        ifn.thenPart.isReachable ← true
        ifn.elsePart.isReachable ← true
        call visitChildren(ifn)
        thenNormal ← ifn.thenPart.terminatesNormally
        elseNormal ← ifn.elsePart.terminatesNormally
        ifn.terminatesNormally ← thenNormal or elseNormal
    end

    procedure visit( WhileLooping wn )
        /⋆   Figure 9.6 on page 352                              ⋆/
    end
    procedure visit( DoWhileLooping dwn )
        /⋆   Figure 9.7 on page 354                              ⋆/
    end
    procedure visit( ForLooping fn )
        /⋆   Figure 9.8 on page 354                              ⋆/
    end
    procedure visit( LabeledStmt ls )                                 ⑦
        ls.stmt.isReachable ← ls.isReachable
        call visitChildren(ls)
        ls.terminatesNormally ← ls.stmt.terminatesNormally
    end

    procedure visit( Continuing cn )                                  ⑧
        cn.terminatesNormally ← false
    end

    procedure visit( Breaking fn )
        /⋆   Figure 9.16 on page 360                             ⋆/
    end
    procedure visit( Returning rn )
        rn.terminatesNormally ← false
    end
end
```

Figure 9.3: Reachability Analysis Visitors (Part 1)

the condition can be resolved at compile-time is explored in Exercise 1.) Thus, the if and then parts are both marked as reachable. An if statement terminates normally if either its then part or its else part terminates normally. Since null statements terminate trivially, an if-then statement (with a null else part) always terminates normally. This analysis is detailed in Figure 9.3, Marker ⑥.

Since we assume all components of an if statement are reachable, any exception thrown in a subtree of an *IfTesting* node can "escape" from the if. We create a method gatherThrows (Figure 9.4, Marker ⑨) that visits all subtrees of an AST node and returns the union of the *throwsSet* found in each subtree. Throws analysis for an if is simply a call to gatherThrows (Figure 9.4, Marker ⑩).

As an example, consider the following statement:

```
if (b) a=1; else a=2;
```

Semantic analysis first checks the condition expression, `b`, which must produce a Boolean value. Then, the then and else parts are checked; these must be valid statements. Since assignment statements always complete normally, so does the if statement.

The modularity of our AST formulation is apparent here. The semantic checks applied to the control expression `b` are the same as the checks used for all expressions. Similarly, the semantic checks applied to the then and else statements are those applied to all statements. Nested if statements cause no difficulties; the same semantic checks are applied each time an *IfTesting* node is encountered.

9.1.3 While, Do, and Repeat Loops

The AST corresponding to a while statement is shown in Figure 9.5. A *WhileLooping* node has two subtrees corresponding to the condition controlling the loop and the loop body. The semantic rules for a while statement are identical to those of an if statement—the condition must be a valid Boolean-valued expression and the loop body must be semantically valid. Semantic analysis is implemented by calls to visitChildren and checkBoolean (Figure 9.1, Marker ③).

The reachability visitor for while loops is shown in Figure 9.6. Because do-forever loops are common, this analysis must consider the special case where the control expression of the loop is a constant. If the control expression is false, then the statement list comprising the loop body is marked as unreachable (Marker ㉓). If the control expression is true, the while loop is marked as abnormally terminating because it is an infinite loop (Marker ㉒). It may be that the loop body contains a reachable break statement. If this is the case,

```
class ThrowsVisitor extends NodeVisitor
    procedure gatherThrows( $n$ )                                          ⑨
        call visitChildren( $n$ )
        $ans \leftarrow \emptyset$
        foreach $c \in n$.getChildren( ) do $ans \leftarrow ans \cup c.throwsSet$
        $n.throwsSet \leftarrow ans$
    end
    procedure visit( IfTesting $ifn$ )                                     ⑩
        call gatherThrows( $ifn$ )
    end
    procedure visit( WhileLooping $wn$ )                                   ⑪
        call gatherThrows( $wn$ )
    end
    procedure visit( DoWhileLooping $dwn$ )                                ⑫
        call gatherThrows( $dwn$ )
    end
    procedure visit( ForLooping $fn$ )                                     ⑬
        call gatherThrows( $fn$ )
    end
    procedure visit( LabeledStmt $ls$ )                                    ⑭
        call gatherThrows( $ls$ )
    end
    procedure visit( Continuing $cn$ )                                     ⑮
        $cn.throwsSet \leftarrow \emptyset$
    end
    procedure visit( Breaking $bn$ )                                       ⑯
        $bn.throwsSet \leftarrow \emptyset$
    end
    procedure visit( Returning $rn$ )                                      ⑰
        call gatherThrows( $rn$ )
    end
    procedure visit( Switching $sn$ )                                      ⑱
        call gatherThrows( $sn$ )
    end
    procedure visit( CaseItem $cn$ )                                       ⑲
        call gatherThrows( $cn$ )
    end
    procedure visit( LabelList $lln$ )                                     ⑳
        /⋆    Constant-valued expressions cannot throw exceptions        ⋆/
        $lln.throwsSet \leftarrow \emptyset$
    end
end
```

Figure 9.4: Throws Analysis Visitors (Part 1)

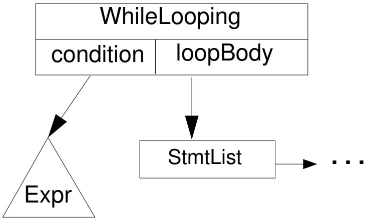


Figure 9.5: Abstract Syntax Tree for a While Statement

```
procedure VISIT( WhileLooping wn )
    wn.terminatesNormally ← true                                   ㉑
    wn.loopBody.isReachable ← true
    constExprVisitor ← new ConstExprVisitor( )
    call wn.condition.ACCEPT( constExprVisitor )
    conditionValue ← wn.condition.exprValue
    if conditionValue = true
    then
        wn.terminatesNormally ← false                              ㉒
    else
        if conditionValue = false
        then
            wn.loopBody.isReachable ← false                        ㉓
    call wn.loopBody.ACCEPT( this )                                ㉔
end
```

Figure 9.6: Reachability Analysis for a While Statement

semantic processing of the break will reset the loop's *terminatesNormally* field to true (Marker ㉔). If the control expression is non-constant, the loop is marked as terminating normally (Marker ㉑). We will assume that we have available a visitor class *ConstExprVisitor* whose methods traverse an expression AST to determine whether it represents a constant-valued expression. If the AST is recognized as a constant expression, then the visitors set the field *exprValue* to the expression value; otherwise *exprValue* is set to null. *ConstExprVisitor* can be very simple, perhaps only evaluating expressions whose operands are AST nodes for literals. With a bit more effort, the symbol table entry for identifiers can be checked, looking for declared constants. As discussed in Section 14.6 on page 623, *constant propagation* can recognize variables that, because of flow of control, must contain a known constant value.

The exceptions potentially thrown in a while loop are those generated by the loop control condition and the loop body. Throws analysis again uses gatherThrows to gather and return each subtree's *throwsSet* (Figure 9.4, Marker ⑪).

As an example, consider the following statement:

```
while (i >= 0) {
   a[i--] = 0; }
```

The control expression, `i >= 0`, is first checked to see if it is a valid Boolean-valued expression. Then, the loop body is checked for possible semantic errors. Since the control expression is non-constant, the loop body is assumed to be reachable and the loop is marked as terminating normally.

Do-While and Repeat Loops

Java, C, and C++ contain a variant of the while loop, the do-while loop. A do-while loop is simply a while loop that evaluates and tests its termination condition after executing the loop body rather than before. Assuming the same AST structure as a while loop, the semantic analysis visitor (Figure 9.1, Marker ④) and throws analysis visitor (Figure 9.4, Marker ⑫) are identical to those of the while loop.

The reachability visitor for a do-while is shown in Figure 9.7. Reachability rules for a do-while loop differ from those of a while loop. Since the loop body always executes at least once, the special case of a false control expression can be ignored. Initially, *terminatesNormally* is set to false (Marker ㉕). It can be reset to true during reachability analysis of the loop body (Marker ㉖) if the body contains a reachable break statement. For non-constant loop control expressions, a do-while loop terminates normally if the loop body does (Marker ㉗).

A number of languages, including Pascal and Modula-3, contain a repeat-until loop. This is essentially a do while loop except for the fact that the loop is terminated when the control condition becomes true rather than false. The semantic analysis of a repeat-until loop is almost identical to that of a do while loop. The only change is that the special case of a non-terminating loop occurs when the control expression is false rather than true.

9.1.4 For Loops

For loops are normally used to step an index variable through a range of values. However, for loops in C, C++, C♯, and Java are really a generalization of while loops. Consider the AST for a for loop, as shown in Figure 9.9.

```
procedure VISIT( DoWhileLooping dwn )
    dwn.loopBody.isReachable ← true
    dwn.terminatesNormally ← false                                        (25)
    call dwn.loopBody.ACCEPT( this )                                      (26)
    constExprVisitor ← new ConstExprVisitor( )
    call dwn.condition.ACCEPT( constExprVisitor )
    conditionValue ← dwn.condition.exprValue
    if conditionValue ≠ true                                              (27)
    then
        bodyNormal ← dwn.loopBody.terminatesNormally
        dwn.terminatesNormally ← dwn.terminatesNormally or bodyNormal
end
```

Figure 9.7: Reachability Analysis for a Do-While Statement

```
procedure VISIT( ForLooping fn )
    fn.terminatesNormally ← true
    fn.loopBody.isReachable ← true
    if fn.condition ≠ null
    then
        constExprVisitor ← new ConstExprVisitor( )
        call fn.condition.ACCEPT( constExprVisitor )
        conditionValue ← fn.condition.exprValue
        if conditionValue = true
        then  fn.terminatesNormally ← false
        else
            if conditionValue = false
            then  fn.loopBody.isReachable ← false
    else    fn.terminatesNormally ← false
    call fn.loopBody.ACCEPT( this )
end
```

Figure 9.8: Reachability Analysis for a For Loop

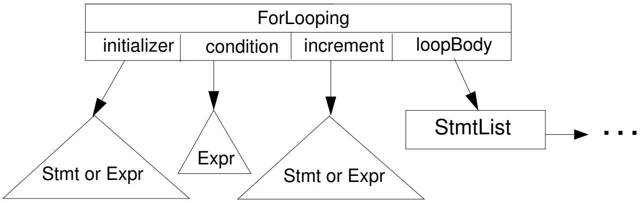


Figure 9.9: Abstract Syntax Tree for a For Loop

As was the case with while loops, the for loop's AST contains subtrees corresponding to the loop's termination condition (*condition*) and its body (*loopBody*). In addition, it contains ASTs corresponding to the loop's initialization (*initializer*) and its end-of-loop increment (*increment*).

There are a few differences that must be properly handled. Unlike the while loop, the for loop's termination condition is optional. (This allows do-forever loops of the form `for (;;) {...}`). In C++, C♯, and Java, an index local to the for loop may be declared, so a new symbol table name scope must be opened, and later closed, during semantic analysis.

The semantic analysis visitor is defined in Figure 9.1 at Marker ⑤. A new name scope is opened in case a loop index is declared in the *initializer* AST. Next, all subtrees are semantically analyzed using visitChildren. If *condition* is non-null, a call to checkBoolean verifies that *condition* is Boolean-valued. Finally, the name scope associated with the for loop is closed.

Reachability analysis, as defined in Figure 9.8, is very similar to that performed for while loops. A null termination condition or a constant control expression equal to true represent a non-terminating loop. In these cases, the for loop is marked as not terminating normally (though a break within the loop body may change this when the loop body is analyzed). A termination condition that is a constant expression equal to false causes the loop body to be marked as unreachable. If the control expression is non-null and non-constant, the loop is marked as terminating normally. The throws analysis visitor for for loops is shown in Figure 9.4 at Marker ⑬. Again, it is just a call to gatherThrows.

As an example, consider the following for loop:

```
for (int i=0; i < 10; i++)
   a[i] = 0;
```

First a new name scope is created for the loop. When the declaration of `i` in the *initializer* AST is processed, it is placed in this new scope (since all new declarations are placed in the innermost open scope). Thus, the references to i in the *condition*, *increment*, and *loopBody* ASTs properly reference the newly declared loop index i. Since the loop termination condition is Boolean-valued and non-constant, the loop is marked as terminating normally, and the loop body is considered reachable. At the end of semantic checking, the scope containing i is closed, guaranteeing that no subsequent references to the loop index are possible.

A number of older languages, including Fortran, Pascal, Ada, Modula-2, and Modula-3, contain a more restrictive form of for loop. Typically, a variable is identified as the "loop index." Initial and final index values are defined and sometimes an increment value is specified. For example, in Pascal a for loop is of the form:

```
for id := intialVal to finalVal do
   loopBody
```

The loop index, id, must already be declared and must be a scalar type (integer or enumeration). The initialVal and finalVal expressions must be semantically valid and have the same type as the loop index. Finally, the loop index may not be changed within the loopBody. This can be enforced by marking id's declaration as "constant" or "read only" while loopBody is being analyzed.

9.1.5 Break, Continue, Return, and Goto Statements

Java contains no goto statement. It does, however, include break and continue statements, which are restricted forms of a goto, as well as a return statement. We will consider the continue statement first.

Continue Statements

As with the continue statement found in C and C++, Java's continue statement attempts to "continue with" the next iteration of a while, do, or for loop. That is, it transfers control to the bottom of a loop where the loop index is iterated (in a for loop), and the termination condition is evaluated and tested.

A continue may only appear within a loop; this must be verified during semantic analysis. Unlike C and C++, a loop label may be specified in a continue statement. An unlabeled continue references the innermost for, while, or do loop in which it is contained. A labeled continue references the enclosing loop that has the corresponding label. Again, semantic analysis must verify that an enclosing loop with the proper label exists.

Any statement in Java may be labeled. As shown in Figure 9.10 we will assume an AST node *LabeledStmt* that contains a string-valued field *stmtLabel*.

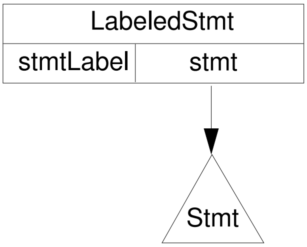


Figure 9.10: Abstract Syntax Tree for a Labeled Statement

```
procedure VISIT( LabeledStmt ls )
    newNode ← new LabelList ( ls.stmtLabel, GETKIND( ls.stmt ), ls.stmt )
    newList ← CONS( newNode, GETLABELLIST( ) )
    call SETLABELLIST( newList )
    call VISITCHILDREN( ls )
    call SETLABELLIST( TAIL( GETLABELLIST( ) ) )
end
```

Figure 9.11: Semantic Analysis for a Labeled Statement

If the statement is labeled, *stmtLabel* contains the label in string form. If the statement is unlabeled, *stmtLabel* is null. *LabeledStmt* also contains a field *stmt* which is the AST node representing the labeled statement. Unlabeled statements need not have *LabeledStmt* as a parent, particularly in contexts where a label is disallowed.

In Java, C, and C++ (and most other programming languages), labels are placed in a different name space than other identifiers. This means that an identifier used as a label may also be used for other purposes (a variable name, a type name, a method name, and so on) without confusion. This is because labels are used in very limited contexts (in continues, breaks, and perhaps gotos). Labels cannot be assigned to variables, returned by functions, read from files, etc.

We will maintain a list of labels that are currently visible to the AST being analyzed. The function GETLABELLIST will return the current label list (possibly null). The procedure SETLABELLIST will set the current label list via its parameter. The label list is set to null when we begin the analysis of a method, constructor body, or a static initializer.

Each *LabelList* node contains three fields. The first is *label* (a string that contains the name of the label). Next is *kind* (one of *iterative*, *switch*, or *other* that indicates the kind of statement that is labeled). The last is *AST* (a link to

```
procedure visit( Continuing cn )
    currentList ← getLabelList( )
    if cn.stmtLabel = null
    then
        while currentList ≠ null do
            currentLabel ← head( currentList )
            if currentLabel.kind = iterative
            then return
            currentList ← tail( currentList )
        call error( "Continue not inside iterative statement" )
    else
        while currentList ≠ null do
            currentLabel ← head( currentList )
            if currentLabel.label = cn.stmtLabel and currentLabel.kind = iterative
            then return
            currentList ← tail( currentList )
        call error( "Continue label doesnot match an iterative statement" )
end
```

Figure 9.12: Semantic Analysis for a Continue

the AST of the labeled statement). Looking at a label list, we can determine the statements that enclose a break or continue, as well as all of the labels currently visible to the break or continue.

The semantic analysis visitor for a *LabeledStmt* is shown in Figure 9.11. The current label list is extended by adding an entry for the current *LabeledStmt* node using its *label* (which may be null), and its *kind* (determined by a call to an auxiliary method, getKind). The subtree is analyzed using the extended label list. After semantic analysis, the label list is returned to its original state by removing its first element. Reachability and throws analyses are defined at Figure 9.3, Marker ⑦ and Figure 9.4, Marker ⑭.

A continue statement without a label references the innermost iterative statement (while, do, or for) within which it is nested. This is easily checked by looking for a node on the label list with *kind* = *iterative* (ignoring the value of the label field).

A continue statement that references a label *L* (stored in AST field *stmtLabel*) must be enclosed by an iterative statement whose label is *L*. If more than one containing statement is labeled with *L*, the nearest (innermost) is used. The semantic analysis visitor for a continue statement is shown in Figure 9.12. Reachability and throws analyses for a continue are defined at Figure 9.3, Marker ⑧ and Figure 9.4, Marker ⑮.

As an example, consider the following code fragment:

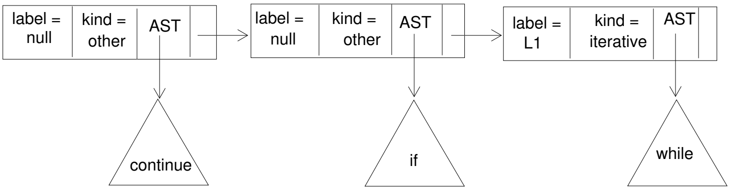


Figure 9.13: Example of a label list in a Continue Statement

```
L1: while (p != null) {
      if (p.val < 0)
         continue
      else ... }
```

The label list in use when the continue is analyzed is shown in Figure 9.13. Since the list contains a node with *kind* = *iterative*, the continue is correct.

In C and C++, semantic analysis is even simpler. Continues do not use labels, so the innermost iterative statement is always selected. This means that we need only the null *stmtLabel* case in Figure 9.12.

Break Statements

In Java, an unlabeled break statement has the same meaning as the break statement found in C, C♯, and C++. The innermost while, do, for, or switch statement is exited, and execution continues with the statement immediately following the exited statement. Thus, a reachable break forces the statement it references to terminate normally.

A labeled break exits the enclosing statement with a matching label (not necessarily a while, do, for, or switch statement), and continues execution with that statement's successor (again, if reachable, it forces normal termination of the labeled statement). For both labeled and unlabeled breaks, semantic analysis must verify that a suitable target statement for the break exists.

We will use the function findBreakTarget, defined in Figure 9.14, to determine the AST node that a break references (illegal breaks return null). The function uses getLabelList, introduced in the last section, to enumerate possible targets. For unlabeled breaks, it searches for a node with *kind* equal to *iterative* or *switch*. For labeled breaks, it tries to find a node with a matching label (its *kind* field does not matter).

Semantic analysis verifies that findBreakTarget can find a valid target (Figure 9.15). Reachability analysis sets the *terminatesNormally* field of the break target to true if the break is marked as reachable (Figure 9.16). This

```
function findBreakTarget( Breaking bn ) returns LabelList
    currentList ← getLabelList( )
    if bn.stmtLabel = null
    then
        while currentList ≠ null do
            currentLabel ← head( currentList )
            if currentLabel.kind = iterative or currentLabel.kind = switch
            then  return (currentLabel)
            currentList ← tail( currentList )
        return (null)
    else
        while currentList ≠ null do
            currentLabel ← head( currentList )
            if currentLabel.label = bn.stmtLabel
            then  return (currentLabel)
            currentList ← tail( currentList )
        return (null)
end
```

Figure 9.14: Function to Find Target of a Break

```
procedure visit( Breaking bn )
    target ← findBreakTarget( bn )
    if target = null
    then
        if bn.stmtLabel = null
        then  call error( "Break not inside iterative or switch statement" )
        else
            call error( "Break label doesnot match any visible statement label" )
end
```

Figure 9.15: Semantic Analysis for a Break

```
procedure visit( Breaking bn )
    bn.terminatesNormally ← false
    target ← findBreakTarget( bn )
    if target ≠ null and bn.isReachable
    then
    target.AST.terminatesNormally ← true
end
```

Figure 9.16: Reachability Analysis for a Break

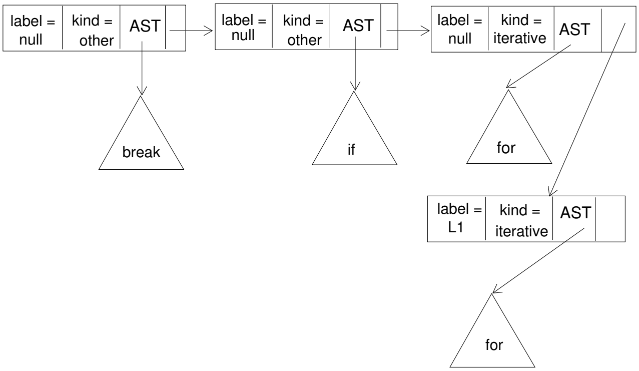


Figure 9.17: Example of a label list in a Break Statement

allows us to properly analyze do-forever loops that terminate by executing a break.

As an example, consider the following code fragment:

```
L1: for (i=0; i < 100; i++)
      for (j=0; j < 100; j++)
          if (a[i][j] == 0)
             break L1;
          else ...
```

The label list in use when the break is analyzed is shown in Figure 9.17. Since the list contains a node with *label* = `L1`, the break is correct. The for loop labeled with `L1` is marked as terminating normally.

Return Statements

An AST rooted by *Returning*, as shown in Figure 9.19, represents a return statement. The field *returnVal* is null if no value is returned; otherwise it is an AST representing an expression to be evaluated and returned.

The semantic rules governing a return statement depend on where the statement appears. A return statement without an expression value may only appear in a void method (a procedure) or in a constructor. A return statement with a return value may only appear in a method whose type may be assigned the return type (this excludes void methods and constructors).

A method declared to return a value (a function) must exit via a return of a value or by throwing an exception. This requirement can be enforced

```
procedure VISIT( Returning rn )
    call VISITCHILDREN( rn )
    currentMethod ← GETCURRENTMETHOD( rn )
    if rn.returnVal ≠ null
    then
        if currentMethod = null
        then
            call ERROR( "A value may not be returned from a constructor" )
        else
            if not ASSIGNABLE( currentMethod.returnType, rn.returnValue.type )
            then  call ERROR( "Illegal return type" )
    else
        if currentMethod ≠ null and currentMethod.returnType ≠ void
        then   call ERROR( "A value must be returned" )
end
```

Figure 9.18: Semantic Analysis for a Return

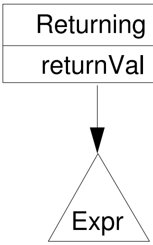


Figure 9.19: Abstract Syntax Tree for a Return Statement

by verifying that the statement list that comprises a function's body has its *terminatesNormally* value set to false.

To determine the validity of a return, we will check the kind of construct (method or constructor) within which it appears. But AST links all point downward; hence looking "upward" is difficult. To assist our analysis, we will assume two methods that return *Attributes* structures, GETCURRENTMETHOD and GETCURRENTCONSTRUCTOR. These methods store information gathered during semantic analysis.

If we are checking an AST node contained within the body of a method, then GETCURRENTMETHOD will tell us which one. If we are analyzing an AST node not within a method, then GETCURRENTMETHOD returns null. The same is true for GETCURRENTCONSTRUCTOR. We can determine which kind of construct we are in (and details of its declaration) by using the reference that is non-null.

The details of semantic analysis for return statements appears in Figure 9.18. We assume GETCURRENTMETHOD().*returnType* gives us the type returned by the method currently being translated (this type may be void). The auxiliary method ASSIGNABLE($T1$,$T2$) tests whether type $T2$ is assignable to type $T1$ (using the assignability rules of the language being compiled).

C♯ and C++ have semantic rules very similar to those of Java. A value may only be returned from a non-void function and the value returned must be assignable to the function's return type. In C, a return without a value is allowed in a non-void function (with undefined behavior).

Goto Statements

Java contains no goto statement, but many other languages, including C, C♯, and C++, do. These languages, and almost all languages that allow gotos, restrict them to be **intraprocedural**. That is, a label and all gotos that reference it must be in the same procedure or function.

As noted above, identifiers used as labels are usually considered distinct from identifiers used for other purposes. Thus, in C, C♯, and C++, the statement:

```
a: a=a+1;
```

is legal. Labels may be kept in a separate symbol table, distinct from the main symbol table used to store ordinary declarations.

Labels need not be defined before they are used; "forward gotos" are allowed. Semantic checking must guarantee that all labels used in gotos are in fact defined somewhere in the current method.

Because of potential forward references, it is a good idea to check labels and gotos in two steps. First, the AST that represents the entire body of a method or subprogram is traversed, gathering all label declarations into a *declaredLabels* table stored as part of the current subprogram's symbol table. Duplicate labels are detected as *declaredLabels* is built.

During normal semantic processing of the body of a subprogram (after *declaredLabels* has been built), an AST for a goto can access *declaredLabels* (through the current subprogram's symbol table). Checking for valid label references (whether forward or not) is straightforward.

A few languages, such as Pascal, allow **nonlocal gotos**. A nonlocal goto transfers control out of a subprogram (forcing a return) to a label in a scope that contains the current procedure. Nonlocal `gotos` can be checked by maintaining a stack (or list) of *declaredLabels* tables, one for each nested procedure. A goto is valid if its target appears in any of the *declaredLabels* tables.

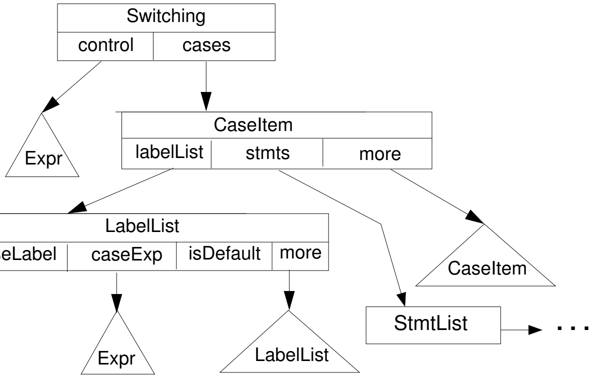


Figure 9.20: Abstract Syntax Tree for a Switch Statement

Finally, some programming languages forbid gotos into a conditional or iterative statement from outside. That is, even if the scope of a label is an entire subprogram, a goto into a loop body, or from a then part to an else part, is forbidden. Such restrictions can be enforced by marking each label in *declaredLabels* as either *active* or *inactive*. Gotos are allowed only to active labels, and a label within a conditional or iterative statement is active only while the AST that contains the label is being processed. Thus, a label *L* within a while loop becomes active when the loop body's AST is checked and is inactive when statements outside the loop body are checked.

9.1.6 Switch and Case Statements

Java, C, C♯, and C++ contain a **switch statement** that allows the selection of one of a number of statements based on the value of a control expression. ML, Ada, and a number of older languages contain a **case statement** that is equivalent. We shall focus on translating switch statements, but our discussion applies equally to case statements.

The AST for a switch statement rooted at *Switching* is shown in Figure 9.20 (fields not needed for semantic analysis are omitted for clarity). In the AST, *control* represents an integer-valued expression; *cases* is a *CaseItem*, representing the cases in the switch. Each *CaseItem* has three fields. *labelList* is a *LabelList* that represents one or more case labels. *stmts* is an AST node representing the statements following a case constant in the switch. *more* is either null or another *CaseItem*, representing the remaining cases in the switch.

A *LabelList* contains an integer field *caseLabel*, an AST *caseExp*, a Boolean field *isDefault* (representing the default case label), and *more*, a field that is

```
class NodeVisitor
    procedure visitChildren( n )
        foreach c ∈ n.getChildren( ) do call c.accept( this )
    end
end

class SemanticsVisitor extends NodeVisitor
    /★  This extends the class definition of Figure 9.1                  ★/

    procedure visit( Switching sn )                                      28
        call sn.control.accept( this )
        if sn.control.type ≠ errorType and not assignable( int, sn.control.type )
        then
            call error( "Illegal type for control expression" )
            call setSwitchType( errorType )
        else  call setSwitchType( sn.control.type )
        call sn.cases.accept( this )
        labelList ← sort( gatherLabels( sn.cases ) )
        call checkForDuplicates( labelList )
        if countDefaults( sn.cases ) > 1
        then call error( "More than one default case label" )
    end

    procedure visit( CaseItem cn )
        call visitChildren( cn )
    end

    procedure visit( LabelList lln )
        call visitChildren( lln )
        lln.caseLabel ← null
        if lln.caseExp.type ≠ errorType
        then
            if not assignable( getSwitchType( ), lln.caseExp.type )
            then call error( "Invalid case label type" )
            else
                constExprVisitor ← new ConstExprVisitor( )
                call lln.caseExp.accept( constExprVisitor )
                labelValue ← lln.caseExp.exprValue
                if labelValue = null
                then call error( "Case label must be a constant expression" )
    end
end
```

Figure 9.21: Semantic Analysis Visitors (Part 2)

either null or another *LabelList* (representing the remainder of the list). The *caseExp* AST represents a constant expression that labels a case within the switch; when it is evaluated, *caseLabel* will hold its value.

A number of steps are needed to check the semantic correctness of a switch statement. The control expression and all the statements in the case body must be type-checked. The control expression must be an integer type (enumerations, if available, are also allowed). Each case label must be a constant expression assignable to the type of the control expression. No two case labels may have the same value. At most, one default label may appear within the switch body.

The semantic visitors used to enforce these rules are shown in Figure 9.21. Utility methods used by the semantic visitors are defined in Figure 9.22. Method GATHERLABELS (defined for both *CaseItem* and *LabelList*) walks an AST for a switch and gathers all labels into an integer list. CHECKFORDUPLICATES takes a sorted label list and compares adjacent list values to find duplicate labels. Method COUNTDEFAULTS (defined for both *CaseItem* and *LabelList*) walks an AST for a switch statement, counting how many default cases have been defined (more than one is illegal). The semantic visitor for *LabelList* uses the visitor class *ConstExprVisitor* introduced in Section 9.1.3. Visitors from this class determine whether an expression AST represents a constant value. If so, the value is placed in field *exprValue*. Otherwise, this field is set to null.

Reachability analysis for `switch` statements is defined in Figure 9.23. A `switch` statement can terminate normally in a number of ways. Although uncommon, an empty `switch` body trivially terminates normally. If the last `switch` group (case labels followed by a statement list) terminates normally, so does the `switch` (since execution "falls through" to the succeeding statement). If any of the statements within a `switch` body contain a reachable break statement, then the entire `switch` can terminate normally.

We first mark the whole `switch` as *not* terminating normally. This will be updated to true if *cases* is null, if a reachable break is encountered while checking the `switch` body, or if the *stmts* AST of the last *CaseItem* in the AST is marked as terminating normally.

As an example of our semantic analysis techniques, consider the following `switch` statement:

```
switch(p) {
  case 2:
  case 3:
  case 5:
  case 7:  isPrime = true; break;
  case 4:
  case 6:
  case 8:
```

```
function GATHERLABELS( CaseItem cn ) returns intList
    if cn.more = null
    then  return (GATHERLABELS( cn.labelList ))
    else
        rest ← GATHERLABELS( cn.more )
        return (APPEND( GATHERLABELS( cn.labelList ), rest ))
end

function GATHERLABELS( LabelList lln ) returns intList
    if lln = null
    then  return (null)
    else
        rest ← GATHERLABELS( llnn.more )
        if lln.caseLabel = null
        then  return (rest)
        else   return (CONS( lln.caseLabel, rest ))
end

procedure CHECKFORDUPLICATES( intList il )
    if LENGTH( il ) > 1
    then
        if HEAD( il ) = HEAD( TAIL( il ) )
        then
        call ERROR( "Duplicate case label : ", HEAD( il ) )
        else
        call CHECKFORDUPLICATES( TAIL( il ) )
end

function COUNTDEFAULTS( CaseItem cn ) returns int
    if cn = null
    then  return (0)
    else   return (COUNTDEFAULTS( cn.labelList ) + COUNTDEFAULTS( cn.more ))
end

function COUNTDEFAULTS( LabelList lln ) returns int
    if lln = null
    then  return (0)
    if lln.isDefault
    then  return (1 + COUNTDEFAULTS( lln.more ))
    else
    return (COUNTDEFAULTS( lln.more ))
end
```

Figure 9.22: Utility Semantic Methods for Switch Statements

```
class ReachabilityVisitor extends NodeVisitor
    /★  This extends the class definition of Figure 9.3                ★/
    procedure visit( Switching sn )
        sn.terminatesNormally ← false
        call visitChildren( sn )
        if sn.cases = null
        then sn.terminatesNormally ← true
        else
            sn.terminatesNormally ← sn.terminatesNormally or sn.cases.terminatesNormally
    end

    procedure visit( CaseItem cn )
        cn.stmts.isReachable ← true
        call visitChildren( cn )
        if cn.more = null
        then cn.terminatesNormally ← cn.stmts.terminatesNormally
        else cn.terminatesNormally ← cn.more.terminatesNormally
    end
end
```

Figure 9.23: Reachability Analysis Visitors (Part 2)

```
        case 9:  isPrime = false; break;
        default: isPrime = checkIfPrime(p);
    }
```

Assume that p is declared as an integer variable. We check p and find it a valid control expression. The label list is built by examining each *CaseItem* and *LabelList* in the AST. We verify that each case label is a valid constant expression that is assignable to p. The case statements are checked and found to be valid. Since the last statement in the switch (the default) terminates normally, so does the entire switch statement. The label list returned by gatherLabels is $\{2, 3, 5, 7, 4, 6, 8, 9\}$. After sorting, we have $\{2, 3, 4, 5, 6, 7, 8, 9\}$. No two adjacent elements in the sorted list are equal. Finally, we count the number of default labels; a count of 1 is valid.

The semantic rules for switch statements in C and C++ are almost identical to those of Java. C♯ adds a requirement that "fall throughs" from one leg of a switch statement to another are illegal. That is, given:

```
switch(p) {
  case 0:  isZero = true;
  case 1:  print(p);
}
```

with a value of 0 for p, after isZero is set, execution attempts to print p's value. This construct is legal in C, C++, and Java, but not C♯. We can check for this

C♯ error by requiring that in each *CaseItem*, *stmts*.*terminatesNormally* be false. Such an analysis is also useful in producing helpful warning messages, since forgetting a break at the end of a case is a common error.

Other languages include a case statement that is similar in structure to the `switch` statement. The latest versions of Java and C♯ allow enumerations as well as integers in case statements.

Ada generalizes a case label to a range of case values (e.g., in Java notation, `case 1..10`, denotes 10 distinct case values). Semantic checks that look for duplicate case values and check for complete coverage of possible control values must be generalized to handle ranges rather than single values.

9.1.7 Exception Handling

Most modern programming languages, including Java and C♯, provide an **exception handling** mechanism. During execution, an exception may be thrown, either explicitly (via a throw statement) or implicitly (due to an execution error). Thrown exceptions may be caught by an exception handler.

Exceptions form a clean and general mechanism for identifying and handling unexpected or erroneous situations. They are clearer and more efficient than using error flags or gotos. Though we will focus on Java and C♯'s exception handling mechanism, most recent language designs, including C++, Ada, and ML, include a very similar exception mechanism.

Java exceptions are *typed*. An exception throws an object that is an instance of class `Throwable` or one of its subclasses. The object thrown may contain fields that characterize the precise nature of the problem the exception represents, or the class may be empty (with its type signifying all necessary information).

Java exceptions are classified as either checked or unchecked. A **checked exception** thrown in a statement *must* be caught in an enclosing try statement or listed in the throws list of the enclosing method or constructor.

An **unchecked exception** (defined as an object assignable to either class `RuntimeException` or class `Error`) may optionally be handled in a try statement. If uncaught, unchecked exceptions will terminate execution. Unchecked exceptions represent errors that may appear almost anywhere (such as accessing a null reference or using an illegal array index). These exceptions usually force termination, so explicit handlers may clutter a program without adding any benefit (termination is the default for uncaught exceptions).

We will first consider the semantic checking needed for a try statement. The AST for a try is shown in Figure 9.24. The AST for the catch clauses found in a try statement is shown in Figure 9.25.

Semantic processing for a try is defined in Figure 9.26 at Marker ㉙. All three components of the try, the try body, the catch clauses, and the optional

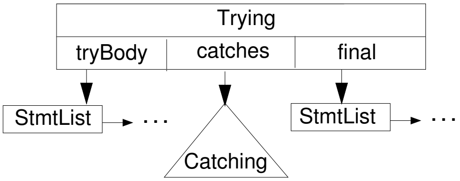


Figure 9.24: Abstract Syntax Tree for a Try Statement

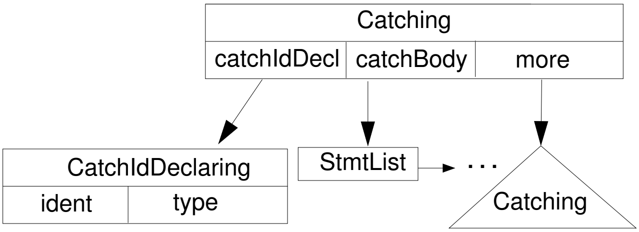


Figure 9.25: Abstract Syntax Tree for a Catch Block

finally statements are analyzed. Semantic analysis of the catch clauses is defined at Marker (30).

Catch clauses require careful analysis. Each clause introduces a new identifier, the parameter of the clause. This identifier must be declared as an exception (of class `Throwable` or a subclass of it). The parameter is made local to the body of the catch clause by opening, and later closing, a new name scope.

Using method SUBSUMESLATERCATCHES, defined in Figure 9.27, we verify that the current catch clause does not "hide" later catches. This is a reachability issue—some exception type must be able to reach, and activate, each of the catch clauses in a try statement.

Throws analysis for try statements is defined in Figure 9.28 at Marker (32). Catch clauses and finally statements are visited to collect the exceptions they might throw. Before the try body is analyzed, we must include the exception types declared in the try's catch clauses. We maintain a "catch list" containing all exception types potentially handled by enclosing try blocks. At the start of analysis for a method or constructor, this list is null. Assume the method GETCATCHLIST gives us the current catch list. Before updating the catch list to include exceptions in the current catch clauses, we save the current catch list into *currentCatchList*. The call to UPDATECATCHLIST adds exceptions in the current catch clauses to the catch list. Now the try body can be analyzed. After its analysis, the catch list is restored using SETCATCHLIST. Once we

```
class NodeVisitor
    procedure visitChildren(n)
        foreach $c \in n$.getChildren( ) do call c.accept(this)
    end
end

class SemanticsVisitor extends NodeVisitor
    /*  This extends the class definition of Figure 9.21  */
    procedure visit( Trying tn )                                           29
        call visitChildren(tn)
    end

    procedure visit( Catching cn )                                         30
        if not assignable(Throwable, cn.catchIdDecl.type)
        then
            call error("Illegal type for catch identifier")
            cn.catchIdDecl.type ← errorType
        else
            if subsumesLaterCatches(cn.catchIdDecl.type, cn.more)
            then call error("This catch hides later catches")
        call openScope( )
        attr.kind ← variableAttributes
        attr.variableType ← cn.catchIdDecl.type.getTypeDescriptor( )
        call currentSymbolTable.enterSymbol(cn.catchIdDecl.ident.name, attr)
        call cn.catchBody.accept(this)
        call closeScope( )
        call cn.more.accept(this)
    end

    procedure visit( Throwing tn )                                         31
        call visitChildren(tn)
        if $tn.thrownVal.type \neq errorType$ and not assignable(Throwable, tn.thrownVal.type)
        then call error("Illegal type for throw")
    end
end
```

Figure 9.26: Semantic Analysis Visitors (Part 3)

```
function subsumesLaterCatches( exceptionType, Catching cn ) returns Boolean
    if cn = null
    then return (false)
    else
        if assignable( exceptionType, cn.catchIdDecl.type )
        then return (true)
        else
        return (subsumesLaterCatches( exceptionType, cn.more ))
end

procedure processCatch( SetOfType throwsSet, Catching cn )
    filteredThrowsSet ← filterThrows( throwsSet, cn.catchIdDecl.type )
    if filteredThrowsSet = throwsSet
    then call error( "No throws reach this catch" )
    else
        if cn.more ≠ null
        then call processCatch( filteredThrowsSet, cn.more )
end

function filterThrows( SetOfType throwsSet, exceptionType ) returns SetOfType
    ans ← ∅
    foreach t ∈ throwsSet do
        if not assignable( exceptionType, t )
        then ans ← ans ∪ t
    return (ans)
end

function filterCatches( SetOfType throwsSet, Catching cn ) returns SetOfType
    if cn.more = null
    then return (filterThrows( throwsSet, cn.catchIdDecl.type ))
    else
    return (filterCatches( filterThrows( throwsSet, cn.catchIdDecl.type ), cn.more ))
end

procedure updateCatchList( Catching cn )
    call extendCatchList( cn.catchIdDecl.type )
    if cn.more ≠ null
    then call updateCatchList( cn.more )
end
```

Figure 9.27: Utility Semantic Methods for Try and Throw Statements

know the exceptions potentially thrown in the try block, we call processCatch (Figure 9.27) to verify that some exception can reach each of the catch clauses in the try. Finally, we call filterCatches (Figure 9.27) to determine the exceptions that can ″escape″ from the current try body (an exception need not be handled locally).

Reachability visitors for try statements are defined in Figure 9.29. The try body, finally statements, and all catch bodies are marked as reachable (processCatch helped verify this). A try terminates normally if the try body or any catch clause can terminate normally *and* the finally statements also terminate normally.

The AST for a throw statement is shown in Figure 9.30. Semantic analysis (Figure 9.26, Marker ㉛) verifies that the type of value thrown is a valid exception (a type assignable to `Throwable`).

Throws analysis (Figure 9.28, Marker ㉞) verifies that if a checked exception is thrown, an enclosing try block can catch the exception *or* the enclosing method or constructor has included the exception in its throws list. We call getCatchList to get the exceptions mentioned in all enclosing try blocks. The exceptions in the current method's throws list is obtained by calling getDeclThrowsList. We join these two lists together and call filterThrows to remove exceptions that can match the thrown exception. If no exceptions are removed, the thrown exception is not properly handled and an error message is issued.

As an example, consider the following Java code fragment:

```
class ExitComputation extends Exception{};

try { ...
     if (cond)
         throw new ExitComputation();
     if (v < 0.0)
         throw new ArithmeticException();
    else a = Math.sqrt(v);
... }

catch (e ExitComputation) {return 0;}
```

A new checked exception, `ExitComputation`, is declared. In the try statement, we first check the catch clause. The current catch list is extended with an entry for type `ExitComputation`. The try body is then checked. Focusing on throw statements, we first process a throw of an `ExitComputation` object. This is a valid subclass of `Throwable`, and `ExitComputation` is on the current catch list, so no errors are detected. Next, the throw of an `ArithmeticException` is checked. It too is a valid exception type. It is an unchecked exception (a

```
class ThrowsVisitor extends NodeVisitor
    /★  This extends the class definition of Figure 9.4  ★/
    procedure gatherThrows( n )
        call visitChildren( n )
        $ans \leftarrow \emptyset$
        foreach $c \in n$.getChildren( ) do $ans \leftarrow ans \cup c.throwsSet$
        $n.throwsSet \leftarrow ans$
    end

    procedure visit( Trying tn )                                    32
        call tn.catches.accept( this )
        call tn.final.accept( this )
        $currentCatchList \leftarrow$ getCatchList( )
        call updateCatchList( tn.catches )
        call tn.tryBody.accept( this )
        call setCatchList( currentCatchList )
        call processCatch( tn.tryBody.throwsSet, tn.catches )
        $tn.throwsSet \leftarrow$ filterCatches( tryBody.throwsSet, tn.catches )
        $tn.throwsSet \leftarrow tn.throwsSet \cup tn.catches.throwsSet \cup tn.final.throwsSet$
    end

    procedure visit( Catching cn )                                  33
        call gatherThrows( cn )
    end

    procedure visit( Throwing tn )                                  34
        call visitChildren( tn )
        $thrownType \leftarrow tn.thrownVal.type$
        $tn.throwsSet \leftarrow tn.thrownVal.throwsSet \cup thrownType$
        if assignable( RuntimeException, thrownType ) or assignable( Error, thrownType )
        then return
        else
            $throwTargets \leftarrow$ getCatchList( ) $\cup$ getDeclThrowsList( )
            $filteredTargets \leftarrow$ filterThrows( throwTargets, thrownType )
            if size( throwTargets ) = size( filteredTargets )
            then
                call error( "Type thrown not found in enclosing catch or declared throws list" )
    end

    procedure visit( Calling cn )                                   35
        call gatherThrows( cn )
        if $cn.calledMethod \neq$ null
        then
            $cn.throwsSet \leftarrow cn.throwsSet \cup cn.calledMethod.declaredThrowsList$
    end
end
```

Figure 9.28: Throws Analysis Visitors (Part 2)

```
class ReachabilityVisitor extends NodeVisitor
    /*  This extends the class definition of Figure 9.23  */
    procedure visit( Trying tn )
        tn.tryBody.isReachable ← true
        tn.final.isReachable ← true
        call visitChildren( tn )
        catchOrTryOK ← tn.catches.terminatesNormally or tn.tryBody.terminatesNormally
        tn.terminatesNormally ← catchOrTryOK and tn.final.terminatesNormally
    end

    procedure visit( Catching cn )
        cn.catchBody.isReachable ← true
        call visitChildren( cn )
        cn.terminatesNormally ← cn.catchBody.terminatesNormally
        if cn.more ≠ null
        then
            cn.terminatesNormally ← cn.terminatesNormally or cn.more.terminatesNormally
    end

    procedure visit( Throwing tn )
        tn.terminatesNormally ← false
    end
    procedure visit( Calling cn )
        cn.terminatesNormally ← true
    end
end
```

Figure 9.29: Reachability Analysis Visitors (Part 3)

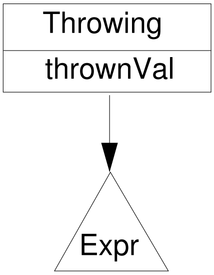


Figure 9.30: Abstract Syntax Tree for a Throw Statement

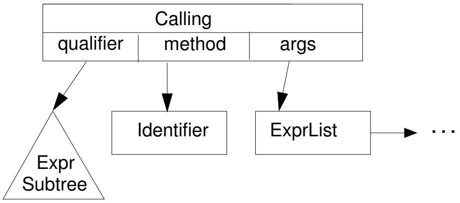


Figure 9.31: Abstract Syntax Tree for a Method Call

subclass of `RuntimeException`), so the throw is valid independent of any try statements that enclose it.

The exception mechanisms of C♯ and C++ are very similar to that of Java, using an almost-identical throw/catch mechanism. The techniques developed in this section are directly applicable.

Other languages, such as ML and Ada, feature a single exception type that is ″raised″ rather than thrown. Exceptions are handled in a ″handle″ clause (ML) or a ″when″ clause (Ada) that can be appended to any expression or begin-end block. Again, semantic processing is very similar to the mechanisms developed here.

9.2 Semantic Analysis of Calls

In this section we investigate the semantic analysis of method calls in Java. The techniques we present are also applicable to constructor and interface calls, as well as calls to methods and subprograms in C♯, C, C++, and related languages.

The AST for *Calling* is shown in Figure 9.31. The field *method* is an identifier that specifies the name of the method to be called. The field *qualifier* is an optional expression that specifies the object or class within which method is to be found. Finally, *args* is an optional expression list that represents the actual parameters to the call.

The first step in analyzing a call is to determine which method definition to use. This determination is by no means trivial in Java and other object-oriented languages because of **inheritance** and **overloading**.

Recall that classes form an inheritance hierarchy, and all classes are derived, directly or indirectly, from `Object`. An object may have access to methods defined in its own class, its parent class, its grandparent class, and so on, all the way up to `Object`. In processing a call, all potential locales of definition must be checked.

Because of overloading, it is valid to define more than one method with the same name. A call must select the ″right″ definition, which informally is the nearest accessible method definition in the inheritance hierarchy whose parameters match the actual parameters provided in the call.

We begin semantic analysis of a call by gathering all the method definitions that might be targets of the call. This lookup process is guided by the kind of method qualifier (if any) that is provided and the access mode of individual methods.

If no qualifier is provided, then we examine the class (call it *C*) that contains the call being analyzed. All methods defined within *C* with the selected method name are accessible. In addition, methods defined in *C*'s superclasses (its parent class, grandparent class, etc.) may be inherited depending on their access qualifiers. Methods marked public or protected are always included, but not private methods which cannot be inherited.

The qualifier, if non-null, also influences the selection of applicable methods. If the qualifier is the reserved word `super`, then a call of *M* in class *C* must reference a method inherited from a superclass (as defined above). (Use of `super` in class `Object` is illegal, because `Object` has no superclass.)

If the qualifier is a type name *T* (which must be a class name), then a call of *M* must reference a static method. (Instance methods are disallowed because the object reference symbol `this` is undefined.) Public or protected static methods may be referenced.

If the qualifier is an expression that computes an object of type *T*, then *T* must be a class marked public. A call of *M* may reference public or protected methods within *T* and its superclasses.

These rules for selecting possible method definitions in a call are codified in GETMETHODS (Figure 9.32). We assume that METHODDEFS(*ID*) returns *Attributes* structures for all of the methods named *ID* in a given class. Similarly, VISIBLEMETHODS(*ID*) returns all the public and protected methods named *ID*. The method GETCURRENTCLASS returns the class currently being compiled.

Once we have determined the set of definitions that are possible, we must *filter* them by comparing each definition with the number and type of expressions that form the call's actual parameters. We will assume that each method definition included in the set of accessible methods is represented as an *Attributes* structure, which contains the fields *returnType*, *signature*, and *classDefinedIn*. Field *returnType* is the type returned by the method; *classDefinedIn* is the class in which the method is defined; *signature* is the type signature of the method. We can build a types list for the actual parameters of a call using the GETARGTYPES method defined in Figure 9.32, Marker ㊱.

Once we have a type list for the actual parameters of a call, we must compare it with the declared parameter type list of each method. A method definition contains the field *signature* that records its parameter types and

```
function getMethods( Calling cn ) returns SetOfAttributes
    currentClass ← getCurrentClass( )
    if cn.qualifier = null
    then  methodSet ← currentClass.methodDefs( cn.method )
    else  methodSet ← ∅
    if cn.qualifier = null or cn.qualifier = superNode
    then  nextClass ← currentClass.parent
    else  nextClass ← cn.qualifier.type
    while nextClass ≠ null do
        if cn.qualifier ≠ null and cn.qualifier ≠ superNode and not nextClass.isPublic
        then  nextClass ← nextClass.parent
              continue
        methodSet ← methodSet ∪ nextClass.visibileMethods( cn.method )
        nextClass ← nextClass.parent
    return (methodSet)
end

function getArgTypes( ExprList el ) returns ListOfType                    36
    typeList ← null
    foreach expr ∈ el do  typeList ← append( typeList, list( expr.type ) )
    return (typeList)
end

function applicable( formalParms, actualParms ) returns Boolean           37
    if formalParms = null and actualParms = null
    then  return (true)
    else
        if formalParms = null or actualParms = null
        then  return (false)
        else
            if bindable( head( formalParms ), head( actualParms ) )
            then  return (applicable( tail( formalParms ), tail( actualParms ) ))
            else  return (false)
end
```

Figure 9.32: Utility Semantic Methods for Method Calls (Part 1)

```
function moreSpecific($def1, def2$) returns Boolean                38
    if bindable($def1.classDefinedIn, def2.classDefinedIn$)
    then
        $arg1 \leftarrow def1.argTypes$
        $arg2 \leftarrow def2.argTypes$
        while $arg1 \neq$ null do
            if bindable(head($arg1$), head($arg2$))
            then
                $arg1 \leftarrow$ tail($arg1$)
                $arg2 \leftarrow$ tail($arg2$)
            else   return ($false$)
        return ($true$)
    else   return ($false$)
end

function filterDefs($methodDefSet$) returns Boolean                39
    $changes \leftarrow true$
    while $changes$ do
        $changes \leftarrow false$
        foreach $def1 \in methodDefSet$ do
            foreach $def2 \in methodDefSet$ do
                if $def1 \neq def2$ and moreSpecific ($def1, def2$)
                then
                    $methodDefSet \leftarrow methodDefSet - \{def1\}$
                    $changes \leftarrow true$
    return ($methodDefSet$)
end
```

Figure 9.33: Utility Semantic Methods for Method Calls (Part 2)

return type. The method getArgs extracts a list of parameter types from the method signature. This list is compared with the list of actual parameter types.

But what exactly defines a match between formal and actual parameters? First, both argument lists must have the same length—this is easy to check. Next, each actual parameter must be "bindable" to its corresponding formal parameter.

Bindable means that it is legal to use an actual parameter whenever the corresponding formal parameter is referenced. In Java, bindable is almost the same as assignable. When interfaces are considered, a class object may sometimes be bound even if it may not be directly assigned.

Now checking the feasibility of using a particular method definition in a call is straightforward (we check that the number of parameters is correct and that each parameter is bindable). This is detailed in Figure 9.32, Marker 37, which defines the method applicable(*formalParms*, *actualParms*). If applicable

```
class NodeVisitor
    procedure visitChildren( n )
        foreach $c \in n$.getChildren( ) do call c.accept( this )
    end
end

class SemanticsVisitor extends NodeVisitor
    /★  This extends the class definition of Figure 9.26                ★/

    procedure visit( Calling cn )                                        40
        call visitChildren( cn )
        $cn.calledMethod \leftarrow$ null
        $methodSet \leftarrow$ getMethods( cn )
        $actualArgsType \leftarrow$ getArgTypes( cn.args )
        foreach $def \in methodSet$ do
            if not applicable (getArgs( def.signature ), actualArgsType )
            then $methodSet \leftarrow methodSet - \{def\}$
        if size( methodSet ) $= 0$
        then call error( "No method matches this call" )
        return
        else
            if size( methodSet ) $> 1$
            then $methodSet \leftarrow$ filterDefs( methodSet )
        if size( methodSet ) $> 1$
        then call error( "More than one method matches this call" )
        else
            Let m be the singleton member of methodSet
            $cn.calledMethod \leftarrow m$
            if $cn.qualifier \neq$ null and $cn.qualifier \neq superNode$ and $m.accessMode \neq static$
            then call error( "Method called must be static" )
            else
                if inExpressionContext( cn ) and $m.returnType = void$
                then call error( "Call must return a value" )
    end
end
```

Figure 9.34: Semantic Analysis Visitors (Part 4)

returns true, a particular method definition can be used; otherwise, it is immediately rejected as not applicable to the call being processed.

After filtering out those method definitions that are not applicable (because of an incorrect argument count or an argument type mismatch), we count the number of method definitions still under consideration. If it is zero, we have an invalid call (no accessible method can be called without error). If the count is one, we have a correct call.

If two or more method definitions are still under consideration, then we need to choose the most appropriate definition. Two issues are involved here. First, if a method is redefined in a subclass, we want to use the redefinition. For example, let `method M()` be defined in both classes `C` and `D`:

```
class C { void M() { ... } }
class D extends C { void M() { ... } }
```

If we call `M()` in an instance of class `D`, we want to use the definition of `M` in `D`, even though `C`'s definition is visible and type-correct.

Second, it may also happen that one definition of a method `M` takes an object of class `A` as a parameter, whereas another definition of `M` takes a subclass of `A` as a parameter. An example of this is:

```
class A { void M(A parm) { ... } }
class B extends A { void M(B parm) { ... } }
```

Now consider a call `M(b)` in class `B`, where `b` is of type `B`. Both definitions of `M` are possible, since an object of class `B` may always be used where a parameter of its parent class (`A`) is expected. In this case we prefer to use the definition of `M(B parm)` in class `B` because it is a ″closer match″ to the call `M(b)` that is being analyzed.

We now formalize the notion of one method definition being a ″closer match″ than another. We define a method definition `D` to be *more specific* than another definition `E` if `D`'s class is bindable to `E`'s class and each of `D`'s parameters is bindable to the corresponding parameter of `E`. This definition captures the notion that we prefer a method definition in a subclass to an otherwise identical definition in a parent class (a subclass may be assigned to a parent class, but not vica versa. Similarly, we prefer arguments that involve a subclass over arguments that involve a parent class (as was the case in the example of `M(A parm)` and `M(B parms)` used above). A method moreSpecific($def1, def2$) that tests whether method definition $def2$ is more specific than method definition $def1$ is presented in Figure 9.33, Marker 38.

If we have more than one accessible method definition that matches a particular argument list in a call, then we will filter out less-specific definitions. If, after filtering, only one definition remains (called the **maximally specific**

definition), we know it is the correct definition to use. Otherwise, the choice of definition is ambiguous and we must issue an error message. The process of filtering out less-specific method definitions is detailed in Figure 9.33, Marker ㊴, which defines the method FILTERDEFS(*methodDefSet*).

After we have reduced the set of possible method definitions down to a single definition, semantic analysis is almost complete. We must check for the following special cases of method calls:

- Method calls qualified by a class name (`className.method`) must be to a static method.
- A call to a method that returns void may not appear in an expression context (where a value is expected).

The complete semantic analysis for a method call, as developed above, is defined in Figure 9.34, Marker ㊵.

As an example of how method calls are checked, consider the call of `M(arg)` in method `test`:

```
class A { void M(A parm) {...}
          void M() {...} }

class B extends A { void M(B parm) {...}
                    void test(B arg) {M(arg);}}
```

At the call of `M(arg)`, three definitions of `M` are visible. All are accessible. Two of the three (those that take one parameter) are applicable to the call. The definition of `M(B parm)` in `B` is more specific than the definition of `M(A parm)` in `A`, so it is selected as the target of the call.

In all of the rules used to select among overloaded definitions, it is important to observe that the result type of a method is *never used* to decide if a definition is applicable. Java does not allow two method definitions with the same name that have identical parameters, but different result types, to coexist. Neither do C♯ or C++. For example, the following two definitions force a multiple definition error:

```
int add(int i, int j) {...}
double add(int i, int j) {...}
```

This form of overloading is disallowed because it significantly complicates the process of deciding which overloaded definition to choose. Not only must the number and types of arguments be considered, but also the context within which result types are used. For example in:

```
int i = 1 - add(2,3);
```

a semantic analyzer would have to conclude that the definition of add that returns a double is inappropriate because a double, subtracted from 1 would yield a double, which cannot be used to initialize an integer variable.

A few languages, such as Ada, *do* allow overloaded method definitions that differ only in their result type. An analysis algorithm that can analyze this more general form of overloading may be found in [Bak82].

Interface and Constructor Calls

In addition to methods, Java and C♯ allow calls to interfaces and constructors. The techniques we have developed apply directly to these constructs. An interface is an abstraction of a class specifying a set of method definitions without their implementations. For purposes of semantic analysis, implementations are unimportant. When a call to an interface is made, the methods declared in the interface (and perhaps its superinterfaces) are searched to find all declarations that are applicable. Once the correct declaration is identified, we can be sure that a corresponding implementation will be available at runtime.

Constructors are similar to methods in definition and structure. Constructors are called in object creation expressions (using new) and in other constructors; they can never be called in expressions or statements. A constructor can be recognized by the fact that it has no result type (not even void). Once a constructor call is recognized as valid (by examining where it appears), the techniques developed above to select the appropriate declaration for a given call can be used.

Subprogram Calls in Other Languages

The chief difference between method calls in Java and C♯ and subprogram calls in languages such as C and C++ is that subprograms need not appear within classes. Rather, subprograms are defined at the global level (within a compilation unit). Languages such as ML and Python also allow subprograms to be declared locally, just like local variables and constants. Some languages allow overloading; others require a unique declaration for a given name.

Processing calls in these languages follows the same pattern as in Java and C♯. Using scoping and visibility rules, possible declarations corresponding to a given call are gathered. If overloading is disallowed, the nearest declaration is used. Otherwise, a set of possible declarations is gathered. The number and type of arguments in the call is matched against the possible declarations. If a single suitable declaration is not selected, a semantic error results.

9.3 Summary

Semantic analysis is an essential component of the translation process. Type checking—the essence of semantic analysis—filters program errors and sets up effective program translation. Because semantic analysis spans many concerns, the use of semantic visitors allows an implementor to divide the overall analysis into small, easily understood components. Reachability analysis, for example, can augment standard type checking without obscuring its essential requirements.

Notions of overloading and inheritance can complicate method calls. It is important to remember that the basic calling mechanism—evaluate parameters and transfer control to a subprogram—is elegant and universal. Succeeding programming languages will undoubtedly add further refinements to semantic analysis, but the fundamental principles developed in this chapter will continue to be at the core of crafting a compiler.

Exercises

1. Extend the semantic analysis, reachability, and throws visitors for if statements (Section 9.1.2) to handle the special case in which the condition expression can be evaluated, at compile time, to true or false.

2. Assume that we add a new kind of conditional statement to C or Java, the *signtest*. Its structure is:

   ```
   signtest ( exp ) {
      neg:  stmts
      zero: stmts
      pos:  stmts
   }
   ```

   The integer expression `exp` is evaluated. If it is negative, the statements following `neg` are executed. If it is zero, the statements following `zero` are executed. If it is positive, the statements following `pos` are executed.

   Show the AST you would use for this construct. Revise the semantic analysis, reachability, and throws visitors for if statements (Section 9.1.2) to handle the signtest.

3. Assume we add a new kind of looping statement, the *exit-when loop* to C or Java. This loop is of the form:

   ```
   loop
      statements1
    exit when expression
      statements2
   end
   ```

   First, the statements in `statements1` are executed. Then, `expression` is evaluated. If it is true, then the loop is exited. Otherwise, the statements in `statements2` and `statements1` are executed. Next `expression` is reevaluated and the loop is conditionally exited. This process repeats until `expression` eventually becomes true (or else the loop iterates forever).

   Show the AST you would use for this construct. Revise the semantic analysis, reachability, and throws visitors for while loops (Section 9.1.3) to handle this form of loop. Be sure the special case of `expression` being constant-valued is handled properly.

4. Some languages, such as Ada, allow `switch` statements to have cases labeled with a *range* of values. For example, with ranges, we might have the following (using Java or C syntax):

   ```
   switch (j) {
      case 1..10,20,30..35 : option = 1; break;
      case 11,13,15,21..29 : option = 2; break;
      case 14,16,36..50    : option = 3; break;
   }
   ```

   How would you change the semantic analysis, reachability, and throws visitors of Section 9.1.6 to allow case label ranges?

5. Consider the following program fragment:

   ```
   ...
   while (a) {
      if (b)
         break;
      else if (c)
         a = update(a);
         continue;
      else return;
      print(a,b,c)
   }
   ...
   ```

   Note that no matter which leg of the if is executed, the print statement cannot be reached. This is quite possibly an error, and certainly deserves a warning message.

   Explain how the `isReachable` and `terminatesNormally` values set during reachability analysis can be used to conclude that the above print statement is unreachable.

6. Recall that in Java and C♯ a method *M* is required to list all checked exceptions that might be thrown to a caller of *M*. In testing *M* it might be helpful to verify that each exception listed in *M*'s ”throws list” *really can* be thrown.

   Explain how to use the techniques of Section 9.1.7 to verify that each listed exception in a throws list of method *M* can potentially reach a caller of *M*. Be sure to include exceptions thrown by methods called from *M* (either directly or indirectly).

7. Some programming languages, such as Ada, require that within a `for` loop, the loop index should be treated like a constant. That is, the only way that a loop index can change is via the loop update mechanism listed in the loop header. Thus (using Java syntax):

   ```
   for (i=1;i<100;i++)
     print(i)
   ```

   is legal, but:

   ```
   for (i=1;i<100;i++)
     print(--i)
   ```

   is not legal.

   Explain how to change the semantic analysis visitor of Section 9.1.4 to enforce read-only access to a loop index within a loop body.

8. When methods are used as functions, calls may be *nested*. Thus, given a method:

   ```
   int t(int a, int b, int c){ ... }
   ```

   the following call is legal:

   ```
   z = t(0, 1, t(2, t(3,4,5), 6));
   ```

   Are the semantic analysis techniques developed in Section 9.2 adequate to handle nested method calls? What if the methods being called are overloaded?

9. (a) Certain data that a method manipulates may require special protection. For example, a password or account number should only be ″touched″ by code we know to be trustworthy.

   Assume that in a program we can mark a variable as `secure`. A secure variable can only be manipulated by methods within the package containing the original declaration of the secure variable.

   Outline how the semantic analysis techniques of this chapter can be used to verify that a secure variable does not ″leak″ out of the package that ″owns″ it.

   (b) The analysis suggested in part (a) may be too restrictive in that it disallows the use of *all* library methods, even very benign ones. Suggest a way of tagging selected library methods as ″trusted.″ We will generalize the security analysis of part (a) by allowing secure data to be passed to trusted library methods. Note that library methods that can print a variable or write it into a file are *never* trusted.

10. One of the problems with the class structure used by Java and C♯ is that field and method declarations (which are terse) are intermixed with method implementations (which can be lengthy and detailed). As a result, it can be hard to casually ”browse” a class definition.

    As an alternative, assume we modify the structure of a class to separate declarations and implementations. A class begins with class declarations. These are variable and constant declarations (completely unchanged) as well as method headers (without method bodies).

    An ”implemented as” section follows, which contains the bodies of each method declared in the class. Each method declared in the class must have a body defined in this section, and no body may be defined unless it has been previously declared. Here is a simple example of this revised class structure:

    ```
    class demo {
         char skip = '\n';
         int f();
         void main();
    implemented as
         f:      {return 10;}
         main:   {print("Ans =",f(),skip); }
    }
    ```

    What changes are needed in the semantic analysis of classes and methods to implement this new class structure?

11. Just as variables and fields may be initialized, some programming languages allow formal parameters in methods to be initialized. An initialized parameter provides a *default value*. In a call of a method, a user may choose to not provide an explicit parameter value, choosing the default instead. For example, given:

    ```
    int power(int base, int expo = 2) {
       /* compute base**expo */}
    ```

    the calls `power(100,2)` and `power(100)` both compute the same value (100^2).

    What changes would be needed in the semantic analysis of method calls to correctly handle initialized formal parameters? Be sure to consider how overload resolution is affected.

12. Most programming languages, including C, C++, C♯, and Java pass parameters *positionally*. That is, the first value in the argument list is the first parameter, the next value is the second parameter, and so on.

    For long parameter lists this approach is tedious and error prone. It is easy to forget the exact order in which parameters must be passed. An alternative to positional parameters is **keyword parameters**. Each parameter value is labeled with the name of the formal parameter it represents. The order in which parameters are passed is now unimportant.

    For example, assume method `M` is declared with four parameters, `a` to `d`. The call `M(1,2,3,4)`, using ordinary positional form, can be rewritten as `M(d:4,a:1,c:3,b:2)`. The two calls have identical effects; only the notation used to match actual parameters to formal parameters differs.

    What changes would be needed in the semantic analysis of calls, as defined in Figure 9.34, to allow keyword parameters?

13. As mentioned in Section 9.1.6, C, C++, and Java allow non-null cases in a `switch` statement to ”fall through” to the next case if they do not end with a break statement. This option is occasionally useful, but far more often leads to unexpected errors. Suggest how the semantic analysis of `switch` statements (Figure 9.21) can be extended to issue a warning for non-null cases that do not end in a break. (The very last case never needs a break since there are no further cases to ”fall into.”)

14. Modern programming languages severely restrict the use of labels and gotos. Java, for example, allows no gotos at all (though labeled breaks and continues are allowed).

    In early programming languages, rules were very different. Gotos were widely used. Moreover, **label variables** were sometimes allowed. That is, a variable of type label could be defined. Label values could be assigned to label variables, and gotos to label variables were allowed. Thus, the following might appear:

    ```
    Label L;
    ...
    if (option)
         L = target1;
    else L = target2;
    ...
    goto L;
    ```

    What changes are needed in the semantic analysis of gotos if label variables are allowed? What can happen if a label inside an active method is allowed to ”escape” outside the method?

15. The method inExpressionContext is used in the visit method for *Calling* nodes (Figure 9.34) to determine if a call is being used in a context where a return value is expected. But the calling context is found *above* the *Calling* node in the AST and these trees have no upward links.

    Suggest how inExpressionContext might be efficiently implemented.

10

Intermediate Representations

Compilers translate programs from their source form into a representation that is suitable for interpretation or execution. The chapters of this book have thus far considered the scanning, parsing, and semantic analysis of programs written in source languages such as Java^{TM} and C++. The **abstract syntax tree** (AST) was introduced in Chapter 7 to represent the source program in a form that omits unnecessary syntactic detail. Semantic information was developed in Chapters 8 and 9 to prepare the AST for code generation. The next and final step of a simple compiler is code generation. Code generation for a virtual machine is considered in Chapter 11. Chapters 12 and 13 consider runtime support and code-generation techniques for low-level targets.

Preparation for generating code for a given architecture should include gaining some familiarity with the architecture. Such studies may include reading a specification of the code that will be generated, examining code sequences from other compilers, and writing some sample sequences by hand. In this chapter, we examine a form of code known as an **intermediate representation** (IR). Where such representations are sufficiently formalized, documented, and widely used, they are often merit designation as **intermediate languages** (ILs).

Intermediate representations and languages are typically more concise and abstract than lower-level target languages. We therefore study intermediate code generation in Chapter 11 before proceeding to lower-level code generation in Chapter 13. This allows us to focus on the code-generation *process* in Chapter 11 without having to explain or understand the details of a machine's instruction set. For example, the **Java Virtual Machine** (JVM) contains an instruction (`invokevirtual`) that performs a virtual method call. This single instruction simplifies the discussion of code-generation strategies in Chapter 11. At a lower level, many instructions must be generated to accomplish the call and its return, as described in Section 13.1.3 on page 496.

The comparatively high level of the instructions found in intermediate representations assumes also that a fair amount of **runtime support** is present in the **virtual machine** that interprets the intermediate representation. Continuing with the example of a virtual method invocation, runtime support must be present for managing the storage that can be accessed by the method. The details of providing such support are covered in Chapter 12.

Intermediate representations have many advantages, and most compilers use one or more levels of intermediate representation before generating their ultimate target code. Section 10.1 considers the rationale of using intermediate representations. Section 10.2 presents an overview of the Java Virtual Machine, which is used extensively in Chapter 11, as an example of intermediate code generation. Section 10.3 presents *static single assignment form* as an intermediate representation with properties conducive to program optimization.

Studying these and other IRs offers insight into programming language design as well as preparation for code generation.

10.1 Overview

Most applications are written in relatively high-level **source languages** such as C++ or Java. Such languages offer extensible data and control abstractions that are conducive to algorithmic expression. However, most computers lack any native comprehension of such high-level languages. They rely instead on compilers to translate source programs into some *target* machine language, the instructions of which typically operate on a dramatically reduced scale. Compilers and other programming language translation tools bridge the **semantic gap** between high- and low-level program representations. That gap is typically traversed as a sequence of steps, each involving an intermediate representation.

For example, a compiler might accept Java programs as input and ultimately produce machine instructions for an Intel® architecture. Interposed between the original source and ultimate target, a **class file** might be generated according to the specification of the JVM. Other intermediate representations may be produced as well, both before and after the class file is generated.

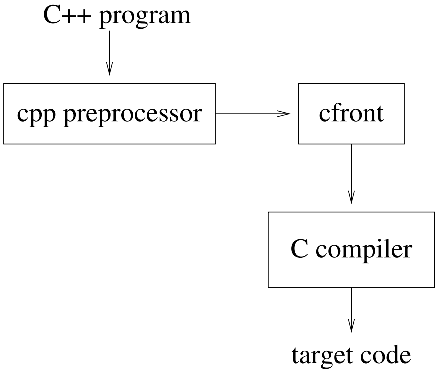


Figure 10.1: Use of `cfront` to translate C++ to C.

10.1.1 Examples

The early C++ compilers did not produce machine code directly [Str94, Str07]. As shown in Figure 10.1, the C++ language was initially translated from source to standard C, with the resulting C program compiled to machine code. In fact, since C++ programs could use the standard C preprocessor, source programs were first translated by C's preprocessor (`cpp`) and then translated into standard C by `cfront`. Thus, in the trek from C++ to machine code, there are two articulated intermediate points and ILss:

- From the perspective of the C and C++ programming languages, which include preprocessor directives, the output of `cpp` is an intermediate language. Although no name is formally given to this language, it is the subset of C or C++ obtained after processing all preprocessor directives. This simplifies construction of the rest of the compiler, which need not worry about any preprocessor directives.

- From the perspective of `cfront`, standard C is an ILs, interposed between C++ and machine code.

The `cfront` approach served nicely to prototype C++, but it was soon supplanted by a more integrated approach that processed C++ directly. Such compilers could better diagnose and report compile-time errors.

As another example, consider the steps by which LaTeX [Lam95]—the language in which this book was authored—is translated into print, as shown in Figure 10.2. The LaTeX document-preparation system does not produce printable pages directly. Instead, LaTeX is translated into a more basic ILs called TeX, which is in turn translated into a device-independent intermediate

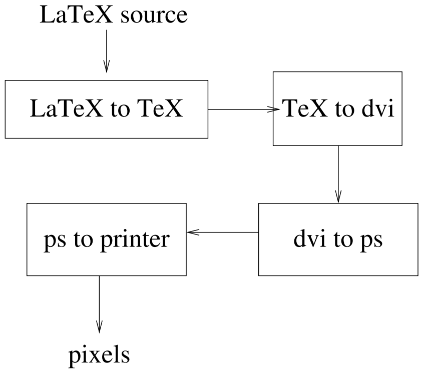


Figure 10.2: Translation from LaTeX into print.

representation called `dvi`, which may in turn undergo several translations before producing print.

Intermediate representations are often deployed to enhance portability. In this case, TeX does not target any one printer, but instead produces a set of binary data structures called `dvi`. A separate program reads the `dvi` intermediate representation and produces viewable or printable pages. Thus, TeX need not undergo modification to accommodate a new kind of printer. Instead, a new program is written (or an old one is modified) to translate the relatively simpler `dvi` representation into a printer's "instruction set" (e.g., PostScript).

In consideration of the above examples, ILss in a translation system face the following challenges:

- An intermediate language must be precisely defined. Failure to define these languages carefully can have the same negative consequences as imprecise definitions of programming languages.

- Translators and processors must be crafted for an ILs and any IRs. Where such tools operate beyond the user's focal plane, care must be taken to make the tools as transparent as possible. For example, C developers may be unaware that the `cpp` preprocessor is invoked before the actual C compiler.

- Connections must be made between levels so that feedback from intermediate steps can be related to the source program. For example, the output from `cpp` may contain errors that are caught downstream by the C compiler. Messages about such errors should reference the *original* source lines, and not the text or line number of an intermediate representation beyond the user's view.

The extra steps associated with use of an intermediate language raise justifiable concerns of efficiency. A given system that uses ILss may not enjoy the performance of a competing product that avoids ILss and takes a more direct approach. The benefits and cost of an ILs must be analyzed and compared. While gratuitous levels of intermediate representation are unwise, thoughtful system designs include ILss to simplify the task at hand as well as reduce the cost of adapting and maintaining the given system. In support of this, we next examine some principles and examples concerning the role of ILss in an effective programming language translation system.

10.1.2 The Middle-End

For a compiler, the terms **front-end** and **back-end** refer to the phases responsible for parsing the input language and generating the target language, respectively. Most compilers are structured with a set of components between the front- and back-ends, commonly called the compiler's **middle-end**. While such a term may seem nonsensical, the collection of phases situated between a compiler's front-end and back-end can greatly simplify the crafting of a compiler. In particular, compiler suites that host multiple source languages and target multiple instruction sets obtain great leverage from a middle-end.

Consider a suite of compilers (such as **GNU Compiler Collection** (GCC)) for s source languages (C++, Fortran, Java, etc.) and t target architectures (Intel, Sparc$^{\text{TM}}$, MIPS$^{®}$, etc.). If a different product is needed for each situation, then this suite might contain $s \times t$ source- and target-specific compilers, as shown in Figure 10.3(a). However, this work can be reduced to $s + t$ effort if an ILs can be introduced between the source and target specifications, as shown in Figure 10.3(b). Now the suite contains s front-ends and t back-ends: each front-end translates its source language to the ILs; each back-end translates the ILs into native code for its architecture. The middle-end processes the ILs in ways that benefit all of the sources and targets.

Additional advantages obtained by crafting a compiler to include a middle-end and formally defining its ILs are as follows:

- An IL allows various system components to interoperate by facilitating access to information about the program undergoing translation.

  For example, the IL may contain symbolic information such as variable names, variable types, and source line numbers; such information could be useful in the debugger. Similarly, program development tools such as class browsers and performance profilers, operating at different points in the software development cycle, can share and utilize program information through the IL.

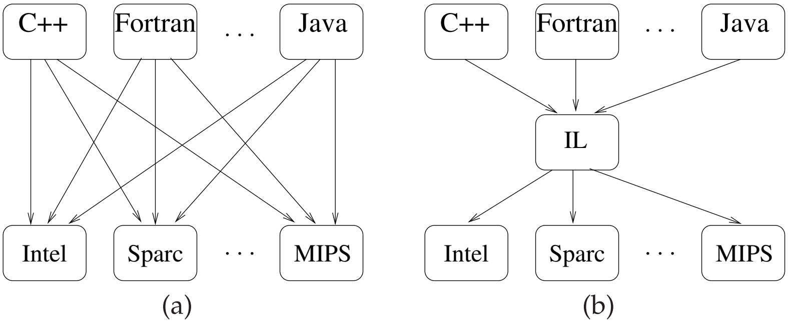


Figure 10.3: A middle-end and its ILs simplify construction of a compiler suite that must support multiple source languages and multiple target architectures.

- An ILs simplifies development and testing of the system's components. The front- and back-ends can be tested independently by artificially synthesizing ILs for the back-end until the front-end is ready.

- The middle-end contains phases that would otherwise be duplicated among the front- and back-ends of a compiler suite. Such phases are generally limited to ILs-to-ILs transformations.

- A carefully designed and suitably formalized ILs allows components and tools to interface with the ILs-bearing product, either by accepting the product's ILs as input for some task, or by acting as a surrogate provider of the ILs.

  In a commercial setting, the articulated ILs allows multiple vendors to share compiler and software toolchain components that can be based on the ILs.

- In a research setting, the ILs can simplify the pioneering and prototyping of new ideas by providing easy access to the requisite infrastructure.

  Consider a compiler writer who wishes to experiment with new ideas for eliminating computational redundancy. The task of developing a complete compiler from scratch is daunting. However, if the idea can instead be prototyped using a compiler's ILs, then the expense of writing front- and back-ends can be avoided. Moreover, if the system is multisource or multitarget, then deploying the optimization at the ILs-level can obtain benefits for multiple languages and multiple target platforms.

- The ILs and its interpreter can serve as a reference definition of a language. Implementation of the interpreter often resolves issues that might be ambiguous or unclear in the formal specification.

For example, the storage model described in the JVM specification has been refined and improved based on common implementations of the specification. The **Descriptive Intermediate Attributed Notation for Ada** (DIANA) was developed as a formal specification of information that should be communicated between the front- and back-end of an Ada compiler.

- Interpreters written for a well defined ILs are helpful in testing compilers and porting a compiler among platforms.
- An ILs enables the crafting of a **retargetable code generator**, which greatly enhances the compiler's **portability**. Numerous compilers have been developed using ILss for this reason [CG83, Ott84]:

Source Language	Intermediate Language
Pascal	P-code
Java	JVM
Ada	DIANA

The most popular and widely ported compiler suite is the GCC, which offers multiple levels of ILss.

In summary, ILss play an important role in reducing the cost and complexity of compilers. Some are designed to support a specific language. For example, the JVM is intended to support interpretation of Java, and DIANA was designed specifically for Ada. However, other ILss are formulated to support diverse front- and back-ends. The GCC includes two ILss, one that represents source programs at a relatively high level and another that represents machine instructions abstractly. The Microsoft® compiler suite uses **Common Intermediate Language** (CIL) as an ILs and the **Common Language Runtime** (CLR) as a generic interpreter of CIL.

We next examine several IRs, with the goal of understanding their structure before proceeding to the code generation and optimization material of Chapters 11, 13, and 14.

10.2 Java Virtual Machine

We next describe some specifics of an ILs that has served as the reference platform for interpreting Java programs. The JVM interprets Java class files, which represent the code and data of a Java class. Although the Java language continues to evolve, the JVM has been relatively stable and serves well as a target for intermediate code generation in Chapter 11.

10.2.1 Introduction and Design Principles

The JVM interprets class files, which are binary encodings of the data and instructions needed to execute a Java program. To simplify exposition, the contents of a class file are typically discussed in printable form. For example, the instruction that specifies integer addition has the numerical value of 96, but we typically refer to it as `iadd`.

Notation for describing various aspects of the JVM class files is borrowed from the JVM reference [JVM] and the Jasmin user manual [Mey]. The JVM is designed with the following principles in mind:

Compactness Because JVMs are deployed in browsers and mobile devices, Java class files are designed to be relatively compact. In particular, the JVM's instructions are in *nearly* **zero-address form** so that most instructions manipulate data at the top of Java's **runtime stack**. We refer to the topmost location as **top-of-stack** (TOS).

For example, the `iadd` instruction pops two items off the stack and pushes their sum onto the TOS. Only a single byte is needed to represent such an operation because the instruction's operands are **implicit**. Generally, such compaction is achieved with a loss of runtime performance: stack manipulation is generally slower than processing operands in a register file.

The goal of compactness drives the design of JVM instructions to include multiple instructions that accomplish the same effect. For example, there are many ways to push 0 on TOS. The shortest instruction, `iconst_0`, takes only a single byte. The most general instruction, `ldc_w 0`, takes 3 bytes for the instruction and consumes a constant-pool entry. While the `iconst_0` instruction is not strictly necessary, its inclusion allows greater code compaction, because pushing 0 on TOS is a frequent operation.

Safety Because the JVM may be deployed in an environment that cannot tolerate badly behaved programs, the JVM's instructions are designed to execute safely: an instruction can reference storage only if said storage is of the type allowed by the instruction and only if the storage is located in an area appropriate for access. Moreover, the instructions are designed so that most safety errors can be caught prior to executing code.

In a pure zero-address form (which the JVM is *not*), a register load is accomplished by computing a register number that is pushed on TOS. A load instruction then pops the register number from the stack, accesses the register's contents, and pushes the contents on TOS.

From a security point of view, the purely zero-address form is problematic because the registers that could be accessed by a load instruction may not be known until runtime. For example, the zero-address form allows a method to compute a register number and leave it on TOS to serve as the operand of a subsequent load instruction. While the zero-address approach is more general, and runtime checks could be deployed to check the validity of a load instruction, a more reliable and efficient approach would check such instructions prior to running the code.

As a compromise, the load instruction in the JVM is not zero-address, but instead specifies the register number as an **immediate operand** of the instruction. For example, the instruction `iload 5` causes the contents of register 5 to be pushed on TOS. When the class file is loaded, the instruction is checked to ensure that register 5 falls within the range of registers its method can access. The instruction will always access register 5, because the immediate operands of an instruction cannot be changed at runtime. Thus, by discovering that the register reference is valid prior to running the code, no further checks of this kind are necessary at runtime.

When a class file is loaded, many other checks are performed by the **bytecode verifier**. The JVM instruction set and class file format are designed to facilitate such checks.

10.2.2 Contents of a Class File

A JVM class file is organized into sections called **attributes** that contain various information about the compiled class. Here we cover only those attributes that are most relevant to the code-generation topics discussed in Chapter 11. Throughout this discussion we describe various aspects of the JVM using the human-readable Jasmin syntax to denote the binary information contained in a class file.

Types

Like Java, the JVM offers primitive and reference types. Types are generally used to specify the signature of fields and methods, and most instructions require inputs of a certain type. Primitive types in the JVM are designated by a single character, as shown in Figure 10.4.

A reference type *t* is designated as `L`*t*`;` with *t* specified as follows. Each dot in the type is replaced by a forward slash, which results in a Unix®-like file path to the class file for the type. The JVM uses such paths to locate a class file at runtime. For example, the `String` type in Java is actually found in the `java.lang` package. Its full type name is therefore `java.lang.String` and it has the type designation `Ljava/lang/String;` in the JVM. Because of its need to represent `String` constants efficiently, the JVM is aware of the `String` reference type, almost as if it were a primitive type.

Type	**JVM designation**
`boolean`	`Z`
`byte`	`B`
`double`	`D`
`float`	`F`
`int`	`I`
`long`	`J`
`short`	`S`
`void`	`V`
Reference type *t*	`L`*t*`;`
Array of type *a*	`[`*a*

Figure 10.4: Java types and their designation in the JVM. All of the integer-valued types are signed. For reference types, *t* is a fully qualified class name. For array types, *a* can be a primitive, reference, or array type.

As shown in Figure 10.4, the JVM can construct an array of a given type (primitive or reference). While the result of the construction is a form of a reference type, it is formally designated using the array character `[`. For aesthetic purposes, it might be nice if the opening bracket of the designation were balanced by a closing bracket, but such is not the case. Remember that the JVM and Jasmin syntax are rarely read or written directly by humans. The syntax is therefore designed to be terse rather than familiar.

Constant Pool

Java programs refer to various runtime constants, and these are usually allocated in a class's **constant pool**. The constant pool is designed as a **tagged union** as discussed in Chapter 8. Each entry represents a constant of a given type, such as `int`, `float`, or `java.lang.String`, and each can take as much room as it needs in the constant pool. An `int` might take only four bytes, but a `String`'s allocated space will depend on its length.

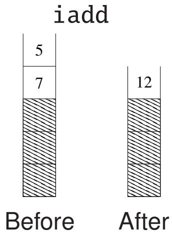
A constant is referenced by its ordinal position (0, 1, 2, etc.) in the constant pool, and not by a byte-offset into the pool (see Exercises 2 and 3). For some instructions (e.g., `ldc`), a constant-pool reference occupies a single byte. Other instructions (e.g., `ldc_w`) provision two bytes for a constant-pool reference.

10.2.3 JVM Instructions

While an exhaustive list of the JVM's instructions appears elsewhere [Mey, JVM], we describe members of various *families* of instructions to provide an overview of the JVM instruction set.

Arithmetic

JVM instructions that compute results based on simple mathematical functions operate by popping their required number of operands (typically 2) from the runtime stack, computing the required result, and finally pushing the result on TOS.



The `iadd` instruction pops the top two elements off of the stack and pushes their sum onto the stack. The instruction expects the operands to be primitive `int` types, and the result is also `int` type. All operations involving `int` types are performed using 32-bit, **two's complement** arithmetic. Such arithmetic never throws any exceptions: overflow or underflow occur silently.

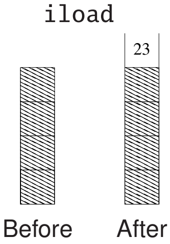
Instructions that perform addition on other primitive types are `fadd` (`float`), `ladd` (`long`), and `dadd` (`double`). There are no instructions for integer types that are shorter than `int`, such as `byte` and `short`. Such arithmetic is performed on `int` types with precision lost as data are stored. Instructions are available for performing the usual arithmetic operations such as subtraction, multiplication, division, and remainder.

Register Traffic

As is frequently the case with IRs, the JVM has (practically) an unlimited number of **virtual** registers it can reference. Each method declares how many registers it can reference, and the JVM sets aside space for those registers for each invocation of the method. Such space is usually allocated in a method's **stack frame** (see Section 12.2 on page 447).

JVM registers typically host a method's **local variables**, which are similar to a machine's architected registers (Section 13.3 on page 505). Registers starting from 0 are set aside for a method's parameters. For **static** methods, register 0 holds the method's first declared parameter. For instance-based methods, register 0 holds **this** (the object's self-reference) and register 1 holds the method's first declared parameter. When a method is invoked, parameter values are automatically popped from the caller's stack and deposited into the low-numbered registers.

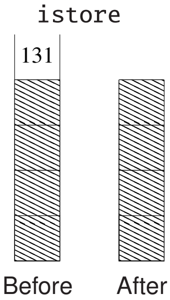
The JVM's registers are **untyped**, so they can hold any kind of value. For values that require two registers (`long` and `double` types), an even-odd pair of registers must be used.



The `iload` instruction pushes the contents of a JVM register on TOS. The instruction must contain an immediate operand designating the register whose contents should be loaded. The register is unaffected by the instruction.

If register 2 contains the value 23, then the example shows the results of executing `iload 2`.

To facilitate compression, the JVM has some single-byte instructions that load low-numbered registers. The `iload 2` instruction takes two bytes: one for the instruction and one for the operand. The operation can be abbreviated as `iload_2`, which takes only a single byte, as the opcode implies that register 2 is loaded.



Instructions are also available for moving data from the stack to a register. The `istore` instruction pops a value from the stack and stores it in the register specified as an immediate operand of the instruction.

In the example, an `istore 10` would pop the value 131 from the stack and store it in register 10. There is no abbreviated form of this instruction because register 10 is beyond the registers provisioned for the single-byte `istore` instructions.

There are variations of `iload` and `istore` for each type of data that can be loaded or stored. For example, `fload` *n* reads a `float` value from register *n* and pushes it on TOS. There are no register instructions specific to the `boolean` type. The JVM uses an `int` representation for `boolean`, with 0 representing `false` and 1 representing `true`. Types `char`, `byte`, and `short` are similarly treated as `int` at the register level, because 32 bits can accommodate their values as well.

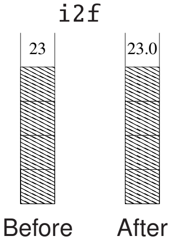
All reference types are loaded and stored using `aload` and `astore` instructions, respectively. An object reference takes the same space as an `int`: 32 bits. The value of 0 is reserved to represent `null`.

A compiler is free to use a method's registers in any manner, provided that register usage respects type consistency. For example, suppose register 4 holds a reference to an object after an `astore 4` instruction. The reference cannot be pushed onto the stack as an `int` using an `iload 4` instruction. Such an error is detected by the **bytecode verification** phase of the JVM.

Registers and Types

Although registers are untyped, most Java execution environments are required to perform **static analysis** (often called **bytecode verification**) on the JVM code to ensure that values flow in and out of registers without compromising Java's type system. Such analysis prevents a JVM program from loading a reference to an object (using `aload`) and subsequently performing math on the reference to trick the JVM into accessing storage inappropriately. In the `iload` example on page 402, analysis can show that the stack's top contains an `int` value. Whatever instruction consumes that value must type-match successfully with the `int` value. For example, an `istore 3` would successfully pop the 23 from TOS and store it in register 3. However, an `fstore 3` would be detected by the analysis as a faulty instruction when the analysis is performed (prior to executing the code).

In this regard, the JVM appears to be stricter than the Java language: an `int` value can be treated as a `float` without casting in Java. However, it is important to understand that the transition from `int` to `float`, while allowed by the language without casting, is nonetheless a type transformation. Semantic analysis as described in Chapters 2 and 8 can insert the `int`-to-`float` **type conversion**, which is then realized by generating an `i2f` instruction.



The JVM type conversion instructions operate by popping a value off the stack and pushing its converted value. The stack must contain a value of the appropriate type at its top,

In the example, the `int` 23 is at the TOS. When the instruction has finished, the value is effectively replaced by its `float` representation.

The bit patterns for 23 and 23.0 are markedly different, with the former in two's complement and the latter in **IEEE floating point** format. Moreover, bytecode verification tracks the type of the stack cells, and the `i2f` instruction leaves a cell of type `float` on TOS.

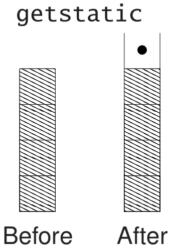
Static Fields

A class's **static fields** are present in every instance of the class. Space for such fields is provisioned when the class is loaded and initialized. Thereafter, a static field can be loaded or stored by the `getstatic` and `putstatic` instructions, respectively.

The action of `getstatic` is similar to the various load instructions (`iload`, `aload`, etc.) in that the result of the fetch is pushed on TOS. The form of a `getfield` instruction as coded in Jasmin is follows:

`getstatic` *name type*

where *name* is the name of the static field, prefaced by its fully qualified class name, and *type* is the *expected* type of the result.



Many Java programs reference `System.out` for console output. Such accesses are actually references to the static field `java.lang.System.out`, whose type is `java.io.PrintStream`.

The example shows that after the `getfield` instruction executes, the stack has a new element at TOS. The • on the stack represents the reference to `System.out`.

The JVM instruction generated for that static field above is therefore

```
getstatic java/lang/System/out Ljava/io/PrintStream;
```

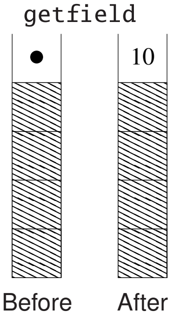
The actual representation of the instruction in the JVM occupies only 3 bytes: one specifies the `getstatic` opcode and the other two form a 16-bit integer specifying a **constant-pool** entry. Recall that constant-pool entries are denoted ordinally (0, 1, 2, etc.) and not by their byte-offset in the constant pool. In this case, the designated constant-pool entry contains the *name* and *type* operand values for the `getfield` instruction.

Static variables can be modified by the `putstatic` instruction, which pops a value from TOS and stores the value at the location specified immediately by the instruction.

Instance Fields

A class can declare **instance fields** for which instance-specific storage is allocated. Every instance of type *t* is provided with storage for *t*'s instance fields. To access those fields, a particular instance of *t* must be provided.

The `getfield` instruction pushes the value of a particular instance field on TOS. The syntax for a `getfield` instruction is exactly the syntax of the `getstatic` instruction: a *name* and *type* are specified as immediate operands. However, because the instruction also requires an instance of the accessed field, the semantics of the instruction specify that the TOS must contain a reference to the instance whose field is to be accessed.



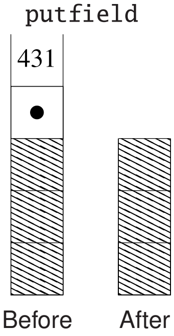
Consider a `Point` class such that each instance has `int` fields: `x` and `y`. Consider a particular instance that has 10 and 20 values for its `x` and `y`, respectively.

The example shows a `Point` reference (•) on TOS. The instruction

```
getfield Point/x I
```

retrieves the value of •'s `x` field, making sure it is an `int`, and pushes the value (10) on TOS.

The `putfield` instruction is the instance-specific version of the `putstatic` instruction.



In the example, the object reference (•) refers to an instance of a `Point` object. The instruction

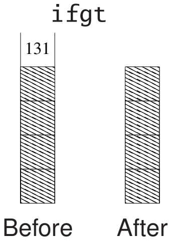
```
putfield Point/x I
```

pops *two* values from TOS. The first is the value (431) that should be stored at the field (`x`) specified in the `putfield` instruction. The second is a reference to a `Point` object, shown as •. When the instruction completes, •'s `x` field will have the value 431, and the stack will have two fewer items.

Branching

The JVM provides instructions to alter the control flow of the executing program. Control can be transferred unconditionally to the instruction at location q by a `goto` instruction. The instruction occupies 3 bytes: one byte specifies the `goto` opcode and the other two bytes are concatenated to form a signed 16-bit offset denoted here as Δ. If p is the location of the current `goto` instruction, control is transferred to the opcode at $q = p + \Delta$. In Jasmin, offsets are computed automatically and targets are specified symbolically using labels. There is also a 5-byte `goto_w` instruction, which provisions 4 bytes to hold Δ (see Exercise 7).

There are several kinds of instructions that accommodate conditional branches. Each such instruction contains an opcode that specifies the conditional test and a branch to be taken if the test is true.

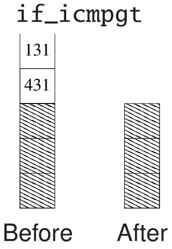


The `ifgt` instruction and its cognates expect that the TOS is a signed `int` value, The branch is taken if the condition (in this case, greater-than-zero) is satisfied.

In the example, 131 is popped and compared against 0. Because $131 > 0$, the branch will be taken. Had the comparison failed, control would pass to the instruction following the `ifgt` instruction.

The JVM contains 6 such instructions, one for every possible comparison of an `int` value against 0: `ifeq` (=), `ifne` (≠), `iflt` (<), `ifle` (≤), `ifgt` (>), and `ifge` (≥). A separate opcode is provisioned for each instruction.

Some programs call for comparisons of non-zero values. While the instructions described above are sufficient for such programs (see Exercise 9), a shorter sequence of instructions can be generated using the following relatively more complex instructions.



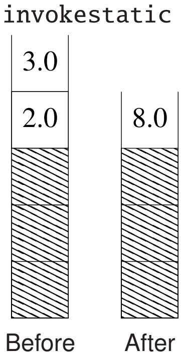
Consider a source program that at some point has $a = 431$ and $b = 131$ and must next evaluate whether $a > b$. After an `iload` instruction is issued for a and b, in that order, the stack appears as shown on the left.

In the example, the `if_icmpgt` instruction pops the top two elements and performs the comparison $431 > 131$. Because this test succeeds, the branch is taken.

There are 6 instructions in this family: `if_icmpeq` ($a = b$), `if_icmpne` ($a \neq b$), `if_icmplt` ($a < b$), `if_icmple` ($a \leq b$), `if_icmpgt` ($a > b$), and `if_icmpge` ($a \geq b$).

Static Method Calls

There are several forms of method-calling instructions in the JVM. In object-oriented languages, a method could be common to all instances of some type t or instance specific. In the former case, Java designates such methods as **static**. A static method of type t, like a static field, is referenced by its type t and does not require an instance of t to be called. An example of a static method is `Math.pow(double a,double b)`, which returns a^b. Such methods are called using the `invokestatic` instruction.



The `Math.pow` static method is called using the Jasmin notation

```
invokestatic java/lang/Math/pow(DD)D
```

which specifies the full path to the method and also contains the **signature** of the method.

In the example, two values (2.0 and 3.0) are popped and supplied to the `Math.pow` static method as its first and second parameters, respectively. When `Math.pow` completes, its result (8.0) is pushed on TOS.

From the above example, it should be clear that a method's parameters are pushed on the stack in left-to-right order. If a static method takes *n* parameters, then its n^{th} parameter is on TOS just as the method is called.

For a **method signature**, the symbols between the parentheses indicate the method's input parameter types, using the notation described in Figure 10.4. In the above example, two `double` parameters (`(DD)`) are expected. A method's return type is specified just after the parentheses. In the above example, the return type is also `double`.

Although the Jasmin notation shows the method and its signature as part of the `invokestatic` instruction, the method and signature descriptive information is actually held in the constant pool. The `invokestatic` instruction occupies 3 bytes, with the second two bytes forming an ordinal index into the constant pool.

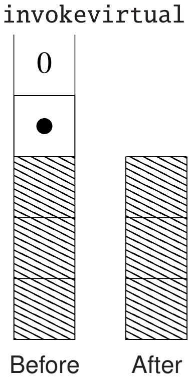
Instance-Specific Method Calls

An instance-specific method, such as `PrintStream.print()`, is invoked in a manner similar to `invokestatic` with the following differences:

- Because the method is instance-specific, an instance must be pushed on the stack before the method's parameters. The instance becomes **this** inside the called method.

- The `invokevirtual` operator is used instead of `invokestatic`.

Thus, an instance-specific method formally declaring that it takes *n* parameters $(p_1, p_2, \ldots p_n)$ actually takes $n+1$ parameters where p_0 is effectively the called method's **this**.



The PrintStream.print(boolean) method is called using the Jasmin notation

```
invokevirtual java/io/PrintStream/print(Z)V
```

which specifies the full path to the method and indicates that the method takes one boolean parameter (indicated by Z) and returns no values (indicated by V after the parentheses).

In the example, the • is an instance of a PrintStream class (e.g., System.out). The 0 on TOS is Java's integer encoding of false.

Although the method declares that it takes only one parameter, it is instance-specific and will consume two values from the stack. An instance of a PrintStream class must be pushed first (shown as •), followed by the parameters declared by the called method. Within the PrintStream.print() method, the • becomes **this**. The method completes but its return type is void so it returns no result. The method has the side effect of printing false to its PrintStream (**this**, inside the method).

Other Method Calls

Almost all methods in Java are called by invokevirtual or invokestatic. However, important exceptions include constructor calls and calls based on super, which are handled by the invokespecial instruction.

- A constructor call is *special* in the following sense. An uninitialized reference to an object instance is pushed on TOS (usually by a new instruction). The method name actually involved is <init> within the type of the object pushed on TOS. The constructor consumes the reference from TOS as its input parameter (**this**) along with any declared parameters. All constructors are void so nothing is returned from the constructor call. A code generator must be aware of this behavior and issue the appropriate instructions to be able to access the instantiated object (see Exercise 10).

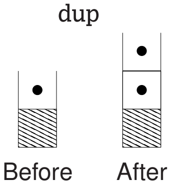
- Methods called by invokevirtual are dispatched based on the actual (runtime) type of the instance on which the method is invoked. If the actual instance type is *t*, then the invoked method will be sought first in class *t* and then in *t*'s parent in the object hierarchy, all the way up to Object (the *superest* of all superclasses).

  A method call based on super begins its search for an appropriate method at the current class's parent in the object hierarchy. In other words, suppose a method FOO is invoked on an object of actual type *t*. If that method is resolved in class *s*, then *s* must be a superclass of *t* (if $s = t$, then *s* is a nonproper superclass of *t*, so the definition works). If `s.`FOO( ) invokes a method BAR using **`super.`**BAR( ), then the search for an appropriate BAR begins at the first proper superclass of *s* (i.e., the parent of *s* in the object hierarchy).

- The `invokespecial` method can also be used to invoke a `private` method, but this appears to be for efficiency reasons only. A `private` method cannot be overriden, so there is no reason to employ a virtual method dispatch. Such methods should be invokable using `invokevirtual` as well.

Stack Operations

The JVM provides some instructions specifically for manipulating items near the TOS. Such instructions may seem superfluous, in that they can be simulated using other instructions and registers. They are included to facilitate shorter instruction sequences for common program fragments.

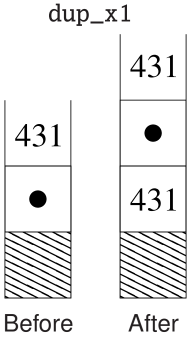


The example depicts the duplication of the cell at TOS.

The instruction works for any 32-bit type (i.e., all types except `long` and `double`). There is a `dup2` instruction that duplicates the top two cells to accommodate `long` and `double` types.

The `dup` instruction nicely accommodates multiple assignments (`x=y=z=value`). It is also useful for constructing new objects. The `new` *t* instruction leaves a reference on TOS to the newly allocated storage of type *t*, but that storage cannot be accessed until a constructor has been invoked. Constructors consume the reference to the allocated storage (along with their other parameters), but they return nothing (they are `void`). Thus, to remember the reference, the TOS is usually `dup`ed before the constructor invocation sequence is generated.

Other stack-manipulating instructions include `pop` and `swap`, which are self-explanatory. However, there are other instructions whose application may not be obvious.



This instruction duplicates the TOS element, but it situates the duplicated cell as shown in the example, two cells below the TOS.

One application of this seemingly bizarre instruction is the duplication of a value that participates in an **embedded assignment**. Consider FOO(**this.**$x \leftarrow y$), where y happens to have the value 431. The example starts with y's value already loaded on the stack. The example ends with the stack prepared for the `putfield` instruction followed by the method call to FOO.

The `dup_x1` instruction duplicates the 431 and places it below the **this** reference (shown as •). Recall that the field assigned by a `putfield` is an immediate operand of the instruction. Thus, when the `putfield` for x completes, the top two elements will be removed from the stack, leaving the duplicated 431 as the parameter value for FOO. This instruction nicely demonstrates that the JVM instruction set was designed not to be simple but to allow for compact code sequences. Exercise 11 explores this in greater detail.

10.3 Static Single Assignment Form

The **static single assignment** (SSA) Form [CFR+91] intermediate representation has properties that are beneficial for program analysis and optimization (Chapter 14). The form is named after a property enjoyed by purely functional languages: **single assignment** means that a name in a program is assigned only once. This property makes the assignment $a \leftarrow b + 1$ a mathematical truth: after $a \leftarrow b + 1$ completes, a mathematically equals $b + 1$ for the rest of the program's execution. Neither a nor b could change value due to the single-assignment rule. In summary, the program assignment $a \leftarrow b + 1$ translates into the predicate $a = b + 1$, which persists indefinitely. This allows algebraic substitution throughout the program of $b + 1$ for a.

Such transparency makes programs arguably easier to analyze and optimize. Some would argue further that functional programs are easier to understand and maintain.

Now consider the assignment $x \leftarrow x + 1$. We assume all names are properly initialized before they are used. For $x \leftarrow x + 1$ to make sense, x must already have been assigned some value. In that case, $x \leftarrow x + 1$ violates the single-assignment rule. Moreover, the program fragment $x \leftarrow x + 1$ does not translate well mathematically, since its corresponding mathematical predicate $x = x + 1$ is always false.

In the discussion that follows, we call an assignment to x a **def** (short for **definition**) of x. Any other reference to x is called a **use** of x. SSA Form was developed as an intermediate representation for programs written in any language. The single assignment rule is relaxed to apply *statically*: assignment to a given name can appear only once in a source program. Once SSA Form is achieved, the value supplied for any given use of a name such as x can be associated with exactly one def of x. This property allows algebraic substitution of program analysis information that flows from a def to each use.

SSA Form allows a statement such as $a \leftarrow b + 1$ to execute multiple times, but that statement can be the only one that defines a. Algorithms that compute and use SSA Form are covered in Chapter 14. We describe the approach more informally here, so that SSA Form can be computed by hand. We consider monolithic programs (no procedure calls) that reference only scalar variables (no arrays) and constants. Extensions are considered in the Exercises 13, 14, and 15.

10.3.1 Renaming and ϕ-functions

The first step in obtaining SSA Form is to **rename** the defs of a program so that each is unique. A simple approach that leaves the program relatively intact is to rename each def using an integer subscript. This task can be done separately for each name in the original program, as shown below for v:

```
$v \leftarrow 4$                      $v_1 \leftarrow 4$
  $\leftarrow v + 5$                    $\leftarrow v_1 + 5$
$v \leftarrow 6$                      $v_2 \leftarrow 6$
  $\leftarrow v + 7$                    $\leftarrow v_2 + 7$
```

When the program on the right is obtained, v_1 and v_2 are treated as distinct names. They are renamed in this way only to show that their original name was v. For code without branches, renaming suffices to compute SSA Form. In other cases, special care must be taken to manage the confluence of values flowing from multiple defs of the same name:

```
if $p$                                if $p$
then $v \leftarrow 4$                 then $v_1 \leftarrow 4$
else $v \leftarrow 6$                 else $v_2 \leftarrow 6$
                                      $v_3 \leftarrow \phi(v_1, v_2)$
  $\leftarrow v + 5$                    $\leftarrow v_3 + 5$
  $\leftarrow v + 7$                    $\leftarrow v_3 + 7$
```

The $\phi(v_1, v_2)$ function articulates the point in the program where v_1 and v_2 converge. Without the ϕ-function, both defs would reach each of the uses that follow. The introduction of the new assignment to v_3 prevents such behavior.

Conceptually, ϕ-functions could be placed anywhere in a program. If placed at point p in a program, then the ϕ-function would have a parameter

```
$i \leftarrow 1$
$j \leftarrow 1$
$k \leftarrow 1$
$l \leftarrow 1$
repeat




    if $p$
    then
        $j \leftarrow i$
        if $q$
        then  $l \leftarrow 2$
        else  $l \leftarrow 3$

        $k \leftarrow k + 1$
    else  $k \leftarrow k + 2$



    call PRINT($i, j, k, l$)
    repeat

        if $r$
        then
            $l \leftarrow l + 4$


    until $s$
    $i \leftarrow i + 6$
until $t$
```

(a)

```
$i_1 \leftarrow 1$
$j_1 \leftarrow 1$
$k_1 \leftarrow 1$
$l_1 \leftarrow 1$
repeat
    $i_2 \leftarrow \phi(i_3, i_1)$
    $j_2 \leftarrow \phi(j_4, j_1)$
    $k_2 \leftarrow \phi(k_5, k_1)$
    $l_2 \leftarrow \phi(l_9, l_1)$
    if $p$
    then
        $j_3 \leftarrow i_2$
        if $q$
        then  $l_3 \leftarrow 2$
        else  $l_4 \leftarrow 3$
        $l_5 \leftarrow \phi(l_3, l_4)$
        $k_3 \leftarrow k_2 + 1$
    else  $k_4 \leftarrow k_2 + 2$
    $j_4 \leftarrow \phi(j_3, j_2)$
    $k_5 \leftarrow \phi(k_3, k_4)$
    $l_6 \leftarrow \phi(l_2, l_5)$
    call PRINT($i_2, j_4, k_5, l_6$)
    repeat
        $l_7 \leftarrow \phi(l_9, l_6)$
        if $r$
        then
            $l_8 \leftarrow l_7 + 4$
        $l_9 \leftarrow \phi(l_8, l_7)$

    until $s$
    $i_3 \leftarrow i_2 + 6$
until $t$
```

(b)

Figure 10.5: SSA Form example taken from [CFR+91]. Program (b) shows the SSA Form for program (a).

for each distinct statement that could execute just before p. In the example above, the **if-then-else** construct results in two statements that could execute just before the ϕ-function, and it therefore has two parameters. The values supplied for the parameters are the values of v transmitted through each statement that could execute just before the ϕ-function.

Clearly, a ϕ-function with a single parameter serves no purpose. The parameter can simply pass through without being renamed. The trick in computing SSA Form is to determine the minimum number of ϕ-functions that must be placed so that each use is reached by a unique def. A more complete example is shown in Figure 10.5.

Exercises

1. Show as many instruction sequences as you can that occupy fewer than 10 bytes and have the effect of pushing 0 on the runtime stack. Your instructions can temporarily modify any storage you wish, but at the end of the sequence, the only noticeable change should be the new top-of-stack cell containing 0.

2. Investigate the layout of the JVM constant pool and design an algorithm to discover the type and value of the i^{th} entry.

3. As described in Section 10.2.2, a constant-pool entry is referenced by its *ordinal* location in the pool. Why was the JVM's constant pool designed in this fashion when it would surely be faster for programs to specify the byte-offset of a constant-pool entry directly?

4. Why are there two instructions (`ldc` and `ldc_w`) for pushing a constant value on top of the runtime stack?

5. The form of the `getstatic` instruction is described in Section 10.2.3 as having both a *name* and a *type*. While the *name* is certainly necessary to access the desired static field, is the *type* information really necessary? Recall that the accessed field has a declared type in the class defining the field.

6. The `getstatic` instruction described in Section 10.2.3 requires that the static field's *name* be specified as an **immediate operand** of the instruction.

   Suppose the JVM instead had a `getarbitrary` instruction that could load a value from some arbitrary location. Instead of specifying the field by its name in an immediate operand, the location's address would be found at TOS.

   What are the implications of such an instruction on the performance and security of the JVM?

7. The JVM has two instructions for unconditional jumps: `goto` and `goto_w`. Determine the appropriate conditions for using each instruction.

8. The `ifeq` instruction provisions only 2 bytes for the branch offset. Unlike `goto`, there is no corresponding `goto_w` instruction for `ifeq`. Explain how you would generate code so that a successful outcome of `ifeq` could reach a target that is too far to be reached by a 16 bit offset.

9. Consider the C or Java **ternary expression**: $(a > b)$ **?** c **:** d , which leaves either c or d on TOS depending on the outcome of the comparison. Assume all variables are type `int`, but do not assume that any of them are 0. Explain how you would generate code for the comparison that uses only the `ifne` instruction for branching (no other `if` or `goto` instructions are allowed).

10. A constructor call consumes the TOS reference to the class instance it should initialize. All constructors are `void`, so they do not return any kind of result. However, Java programs expect to obtain the result of the constructor call on TOS after the constructor has finished. By what JVM instruction sequence can this be accomplished?

11. The text includes a discussion of the `dup_x1` instruction as applied to the code fragment: FOO(**this.**$x \leftarrow y$). Develop a code sequence that leaves y's value (431) on the stack after the embedded assignment, without using any of the `dup` instructions. Note that 431 is *not* a constant here: it happens to be the value that was loaded for y.

12. Figure 10.5 shows a sequence of ϕ-functions before the call to the PRINT method. Why is a ϕ-function not needed for i at that point?

13. Investigate how arrays are treated in SSA Form.

14. Investigate how heap-allocated storage is treated in SSA Form.

15. Investigate how method calls are treated in SSA Form.

16. What is the difference between a **def** of x and its L-value as described in Section 7.6 on page 261?

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11

Code Generation for a Virtual Machine

In this chapter, we take a final step in program translation by traversing an **abstract syntax tree** (AST) and generating a form of code that is suitable for a **virtual machine**. The construction of an AST (Chapter 7) and its subsequent semantic processing (Chapters 8 and 9) have developed all of the information that is necessary to translate a source program into some form of interpretable or executable code. The AST serves well for expressing the structure and the meaning of a source program. However, its design is purposefully abstract, and thus independent of any particular architecture specification. Moreover, the AST nicely represents the nested structure of programs written in a modern programming language, while the instructions executed by most architectures are more linear in nature.

In Chapters 5 and 6, parsing techniques are presented that check an input program's syntax based on a programming language's grammar. While the grammar provides an automatic structure for regulating the parser's activity, the translation of the source program into a suitable AST requires actions that are inserted by hand. In this chapter, code generation is essentially the inverse of the parsing process. A program's AST provides a structure that can be traversed automatically, but the actions required to synthesize code are formulated by hand.

Code-generation issues are discussed here and in Chapter 13, and the differences in treatment are as follows:

- The target of code generation here is **virtual machine** (VM) code, which is fairly close in form and semantics to a source language. Chapter 13 considers machine-level targets that bear little resemblance to source languages. For example, the VMs considered in this chapter offer intrinsic treatment of objects, virtual method calls, and Java$^{\mathrm{TM}}$ data types such as `String` and `boolean`. In Chapter 13, translation strategies must be introduced to implement such features properly.
- The resource issues that must be addressed in code generation are relatively simple for VMs, but require a more sophisticated treatment in Chapter 13. For example, the VM here can reference an almost unlimited number of registers, while the targets in Chapter 13 have a relatively small number of architected registers.

While a reader concerned mostly with native code generation may be tempted to skip this material and jump to Chapters 12 and 13, we recommend studying this chapter first as a relatively gentle introduction to the techniques of code generation and as a foundation for the material presented in Chapter 13.

11.1 Visitors for Code Generation

As was the case with semantic processing, code generation makes extensive use of the **visitor pattern** presented in Chapter 7, which allows method dispatch to be based on a given visitor and the actual type of a given node. Code based on the visitor pattern can be authored in a single class, so that the tasks ascribed to a given visitor are easily aggregated across AST node types. The actual code executed within a visitor is based on the runtime type of the visited node (binary addition, local variable reference, etc.) *and* the type of the visitor itself.

A *reflective* mechanism for achieving such **double dispatch** is presented in Section 7.7.3 on page 268. A review of that material may be helpful before proceeding with this chapter.

A given visitor is typically tasked with performing a relatively narrow set of activities. For the purposes of code generation, it is helpful to organize the code-generating phase using the following visitors:

TopVisitor is the top-level visitor for processing an AST's nodes. It is responsible for processing class and method declarations, and it also initiates processing of each method's contents.

MethodBodyVisitor generates code for the constructs found within a method. The visitor accepts a label at which the method's *postlude* code will be generated, so that proper method termination can be achieved from any point in the method.

While this visitor bears most of the responsibility of code generation, some exceptional circumstances must be handled by the other visitors described below.

LHSVisitor is responsible for generating code for the left-hand side of assignment statements. Recall from semantic analysis (Chapter 9) that the meaning of a variable name changes across an assignment operator (e.g., = in Java). On the left side, a name means the *address* of the variable; nearly everywhere else, a name means the *value* of the variable. As another example, some languages (such as C++ and Pascal) include notation for **reference parameters**, which are transmitted using their address instead of their value.

The *LHSVisitor* will be directed to process portions of an AST in which a name denotes its address instead of its value. Within such subtrees, other names may refer to values, as dictated by a particular programming language's semantics. The visitors call each other as needed to develop the requisite addresses and values of names.

SignatureVisitor is responsible for visiting an AST subtree that corresponds to a method definition or method invocation and developing the **signature** of the associated method. The signature typically includes the name of the method, a representation of the number and type of the method's parameters, and the return type of the method.

This visitor is necessary when a method's signature is required, as the code-generating visitor would otherwise generate code to invoke the method. More detail about developing a method's type signature can be found in Section 8.4.2 on page 294 and Section 8.7 on page 316.

The visitors facilitate organization of the code generator into sections of code with related function. For each AST construct of interest, we present algorithm-style code in the form of a visit method that illustrates one strategy for generating code for the construct. Exercises at the end of this chapter explore alternative strategies.

At some point in each of the code-generation visit methods, actual code must be emitted. The syntax and specification of such code depend on the actual form of the VM code. To illustrate the principles of code generation with concrete code sequences, we show the results of code generation using instructions and directives from the **Java Virtual Machine** (JVM). Section 10.2 on page 397 describes the format of those instructions and provides resources for further investigation of the JVM instruction set.

The coding convention within the visitors is to pass a visitor a node using the node's accept method. When a node **accepts a visitor**, actions are performed that are appropriate to both the visitor and the node at hand. For example, the action taken to visit a node's children at Marker ① causes each

```
class NodeVisitor
    procedure visitChildren(n)
        foreach $c \in n$.getChildren( ) do
            call c.accept(this)                                   ①
    end
end

class TopVisitor extends NodeVisitor
    procedure visit(ClassDeclaring cd)                            ②
        /★   Section 11.2.1 on page 422                      ★/
    end
    procedure visit(MethodDeclaring md)                           ③
        /★   Section 11.2.2 on page 424                      ★/
    end
end
/★   Continued in Figure 11.2                                ★/
```

Figure 11.1: Structure of the code-generation visitors, with references to sections addressing specific constructs.

child to accept the current visitor. Such a visitor is an instance of the type *NodeVisitor*, such as a *TopVisitor* or a *MethodVisitor*.

Within a visitor, access to a particular node's contents is specified using accessor methods, which typically include the word GET. For example, the name of a class is retrieved using the method GETCLASSNAME at Marker ⑭.

11.2 Class and Method Declarations

The outermost portions of an AST contain class and method declarations. The *TopVisitor* shown in Figure 11.1 is responsible for processing each class and method declarations. Section 11.2.1 explains how classes are processed, including their field and static declarations. Section 11.2.2 covers the initial processing of a method declaration.

The superclass *NodeVisitor* provides a useful method for visiting a node n's children, passing the current visitor to each in turn. To be consistent with the semantics of most programming languages, the visitor is passed to the children in left-to-right order of their appearance in the AST. The code shown at Marker ① is typical of visitors that call for recursive processing of AST subtrees. By calling for each child c of n to accept **this** visitor, the code at Marker ① causes the current visitor to process c recursively.

```
/⋆  Continued from Figure 11.1                          ⋆/

class MethodBodyVisitor extends NodeVisitor
    procedure visit( ConstReferencing n )                      ④
        /⋆  Section 11.3.1 on page 425                  ⋆/
    end
    procedure visit( LocalReferencing n )                      ⑤
        /⋆  Section 11.3.2 on page 426                  ⋆/
    end
    procedure visit( StaticReferencing n )                     ⑥
        /⋆  Section 11.3.3 on page 427                  ⋆/
    end
    procedure visit( Computing n )                             ⑦
        /⋆  Section 11.3.4 on page 427                  ⋆/
    end
    procedure visit( Assigning n )                             ⑧
        /⋆  Section 11.3.5 on page 429                  ⋆/
    end
    procedure visit( Invoking n )                              ⑨
        /⋆  Section 11.3.6 on page 430                  ⋆/
    end
    procedure visit( FieldReferencing n )                      ⑩
        /⋆  Section 11.3.7 on page 432                  ⋆/
    end
    procedure visit( ArrayReferencing n )                      ⑪
        /⋆  Section 11.3.8 on page 433                  ⋆/
    end
    procedure visit( CondTesting n )                           ⑫
        /⋆  Section 11.3.9 on page 435                  ⋆/
    end
    procedure visit( WhileTesting n )                          ⑬
        /⋆  Section 11.3.10 on page 436                 ⋆/
    end
end
```

Figure 11.2: Continuation of the code-generation visitors from Figure 11.1.

11.2.1 Class Declarations

```
/⋆  Visitor code for Marker ②                                          ⋆/
procedure visit( ClassDeclaring cd )
    call emitClassName( cd.getClassName( ) )                             ⑭
    foreach superclass ∈ cd.getSuperClasses( ) do
        call emitExtends( superclass )                                   ⑮
    foreach field ∈ cd.getFields( ) do
        call emitFieldDeclaration( field )                               ⑯
    foreach static ∈ cd.getStatics( ) do
        call emitStaticDeclaration( static )                             ⑰
    foreach node ∈ cd.getMethods( ) do node.accept( this )               ⑱
end
```

The AST contains a subtree for each class defined in the source program. The root of each subtree implements the *ClassDeclaring* interface, which provides important information about the class that must be transcribed by the visitor when generating code so that instances of the class can be properly instantiated. Such information specifies how the class type fits into the namespace and inheritance structure of all classes.

The code in the *ClassDeclaring* visitor serves well to document the conventions we follow in writing visitor methods:

- Generally, methods in our visitors that begin with GET access such information through the interface provided by the visited node.
- Invocations of ACCEPT trigger traversal of an AST subtree with the effect of generating code for whatever is found there.
- Methods that begin with EMIT generate the VM instructions.

Code generation on behalf of a class declaration includes the following:

Name: code emitted at Marker ⑭ reflects the *name* of the class represented by *cd*. The name must include any contextual information that is required to reference instances of *cd*. Such information can also serve to allow instances of *cd* to access data within their purview. For example, the package that hosts a Java class appears as a prefix of the class name.

Some classes are meant to be practically invisible across separate compilations. Examples include **inner** and **anonymous classes**. The name for such classes is typically generated as a distinguishing variation of the primary class's name. If *cd*'s name is *Name*, then an anonymous inner class of *cd* might be dubbed *Name$001*.

For most classes, where visibility of the class is essential across separate compilations, the name must be consistently formulated at Marker ⑭ and everywhere the class is referenced. Languages like C++ require that parametric type values be included in the class name, so that the code generated for *Vector*<*int*> can be distinguished from the code for *Vector*<*double*>.

Inheritance: a specification of *cd*'s superclass(es) is emitted at Marker ⑮. Such information is necessary to realize the method calls and instance variables that are inherited by *cd*.

All Java objects (except *Object*) extend exactly one Java class. Thus, inheritance in Java can be specified as a single superclass extended by *cd* and a set of interfaces implemented by *cd*.

For languages like C++ that offer **multiple inheritance**, code must be generated to include information from all of *cd*'s superclasses.

Instance variables (fields): for each field declared in the class, information about the field's name, type, and access permission is generated at Marker ⑯. When combined with information from *cd*'s superclasses, such information allows provisioning of the data area required for each instance of *cd*.

Static variables: these are processed at Marker ⑰ like instance variables, but they are allocated just once. All instances of *cd* access the same storage for static variables.

For JVM code generation, the class instantiation information can be described succinctly, relying on runtime interpretation of the descriptions to initialize class instances appropriately. A more thorough treatment of such initialization—including static and instance variable allocation and virtual method table construction—is presented in Section 13.1.3 on page 496.

As its final step, the VISIT(*ClassDeclaring*) method initiates translation of each method defined in class *cd*. By calling for each method *node* to accept **this** visitor, the code at Marker ⑱ causes the current visitor (*TopVisitor*) to process *node* recursively. This action triggers the processing of *node* by the VISIT(*MethodDeclaring*) code presented next.

11.2.2 Method Declarations

```
/★  Visitor code for Marker ③                                          ★/
procedure VISIT( MethodDeclaring md )
    sigVisitor ← new SignatureVisitor( )                                    ⑲
    call md.ACCEPT(sigVisitor)
    signature ← sigVisitor.GETSIGNATURE( )
    call EMITMETHODNAME(signature)                                          ⑳
    call EMITMETHODALLOC(md.GETLOCALS( ), md.GETSTACK( ))                   ㉑
    postludeLabel ← GENLABEL( )
    bodyVisitor ← new MethodBodyVisitor(postludeLabel)                      ㉒
    md.GETBODY( ).ACCEPT(bodyVisitor)
    call EMITMETHODPOSTLUDE(postludeLabel)                                  ㉓
end
```

Method signature: The code beginning at Marker ⑲ runs the *SignatureVisitor* on node *md* to develop its declared signature. The signature includes the name of the method, the types of its parameters, and its return type. Such information is needed to define or call the method represented by *md*.

Method prelude: Before code is generated for the contents of a method, the compiler must generate the method's **prelude**. The prelude code establishes a runtime context for executing the method. Marker ⑳ begins the prelude by emitting code based on the method's full signature. Marker ㉑ generates code that allocates space needed by the method during its execution. Such space typically includes the method's **local variables** and stack space for method calls and intermediate computations. The space required for local variables can be determined from the method's **symbol table** as discussed in Chapter 8. The space required for the runtime stack depends on the method's operations and how deeply they push operands before popping them. Java's language definition contains rules that ensure that the stack is used consistently and predictably within a method. A simple form of data flow analysis (Exercise 67 on page 649) can determine the maximum stack depth for a method.

Method body code: We are now in a position to generate the main portion of a method's code. The execution of certain constructs or exceptions may cause the code at some point to conclude the method's execution. We therefore generate a label for the method's *postlude* code and pass that to the visitor that generates the method body code. This allows the visitor to jump to the postlude sequence if the method should cease execution.

Method postlude: As a method finishes execution, some code may be required to prepare a return value, propagate an exception, or manage structures that were created when the method began. Marker ㉓ generates such postlude code, which begins at the supplied label.

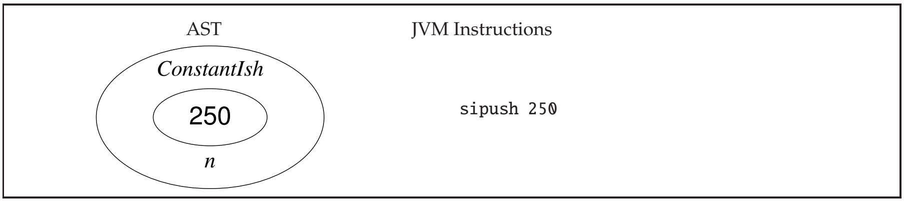
11.3 The *MethodBodyVisitor*

A *MethodBodyVisitor* is instantiated with the label of a sequence of code that serves as *postlude* code for the method. This arrangement enables the code generator to effect a method-return by transferring control to the specified label. We present the elements of this visitor in an order that allows us to construct a running example of how code is generated recursively by the visitor.

11.3.1 Constants

```
/⋆  Visitor code for Marker ④                                        ⋆/
procedure visit( ConstReferencing n )
    loc ← allocLocal( )                                              ㉔
    n.setResultLocal( loc )
    call emitConstantLoad( loc, n.getConstantValue( ) )              ㉕
end
```

Programs reference various constants, whose values can often be included directly in an **immediate instruction**. Marker ㉔ computes a resource (register or stack cell location) to hold the constant value, and the value is itself produced by the code generated at Marker ㉕. For a stack architecture like the JVM, the resource to hold the result is the **top-of-stack** (TOS).



```
sipush 250
```

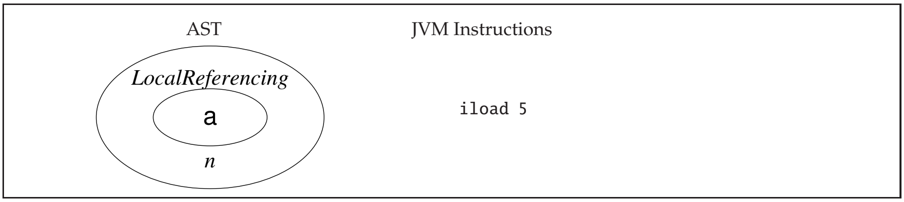
There may be multiple instruction sequences that could generate a given constant value. The instruction that takes the least time or instruction space is typically chosen for this task. The box above features the `sipush` instruction, which can handle a signed value that fits in 16 bits. A single byte (`bipush`) cannot hold the constant value `250`, so the `sipush` instruction is used instead.

Because instruction specifications vary from VM to VM, we make use of methods like emitConstantLoad at Marker ㉕ to serve as an abstraction for the actual VM instructions that are generated. For example, emitConstantLoad is responsible for determining the appropriate instruction for loading the constant represented by node *n*. For the JVM, choices for integer constants include `bipush`, `sipush`, `ldc`, and `iconst`.

11.3.2 References to Local Storage

```
/⋆   Visitor code for Marker ⑤                                  ⋆/
procedure visit( LocalReferencing n )
    loc ← allocLocal( )                                            ㉖
    n.setResultLocal(loc)
    call emitLocalLoad(loc, n.getLocation( ))                      ㉗
end
```

The code generated for a local storage reference causes the value stored at the named reference to be fetched and placed in an accessible place for subsequent operations. Marker ㉖ allocates the register to receive the value, and the code to perform the fetch is generated at Marker ㉗.



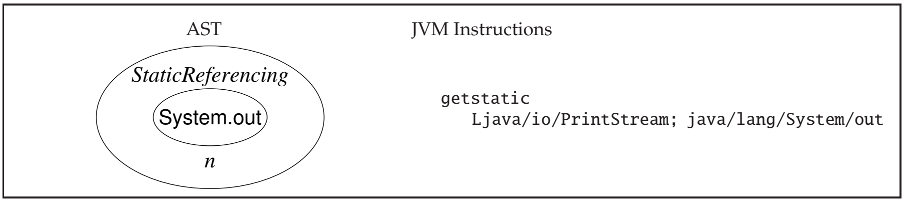
```
iload 5
```

The box shows the resulting instruction generated for the JVM on behalf of a local reference to the variable `a`. If the method's assigned storage location for `a` is local number 5, then the `iload 5` instruction fetches the value from local 5 and pushes it on TOS. A method can assign storage locations for its locals by a visitor that intercepts *LocalDeclaring*. Each visit to a *LocalDeclaring* assigns the next available storage location. The number of storage locations allocated to the *LocalDeclaring* depends on the type of the local. For example, all types on the JVM take four bytes except `double` and `long` types, which take eight bytes.

11.3.3 Static References

```
/*   Visitor code for Marker ⑥                                 */
procedure visit( StaticReferencing n )
    call emitStaticReference(n.getType( ), n.getName( ))
end
```

The *StaticReferencing* AST node represents an access to a global storage name that is not associated with a particular instance of any class. Languages vary in their treatment of static names, but some mechanism is typically present to help organize such names and to encapsulate them according to their intended use. In Java, a static name is associated with a class type. For example, the Java class `java.lang.System` contains the static field `out` to facilitate program output to a standard-output stream. The declared type of `out` is `java.io.PrintStream`.



```
getstatic
   Ljava/io/PrintStream; java/lang/System/out
```

The code generated in the box above specifies the type and name of the static field reference. As described in Figure 10.4 on page 400, an object type in the JVM is prefixed by `L` and terminated by a semicolon. In a fully qualified class name, the package and class components are separated by a forward slash, even though a dot is used in the source language for that purpose. Thus, for the static field `java.lang.System.out`, the name is actually specified as `java/lang/System/out`. Its type is denoted as `Ljava/io/PrintStream;`, which is the fully qualified name of the `PrintStream` class.

For some VMs, and for native code generation, a static field reference is computed more directly as the address of the reference. Exercise 2 explores this issue in more detail.

11.3.4 Expressions

```
/*   Visitor code for Marker ⑦                                 */
procedure visit( Computing n )
    visitChildren(n)                                              ㉘
    loc ← allocLocal( )                                           ㉙
    n.setResultLocal(loc)
    call emitOperation(n)                                         ㉚
end
```

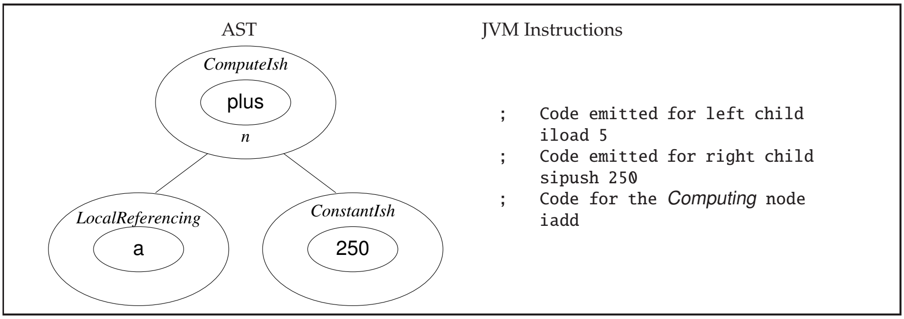
While many nodes fall into the *Computing* category, the strategy for generating code for such nodes can be simply stated as follows:

- Marker ㉘ calls for the generation of code for each child (processed left to right) of the *Computing* node. The generated code evaluates each of the operands required to compute the operation for node *n*.

  After evaluation, the operands' results can be found in the resource allocated when they were visited. For a zero-address target, such as the JVM, the operands' results are available at the TOS.

- Marker ㉙ sets aside space for the value computed by node *n*. For some VMs, the location may be implicit, such as the TOS or in a predetermined register. For example, the JVM obtains an instruction's operands from the TOS and leaves the result of the instruction (if any) on TOS. Other targets require management of registers or other local storage.

- Marker ㉚ emits code to compute the value of the expression associated with AST node *n*. That code will reference the values of *n*'s operands, each already evaluated (recursively) into its own result-holding local storage. The code generated on behalf of node *n*'s operation may be a single instruction, a sequence of instructions, or an invocation of a runtime method to realize the operation.



```
;   Code emitted for left child
    iload 5
;   Code emitted for right child
    sipush 250
;   Code for the Computing node
    iadd
```

The box above shows an AST for the binary `plus` node. The code generated for the JVM by visiting each of *n*'s two children leaves a value on the stack for each child. The `iadd` instruction pops the top two values, computes their sum, and pushes the result onto the stack. The type of instruction, *integer* addition, is determined by the type associated with the *Computing* node during semantic analysis (Section 8.8 on page 323).

11.3.5 Assignment

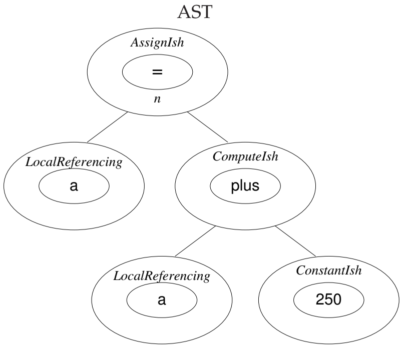
```
/⋆  Visitor code for Marker (8)                                    ⋆/
procedure VISIT( Assigning n )
    lhsVisitor ← new LHSVisitor( this )                              (31)
    call n.GETLHS( ).ACCEPT( lhsVisitor )                            (32)
    call n.GETRHS( ).ACCEPT( this )                                  (33)
    call lhsVisitor.EMITSTORE( n.GETRHS( ).GETRESULTLOCAL( ) )       (34)
end
```

For most languages, evaluation of an assignment statement begins with its left-hand side, which could involve function calls, field references, and array index computations. In left-to-right evaluation order, those expressions must be evaluated before the statement's right-hand side can be evaluated. A special **left-hand side** (LHS) visitor is needed, because the name that is assigned by the AST node refers to that name's location and not its value. That visitor is constructed at Marker (31), and the details of that visitor are discussed in Section 11.4. The left-hand side visitor processes the subtree corresponding to the assignment's left-hand side at Marker (32).

Many VMs are **assignment safe**, in the sense that programs cannot modify storage arbitrarily. To limit the impact of an assignment statement, some VMs restrict the form of an assignment's left-hand side so that safety checks can be performed at compile-time. For safety issues that cannot be checked at compile-time, code is generated to perform the check at runtime. This distinction in meaning is best addressed by a specialized visitor. After Marker (32) finishes, all code for processing the left-hand side is generated, except the final store instruction.

Next, the right-hand side of an assignment statement is an expression whose code can be generated as usual by the code-generation visitor. Names appearing on an assignment statement's right-hand side denote their value. The code generated on behalf of a *LocalReferencing, FieldReferencing* or *ArrayReferencing* AST node causes such values to be loaded into the appropriate resource (register or stack cell).

After both sides of the assignment have been processed, the appropriate instruction is generated at Marker (34) to cause the value to be written to the appropriate location.



JVM Instructions

```
;   Code emitted for Computing
    iload 5
    sipush 250
    iadd
;   Code emitted by LHSVisitor
    istore 5
```

The results of processing a sample *Assigning* node are shown in the box above. The *LHSVisitor* determines at Marker (32) that the assignment will store into the local variable associated with a, but no code is generated yet. All but the last instruction are generated by recursive application of the visitor to the right subtree of the *Assigning* node, initiated at Marker (33). The final instruction is generated at Marker (34). The details of the *LHSVisitor* are discussed in Section 11.4.

11.3.6 Method Calls

```
/⋆  Visitor code for Marker (9)                                        ⋆/
procedure VISIT( Invoking n )
    sigVisitor ← new SignatureVisitor( )
    call n.ACCEPT(sigVisitor)
    usageSignature ← sigVisitor.GETSIGNATURE( )                        (35)
    matchedSignature ← FINDSIGNATURE(usageSignature)                   (36)
    if not n.ISVOID( )                                                 (37)
    then
        loc ← ALLOCLOCAL( )
        call n.SETRESULTLOCAL(loc)
    if not n.ISSTATIC( )
    then
        call n.GETINSTANCE( ).ACCEPT(this)                             (38)
    foreach param ∈ n.GETPARAMS( ) do call param.ACCEPT(this)          (39)
    if n.ISVIRTUAL( )                                                  (40)
    then call EMITVIRTUALMETHODCALL(n)
    else call EMITNONVIRTUALMETHODCALL(n)
end
```

Before discussing the specifics of code generation for method calls, we point out that the interface offered by an *Invoking* node serves to decouple it from any specific AST representation. For example, n.getInstance( ) returns the node whose computation evaluates to the instance on which the method should be called, if it is a non-`static` method. Similarly, n.getParams( ) returns an iteration of the method's explicit parameters. Decoupling the AST's representation from its interfaces follows sound software-engineering principles and is observed wherever possible in this chapter.

For most languages, invoking a method requires not only the name of the method, but also some information about the method's parameters, return value, and object instance on which the call is made. The *SignatureVisitor* accepts the *Invoking* node to develop the **method signature**, which is retrieved at Marker ㉟. For some languages, the method call's signature is just the beginning of identifying the actual method that will be invoked. For example, if a parameter is supplied as type `int` and the only method that otherwise matches the signature expects that parameter as type `double`, then that method may suffice for the call by **widening** the actual parameter an `int` to a `double`. Marker ㊱ is charged with finding the method that suffices for the signature developed at Marker ㉟. If none is available, or if the choice of method is ambiguous, then an error is reported.

The code generated for a method call depends on the following attributes of an *Invoking* node:

- The method may return a value, in which case a resource must be allocated to hold the results. Marker ㊲ sets aside a resource if the invoked method is not `void`.

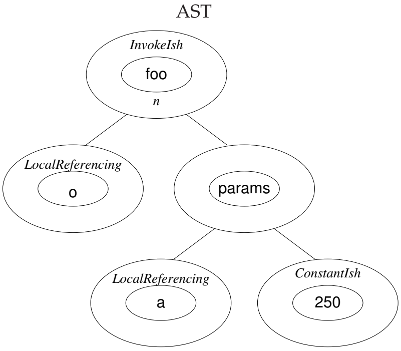
  For zero-address targets such as the JVM, the result will be returned on TOS when the invoked method returns. In that case, no resource needs to be reserved explicitly for the result.

- The method may be `static`, in which case the parameters are exactly as presented in the method call, or the method may be invoked on an object instance, denoted here as the method's **receiver**. The code for evaluating the receiver is generated at Marker ㊳.

  Most runtime architectures treat the receiver as an extra parameter, in addition to the parameters explicitly provided. For example, the JVM architecture specifies that the receiver should be passed as the first parameter to a non-`static` method.

- The method may be **virtual**, in which case the runtime type of the receiver plays an important role in the method call. The VM may have internal support for determining a method call based on the receiver; for example, the JVM has an `invokevirtual` instruction. If no such instruction is available, then a small table is typically generated that is indexed by the receiver's runtime type. The table contains method pointers so that the appropriate method is dispatched based on the receiver type. More detail on this is presented in Section 13.1.3 on page 496.

We use `o.foo(a, 250)` as an example to illustrate the code generated on behalf of an *Invoking* node. We assume `o` is an instance of `MyClass`, and is allocated to local 4. The method `foo` in `MyClass` accepts two `int` parameters and returns a `boolean`. An AST for `o.foo(a, 250)` is shown in the box below. The code generated for the AST treats `o` as if it were an extra parameter to `foo`, evaluating it ahead of `foo`'s specified parameters, and causing the contents of local 4 to be pushed on the stack as an address (`aload 4`). Marker 39 visits the parameters in left-to-right order, generating code to evaluate each and pushing the resulting value on TOS.



JVM Instructions

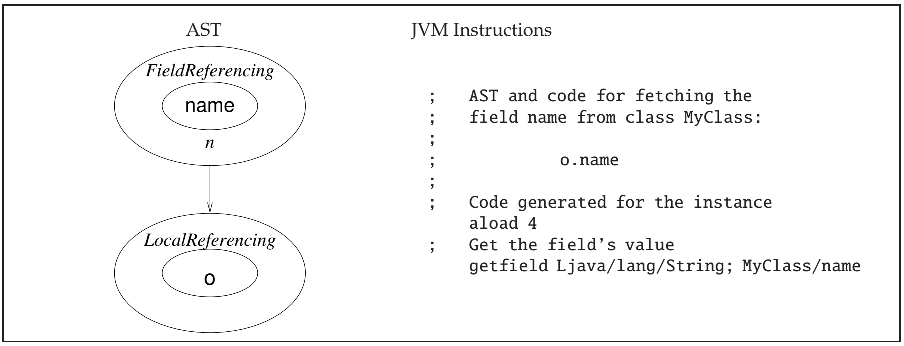
```
;    AST and code for invoking the
;    method foo from class MyClass:
;
;           o.foo(a, 250)
;
;    Marker 38 emits code
;    for the instance o:
     aload 4
;    Marker 39 emits code
;    for the parameters:
     iload 5
     sipush 250
;    The method call
     invokevirtual MyClass/foo(II)Z
```

The JVM specified signature for `foo` is `(II)Z`. This indicates that `foo` has two explicit `int` parameters (denoted by `(II)`) and a `boolean` return value (denoted by `Z`).

11.3.7 Field References

```
/★   Visitor code for Marker 10                                   ★/
procedure visit( FieldReferencing n )
    call n.getInstance( ).accept( this )                            41
    call emitFieldReference( n.getType( ), n.getName( ) )
end
```

A field reference resembles a static reference, with the addition of an object instance that hosts the field. The expression for the object instance could be a simple local reference, but it could have arbitrary complexity, including method calls and other field and static references. Marker ㊶ therefore passes the visitor to the subtree representing the object instance, so that code can be generated to compute the object instance. The ensuing `getfield` instruction has the same form as the `getstatic` instruction, but the execution of `getfield` will use the object instance to find the specified field.



JVM Instructions

```
;   AST and code for fetching the
;   field name from class MyClass:
;
;           o.name
;
;   Code generated for the instance
    aload 4
;   Get the field's value
    getfield Ljava/lang/String; MyClass/name
```

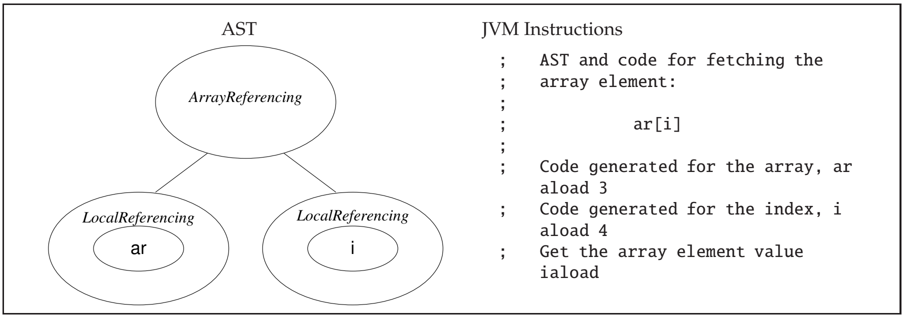
The code generated above assumes that the local reference `o` is in local 4 and is of type `java.lang.String`. The visit initiated at Marker ㊶ is dispatched to the *LocalReferencing* visit method, which generates code to push local 4's value onto the stack. The type of that reference should be `MyClass`. The `getfield` instruction is then emitted to retrieve the value of field `name` from instance `o`. The instruction specifies the expected type of the field reference: `java.lang.String`.

11.3.8 Array References

```
/⋆  Visitor code for Marker ⑪                                    ⋆/
procedure VISIT( ArrayReferencing n )
    call n.GETARRAY( ).ACCEPT( this )                               ㊷
    call n.GETINDEX( ).ACCEPT( this )                               ㊸
    call EMITARRAYREFERENCE( n.GETARRAY( ).GETTYPE( ) )             ㊹
end
```

An array reference contains two components, the name of an array and an index value. In Java, an array name is always a reference to an array object. In languages like C and C++, an array name can be a global, local, or heap address. At Marker ㊷ we visit the subtree representing the array and load (or compute) its address. For the JVM, an object reference will be pushed onto the stack. Code generators for other architectures will load an array address into a register.

Next, at Marker 43, the array index is computed. The JVM will push its value onto the stack; other architectures will load or compute the index into a register. Finally, at Marker 44, an array element is loaded. On the JVM this is simple — special array load instructions use the array reference and index values at the TOS to compute an array value and push it onto the stack. For other architectures, several instructions may need to be generated. Using the formulas detailed in Section 12.3.1 on page 460, the address of an array element is computed using the address of the array, the value of the index, and the size of individual array elements. The selected value is then loaded into a register. If array bounds checking is activated, the index value is also compared against the array's lower and upper bounds. In Java, bounds checking is automatically included as part of the index operation.



```
;   AST and code for fetching the
;   array element:
;
;             ar[i]
;
;   Code generated for the array, ar
    aload 3
;   Code generated for the index, i
    aload 4
;   Get the array element value
    iaload
```

In the above example, assume array `ar` is local variable 3 and that index `i` is local 4. These two values are pushed onto the stack. The instruction `iaload` pops the array and index values and replaces them with the value of `ar[i]`. A variety of array load instructions exist, depending of the type of the array element. The ”i” prefex indicates that an integer is to be loaded.

11.3.9 Conditional Execution

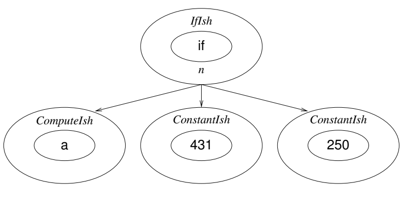
```
/*  Visitor code for Marker ⑫                                        */
procedure visit( CondTesting n )
    falseLabel ← genLabel( )                                          ㊺
    endLabel ← genLabel( )
    call n.getPredicate( ).accept( this )                              ㊻
    predicateResult ← n.getPredicate( ).getResultLocal( )
    call emitBranchIfFalse( predicateResult, falseLabel )              ㊼
    call n.getTrueBranch( ).accept( this )                             ㊽
    call emitBranch( endLabel )
    call emitLabelDef( falseLabel )
    call n.getFalseBranch( ).accept( this )                            ㊾
    call emitLabelDef( endLabel )
end
```

A *CondTesting* node represents conditional execution based on the outcome of some predicate. Languages like Java and C allow conditional execution among statements using the `if` construct. Exercise 5 explores other syntax with similar semantics.

The VM code that is generated for an AST is a linear form, in the sense that the conclusion of one instruction typically causes the next instruction in sequence to begin execution, unless the flow of control is interrupted. Code involving conditional execution therefore requires the use of *jumps*: to transfer control between instructions that should execute, and to skip over instructions that should not execute.

Marker ㊺ reserves two labels for use in generating the code for an *CondTesting* node. One label (*falselabel*) is used to receive control following the predicate's test, should the predicate evaluate to `false`. The other label marks the end of code generated for the *CondTesting* node.

AST



JVM Instructions

```
;   Predicate code (Marker ㊻)
    iload 5
;   Is the predicate false?
    ifeq falseLabel
;   True branch (Marker ㊽)
    sipush 431
    goto endLabel
falseLabel:
;   False branch (Marker ㊾)
    sipush 250
endLabel:
```

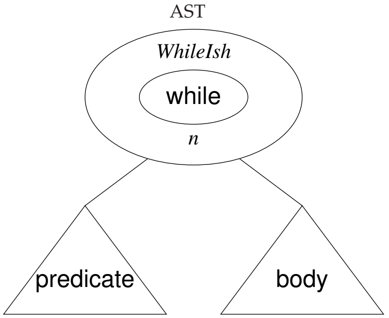
The generation of code for conditional execution follows the code layout shown in the box above. To accommodate the jumps necessary for the code, two labels are generated at Marker (45). Code is generated at Marker (46) to evaluate the predicate. Based on the evaluation of the predicate at runtime, a branch is generated at Marker (47) that skips around the true-branch code if the predicate evaluates to *false*. Generally, a value of 0 represents `false` and some non-zero value (typically 1) represents `true`. If the predicate evaluates to `false`, then the branch is not taken and the code generated at Marker (48) executes, followed by an unconditional branch to the end of the *CondTesting* code.

If control reaches *falseLabel*, then the code generated by Marker (49) executes. The end of the *Ifish* code is then reached.

11.3.10 Loops

```
/★  Visitor code for Marker (13)                                        ★/
procedure VISIT( WhileTesting n )
    doneLabel ← GENLABEL( )                                            (50)
    loopLabel ← GENLABEL( )
    call EMITLABELDEF( loopLabel )
    n.GETPREDICATE( ).ACCEPT( this )                                   (51)
    predicateResult ← n.GETPREDICATE( ).GETRESULTLOCAL( )
    call EMITBRANCHIFFALSE( predicateResult, doneLabel )
    n.GETLOOPBODY( ).ACCEPT( this )                                    (52)
    call EMITBRANCH( loopLabel )
    call EMITLABELDEF( doneLabel )
end
```

The code generated for a *WhileTesting* loop is similar to code generated for an *CondTesting* node. Two labels are obtained at Marker (50). One serves as the exit for the loop, and the other serves to repeat the loop at its predicate test.



JVM Instructions

```
loopLabel:
;   Predicate code (Marker (51))
;   Predicate false?  → exit the loop
;   ifeq doneLabel
;   Loop body code(Marker (52))
    goto loopLabel
endLabel:
    Done with loop
```

The predicate and loop body are shown abstractly in the above box, as each could involve a nontrivial amount of generated code. However, the visitor is recursively applied to the root of each of the subtrees, so that the constructs already considered can trigger proper code generation.

11.4 The *LHSVisitor*

The visitors presented thus far are *value-oriented*, in the sense that when they encounter a name in a program, they generate code to compute the *value* of that name. In some cases, such as on the left-hand side of an assignment statement, a name denotes its location instead of its value.

However, it is *not* the the case that all names appearing left of an assignment operator denote locations and stores. For example, consider the assignment statement `a.b.c = 5`, where `a`, `b`, and `c` are object references. The only actual store is to field `c`. The references to `a` and `b` are loads, not stores.

The code sketch for the *LHSVisitor* is shown in Figure 11.3. The constructor for the *LHSVisitor* is passed the instance of a *MethodBodyVisitor* that is currently processing the AST. That instance can be used in the *LHSVisitor* whenever a value needs to be generated. We use the active instance of a *MethodBodyVisitor* rather than a new one, so that the state of the code generator is available. For example, the active instance of a *MethodBodyVisitor* contains a reference to the postlude label for the current method. That label is necessary to facilitate a clean method-return in the generated code.

For Java and the JVM, there are three cases that must be handled by the *LHSVisitor* in terms of the type of AST node that can appear as the target of an assignment statement, and these are discussed below.

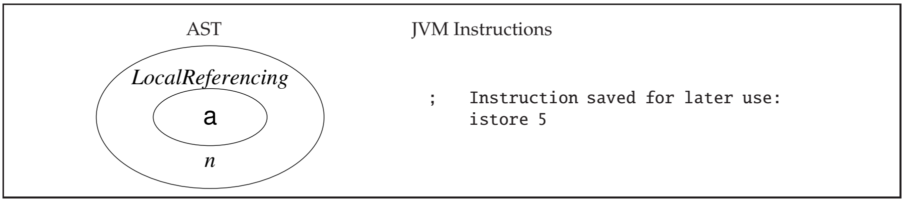
11.4.1 Local References

```
/⋆  Visitor code for Marker (54)                                   ⋆/
procedure VISIT( LocalReferencing $n$ )
    call SETSTORE( new LocalStore ( $n$.GETTYPE( ), $n$.GETLOCATION( ) ) )    (58)
end
```

An assignment to a local name requires no further computation. Marker (58) sets the store instruction to be a local store, parameterized by the type of the instruction (`int`, `double`, object reference, etc.) and the location associated with the local reference. Recall that Marker (34) retrieves the store instruction from the *LHSVisitor* after the left-hand side has been processed.

```
class LHSVisitor extends NodeVisitor
    constructor LHSVisitor( MethodBodyVisitor valueVisitor )
        this.valueVisitor ← valueVisitor                          53
    end
    procedure visit( LocalReferencing n )                         54
        /★  Section 11.4.1 on page 437                    ★/
    end
    procedure visit( StaticReferencing n )                        55
        /★  Section 11.4.2 on page 438                    ★/
    end
    procedure visit( FieldReferencing n )                         56
        /★  Section 11.4.3 on page 439                    ★/
    end
    procedure visit( ArrayReferencing n )                         57
        /★  Section 11.4.4 on page 439                    ★/
    end
end
```

Figure 11.3: Structure of the left-hand side visitor.



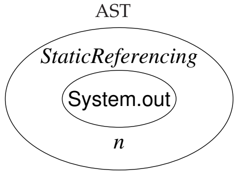
```
;   Instruction saved for later use:
    istore 5
```

The box shows the instruction saved at Marker 58 if the local reference a is associated with local 5 and its type is `int`.

11.4.2 Static References

```
/★  Visitor code for Marker 55                            ★/
procedure visit( StaticReferencing n )
    call setStore( new StaticStore( n.getType( ), n.getName( ) ) )      59
end
```

Code generation for assignments to statics resembles assignments to locals. Marker 59 creates an instruction that will assign the static name contained in the *StaticReferencing* node. The type of that reference may be required to generate the appropriate instruction.



JVM Instructions

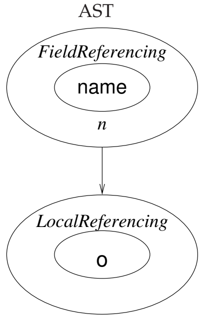
```
;   Instruction saved for later use:
  putstatic
     Ljava/io/PrintStream; java/lang/System/out
```

The box shows the instruction saved by Marker (59) for an assignment to the static field `out` of the `java.lang.System` class.

11.4.3 Field References

```
/*  Visitor code for Marker (56)                                      */
procedure VISIT( FieldReferencing n )
    call n.GETINSTANCE( ).ACCEPT( valueVisitor )                      (60)
    call SETSTORE( new FieldStore( n.GETTYPE( ), n.GETNAME( ) ) )     (61)
end
```

For a field reference, the instance containing the field must be evaluated first. Marker (60) causes the relevant code to be generated by having the AST subtree that represents the object instance accept the *MethodBodyVisitor* that was captured at Marker (53). The instruction for storing the field is then saved for later use at Marker (61).



JVM Instructions

```
;   AST and code for storing the
;   field name from class MyClass:
;
;           o.name
;
;   Code generated for the instance
    aload 4
;   Instruction saved for later use:
    putfield Ljava/lang/String; MyClass/name
```

11.4.4 Array References

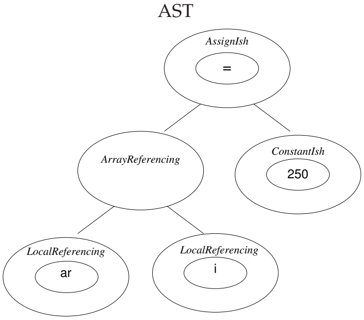
```
/*  Visitor code for Marker (57)                                      */
procedure VISIT( ArrayReferencing n )
    call n.GETARRAY( ).ACCEPT( valueVisitor )                         (62)
    call n.GETINDEX( ).ACCEPT( valueVisitor )                         (63)
    call SETSTORE( new ArrayStore( n.GETARRAY( ).GETTYPE( ) ) )       (64)
end
```

To translate an array reference used as a left-hand side, we first visit the array name, Marker 62. In Java this will push a reference to an array object. In languages like C and C++ an array address will be loaded into a register.

Next, at Marker 63, the array index is computed. The JVM will push its value onto the stack; other architectures will load or compute the index into a register.

Finally, at Marker 64, we prepare a store instruction that will be issued once the value to be stored is determined. On the JVM this is simple — special array store instructions use the array reference, index value and right-hand side value found at the TOS to store the right-hand side value into the proper array element. Array bounds checking is included too.

For other architectures, several instructions may need to be generated to compute the *address* of the selected array element. Again, we use the formulas detailed in Section 12.3.1 on page 460. The selected element address is placed in a register. If array bounds checking is activated, the index value is also compared againt the array's lower and upper bounds. Then a suitable store instruction is saved, and later generated (when the value to be stored into the array element is known).



JVM Instructions

```
;   AST and code for the
;   assignment:
;
;           ar[i] = 250
;
;   Code generated for the array, ar
    aload 3
;   Code generated for the index, i
    aload 4
;   Code generated for 250
    sipush 250
;   Code to store into the array element
    iastore
```

In the above example, assume array `ar` is local variable 3 and that index `i` is local 4. Since this is an assignment, code generastion begins with the *Assignish* visitor defined at Marker 31. The *LHSVistor* for *ArrayReferencing* is activated. It pushes first a reference to the array `ar` and then the value of the index, `i`. It then stores away, for later generation, an array store instruction, `iastore`.

Control returns to the *Assignish* visitor, which visits the assignment's right-hand side. This pushes the constant 250 onto the stack. At this point, a rererence to `ar`, the value of `i` and 250 are on the stack. Now the array store instruction we earlier saved is generated (Marker 34). The top three stack values are popped, and the selected array element is updated.

Exercises

1. In most object-oriented languages, objects can reference themselves using a reserved keyword such as `self` or `this`.

   (a) Assuming that an object's self-reference is in local 0, write a visit method for *MethodBodyVisitor*, VISIT(*ThisReferencing*), that generates code to compute the value of the self-reference.

   (b) Write the corresponding VISIT(*ThisReferencing*) method for the *LHSVisitor*.

2. As described in Section 11.3.3, a static reference has the following two components in the JVM:

   - The *type* of the reference;
   - The fully qualified name of the reference, including its host class.

   From a security perspective for the JVM, investigate why static references are as described above.

   (a) Why is the type of the reference included with each `getstatic` instruction?

   (b) It might be more efficient to use the relative offset of a static field within a class in lieu of its name. Why are offsets not used in the JVM `getstatic` instruction?

3. Design an AST node that implements the *CondTesting* interface to accommodate an `if` statement with no `else` clause.

   (a) How do you satisfy the *CondTesting* interface while indicating that the `if` statement has no `else` clause?

   (b) How is the code-generation strategy presented in Section 11.3.9 affected by your design?

4. Consider the code-generation strategy presented in Section 11.3.9. A conditional jump to *falseLabel* is taken, when the predicate is `false`, to skip over code that should execute only when the predicate is `true`.

   Rewrite the code-generation strategy so that the conditional jump is taken when the predicate is `true`, to skip over code that should execute only when the predicate is `false`.

5. Languages like Java and C allow conditional execution among statements using the `if` construct. Those languages also allow conditional selection of an expression value using the **ternary operator**, which chooses one of two expressions based on the truth of a predicate. For example, the expression `b ? 3 : 5` has value `3` if `b` is **true**; otherwise, it has value `5`.

    (a) Can the ternary operator be represented by an *CondTesting* node in the AST? If not, design an appropriate AST node for representing the ternary operator.

    (b) How does the treatment of the ternary operator differ from the treatment of an `if` statement during semantic analysis?

    (c) How does code generation differ for the two constructs?

6. Lisp offers a generalization of an `if` statement known as **`cond`**.

    (a) Investigate the syntax and semantics of the `cond` construct and document your findings. Aim for a general treatment based on examining specific languages that offer `cond`-like constructs.

    (b) Design an AST representation for `cond` and define interfaces to access the components of a `cond` construct.

    (c) Write a code-generation visitor for `cond` based on your AST interfaces.

7. Languages like C++ and Java offer the `switch` statement as another construct for managing conditional execution.

    (a) Investigate the syntax and semantics of the `switch` and document your findings. Be mindful of the semantics of `break` and its role in the execution of a `switch` statement. Note any type restrictions on the expression supplied to a `switch` statement.

    (b) Design an AST representation for `switch` and define interfaces to access the components of a `cond` construct.

    (c) Write a code-generation visitor for `cond` based on your AST interfaces. Generate code based on one of the following strategies:

        - Conditional execution is managed by predicate tests and jumps, in the style of an `if` statement.
        - A **lookup table** is generated with (*value*, *label*) pairs (cf. JVM `lookupswitch` instruction). If the `switch`'s expression has a given value, then execution proceeds at its associated label. A default label is also provided as the outcome for expressions whose value does not appear in the table.

- A **jump table** is generated containing only labels (cf. JVM `tableswitch` instruction). The `switch` expression value is used to index the table, starting at 0, and execution proceeds with the resulting label. A default label is provisioned and is chosen as the outcome if the expression's value is larger than accommodated by the table.

(d) Compare and contrast the above code-generation strategies for the `switch` statement. Under which conditions is a given strategy better or worse than the others?

8. The semantics of a *WhileTesting* node accommodate Java and C's `while` constructs. Languages like Java have other syntax for itneration, such as Java's `do-while` construct. The body of the loop is executed, and the predicate is then tested. If the predicate is `true`, then the body is executed again. Execution continues to loop in this manner until the predicate becomes `false`.

   The sense of the predicate is consistent, in that iteration ceases when the predicate test is `false`. However, the `do-while` construct executes the loop body before the test. The *CondTesting* node represents constructs where the predicate is tested *before* the loop body is executed.

   How would you accommodate the `do-while` statement? What changes are necessary across a compiler's phases, from syntax analysis to code generation?

9. Some languages offer iteration constructs like C's `for` statment, which aggregates the loop's initialization, termination, and repetition constructs into a single construct.

   Investigate the syntax and semantics of C's `for` statement. Develop an AST node and suitable interface to represent its components. Design and implement its code-generation visitor.

10. Later versions of Java offer the so-called "enhanced `for`" statement. Investigate the syntax and semantics of that statement. Develop an AST node and suitable interface to represent the statement. Design and implement its code-generation visitor.

11. Section 11.3.10 generates code that emits the predicate test as instructions that occur before the loop body. Write a visitor method that generates the loop body's instructions first, but preserves the semantics of the *WhileTesting* node (the predicate must execute first).

12. Write a visitor method to generate code for a *Returning* AST node. Recall that the node may, or may not, require evaluation of the return value.

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12

Runtime Support

We now consider how program structures are implemented in a computer's memory. The evolution of programming language design has led to the creation of increasingly sophisticated methods of runtime storage organization. Arrays, for example, can be allocated as a single fixed-size block of memory. Newer languages allow array sizes to be set during execution. **Flex arrays** can even be expanded as needed by a program.

Originally, all data were *global*, with a lifetime that spanned the entire program. Correspondingly, all storage allocation was *static*. During translation, a data object or instruction sequence was simply placed at a fixed memory address for the entire execution of a program.

In the 1960's, languages like Lisp and Algol 60 introduced **local variables**, which are accessible only during the execution of a subprogram. This feature led to the notion of **stack allocation**. When a procedure or method is called, a new **frame** is pushed on a runtime stack. A frame contains space for all of the local variables in a particular procedure. When a procedure returns, its frame is popped and the space taken by its locals is reclaimed. Therefore, only procedures that are actually executing are allocated space. Inactive procedures claim no data space, making large programs far more space efficient than in earlier translations, which used only static allocation. Moreover, **recursive procedures**, which require multiple frames for separate nested activations of the same procedure, can be implemented cleanly and naturally.

First Lisp and subsequently languages like C, C++, C♯, and Java^{TM}, popularized **dynamically allocated data** that can be created or released at any time during execution. Dynamic data require **heap allocation**, which allows memory blocks to be allocated and freed at any time and in any order during program execution. With dynamic allocation, the number and size of data objects need not be fixed in advance. Each program execution can "customize" its memory allocation needs.

All memory allocation techniques utilize the notion of a **data area**. A data area is a block of storage known by the compiler to have uniform storage allocation requirements. That is, all objects in a data area share the same data allocation policy. The global variables of a program can comprise a data area. Space for all variables is allocated when execution of a program begins, and variables remain allocated until execution terminates. Similarly, a block of data allocated by a call to `new` or `malloc` forms a single data area, because the entire block remains allocated until it is explicitly freed or collected.

We will begin our study of memory allocation with static allocation in Section 12.1. Stack-based memory allocation is investigated in Section 12.2. The structure and layout of arrays are considered in Section 12.3 and heap storage is studied in Section 12.4.

12.1 Static Allocation

In the earliest programming languages, including all assembly languages, COBOL, and Fortran, all storage allocation was static. Space for data objects was allocated at a fixed memory address for the lifetime of a program. Use of static allocation is feasible only when the number and size of all objects to be allocated is known at compile-time. Static allocation makes storage allocation almost trivial, but it can also be quite wasteful of space. As a result, programmers sometimes found it necessary to *overlay* variables. In Fortran, for example, the `equivalence` statement was commonly used to reduce storage needs by forcing two variables to share the same memory locations. (The C/C++ `union` construct can do this too.) Overlaying impairs program readability because assignment to one variable implicitly changes the value of another. As a consequence, it can also lead to subtle programming errors.

In more modern languages, static allocation is used for global variables that are fixed in size and accessible throughout program execution. It is also used for program literals (constants) that need to be fixed throughout execution. Static allocation is used for `static` and `extern` variables in C/C++ and for static fields in C♯ and Java classes. Static allocation is also routinely used for program code, since fixed runtime addresses are required in branch and call instructions. Also, since control flow within a program is very hard to predict, it is difficult to know which instructions will be needed next. Accordingly, if code is statically allocated, any execution order can be accommodated. Java and C♯ allow classes to be dynamically loaded or compiled; but once program code is made executable, it is static.

Conceptually, we can bind static objects to absolute memory addresses. Thus, if we generate an assembly language translation of a program, a global variable or program statement can be given a symbolic label that denotes a fixed memory address. It is often preferable to address a static data object as a pair (*DataArea*, *Offset*). *Offset* is fixed at compile-time, but the address of *DataArea* can be deferred to link- or runtime. In Fortran, for example, *DataArea* can be the start of one of many `common` blocks. In C, *DataArea* can be the start of a block of storage for the variables local to a module (a "`.c`" file). In Java, *DataArea* can be the start of a block of storage for the static fields of a class. Typically these addresses are bound when the program is linked. Address binding must be deferred until link-time or runtime because subroutines and classes may be compiled independently, making it impossible for the compiler to know about all the data areas in a program.

Alternatively, the address of *DataArea* can be loaded into a register (the **global pointer**), which allows a static data item to be addressed as (*Register*, *Offset*). This addressing form is available on almost every machine. The advantage of addressing a piece of static data with a global pointer is that we can load or store a global value in one instruction. Since global addresses occupy 32 (or 64) bits, they normally "don't fit" in a single instruction on machines in which an instruction is the same number of bits as an address, like MIPS®, PowerPC®, and Sparc™. If a global pointer is not available, global addresses must be formed in several steps, first loading the high-order bits, and then masking in the remaining low-order bits.

12.2 Stack Allocation

Almost all modern programming languages allow **recursive subprograms**. Recursion requires **dynamic memory allocation**. Each recursive call requires the allocation of memory for a new copy of a routine's local variables; thus the number of memory allocations required during program execution is not known at compile-time. To implement recursion, all of the data space required for a routine (a procedure, function, or method) is treated as a data area that, because of the special way it is handled, is called a **frame** or **activation record**.

A frame holds local data for a subprogram activation, and is accessible only for the duration of that activation. In mainstream languages like C, C++, C♯, and Java, subprogram activations obey a stack discipline: the most recently called subprogram must be the first to return. In terms of implementation, a frame will be pushed onto a runtime stack when a routine is called (activated). When the routine returns, the frame is popped from the stack, freeing the

```
p(int a) {
   double b;
   double c[10];
   b = c[a] * 2.51;
}
```

Figure 12.1: A Simple Subprogram

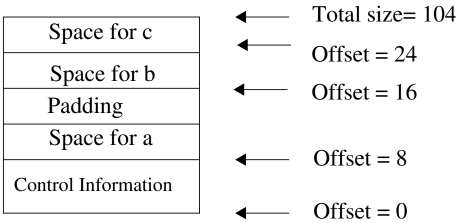


Figure 12.2: Frame for Procedure p

routine's local data. To see how stack allocation works, consider the C++ subprogram shown in Figure 12.1. Procedure p requires space for the parameter a as well as the local variables b and c. It also needs space for control information, such as the **return address** (a subprogram may be called from many different places). As the procedure is compiled, the space requirements of the procedure are recorded (in the procedure's symbol table). In particular, the *offset* of each data item relative to the beginning of the frame is stored in the symbol table. The total amount of space needed, and thus the size of the frame, is also recorded. The memory required for each individual variable (and hence an entire frame) is machine dependent. Different architectures may assume different sizes for primitive values like integers or addresses. It is wise to avoid "hard coding" machine dependent quantities in a compiler. Instead, calls to a *target environment* class can be made.

In our example, assume p's control information requires 8 bytes (this size is usually the same for all methods and subprograms). Assume parameter a requires 4 bytes, local variable b requires 8 bytes, and local array c requires 80 bytes. Because many machines require that word and doubleword data be aligned, it is common practice to pad a frame (if necessary) so that its size is a multiple of 4 or 8 bytes. This guarantees a useful invariant: at all times the top of the stack is properly aligned. Figure 12.2 shows p's frame.

Within p, each local data object is addressed by its offset relative to the start of the frame. This offset is a fixed constant, determined at compile-time. Because we normally store the start of the frame in a register, each piece of data can be addressed as a (*Register*, *Offset*) pair, which is a standard addressing mode in almost all computer architectures. For example, if register R points to the beginning of p's frame, variable b can be addressed as $(R, 16)$, with the offset value of 16 actually being added to the contents of R at runtime as instructions are decoded and executed. Normally, the literal 2.51 of procedure p is not stored in p's frame because the values of local data that are stored in a frame disappear with it at the end of a call. If 2.51 were stored in the frame, its value would have to be initialized before each call. It is easier and more efficient to allocate literals in a static area, often called a *literal pool* or *constant pool*. Java uses a constant pool to store literals, type, method, and interface information as well as class and field names.

12.2.1 Field Access in Classes and Structs

Just as local variables are assigned an offset within the current frame, fields within a class or struct definition are also assigned offsets relative to the beginning of the data object. Consider the following struct definition:

```
struct s {
   int a;
   double b;
   double c[10];
}
```

As each field is processed it is assigned an offset, starting at zero. Thus a is given an offset of 0. b's offset is determined by the size of fields that precede it, augmented with alignment restrictions. Integers typically require 4 bytes, and doubles must be allocated at addresses that are a multiple of 8, so b gets an offset of 8. Array c is assigned an offset of 16, and the size of the entire struct is 96 bytes.

Using this simple scheme, we may use the following formula to compute the address of a field within a class or struct:

$$address(struct.field) = address(struct) + offset(field)$$

For example, if we declare:

```
struct s  var;
```

and s is assigned a static address of 4000, then the address of var.b is $4000+8 = 4008$.

This approach is valid for structs and classes allocated on the runtime stack and heap too. Using our earlier example, if var, of type s, is a local variable within a method, then s will be assigned an offset within the current frame. The offset in a frame for a field within s is s's frame offset plus the field's own offset within its struct or class. Similarly, for a struct allocated on the heap, the address of a field is the field's offset plus the struct's heap address.

Classes generalize structs by allowing members that are methods as well as data fields. However, the code for methods is not allocated within the data allocation for a class object. As described in Section 12.2.3, only one translation of each method is created; it is used with all instances of the class object it is defined for. Thus when we translate fields within classes, we ignore method definitions, making classes and structs effectively identical in translation.

12.2.2 Accessing Frames at Runtime

During execution there will be many frames on the stack. This is because when a procedure A calls a procedure B, a frame for B's local variables is pushed on the stack, covering A's frame. A's frame cannot be popped off because A will resume execution after B returns. In the case of recursive routines, there can be hundreds or even thousands of frames on the stack. All frames except the topmost represent suspended subroutine activations that are waiting for a call to return.

The topmost frame corresponds to the currently active subroutine, and it is important to be able to access it directly. Since the frame is at the top of the stack, the **stack top register** could be used to access it. But the runtime stack may also be used to hold data other than frames, including temporary values or return values too large to fit within a register (compound values like arrays, structures, and strings).

It is therefore unwise to require that the currently active frame always be at exactly the top of the stack. Instead a distinct register, often called the **frame pointer**, is used to access the current frame. This allows local variables to be accessed directly as offset plus frame pointer, using the indexed addressing mode found on all modern machines.

As an example, consider the following recursive function that computes factorials:

```
int fact(int n) {
   if (n > 1)
      return n * fact(n-1);
   else    return 1;
}
```

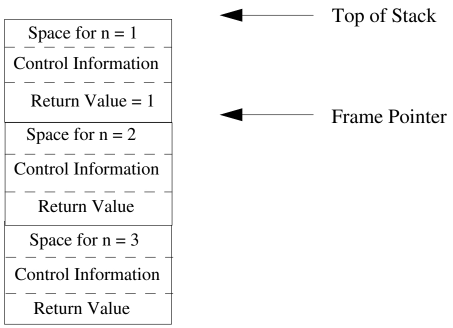


Figure 12.3: Runtime Stack for a Call of `fact(3)`

The runtime stack corresponding to the call `fact(3)` is shown in Figure 12.3 at the point where the call of `fact(1)` is about to return. In our example we show a slot for the function's return value at the very beginning of the frame. This means that upon return, the return value is conveniently placed on the stack, just beyond the end of the caller's frame. As an optimization, many compilers try to return scalar values in specially designated registers. This helps to eliminate unnecessary loads and stores. For function values too large to fit in a register (e.g., a `struct` passed by value), the stack is the natural choice.

When a subroutine returns, its frame must be popped from the stack and the frame pointer must be reset to point to the caller's frame. In simple cases this can be done by adjusting the frame pointer by the size of the current frame. Because the stack may contain more than just frames (e.g., function return values or registers saved across calls), it is common practice to save the caller's frame pointer as part of the callee's control information. Thus each frame points to the preceding frame on the stack. This pointer is often called a **dynamic link** because it links a frame to its dynamic (runtime) predecessor. The runtime stack corresponding to a call of `fact(3)`, with dynamic links included, is shown in Figure 12.4.

12.2.3 Handling Classes and Objects

C, C++, C♯, and Java do not allow procedures or methods to nest. That is, a procedure may not be declared within another procedure. This simplifies runtime data access; all variables are either global or local to the currently executing procedure. Global variables are statically allocated. Local variables are part of a single frame, accessed through the frame pointer.

Languages often need to support simultaneous access to variables in mul-

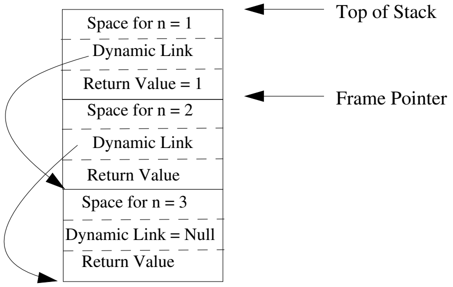


Figure 12.4: Runtime Stack for a Call of `fact(3)`with Dynamic Links

tiple scopes. Java, C++, and C♯, for example, allow classes to have member functions that have direct access to all instance variables. Consider the following Java class:

```
class k {
    int a;
    int sum(){
        int b = 42;
        return a+b;
}   }
```

Each object that is an instance of class k contains a member function sum. Only one translation of sum is created; it is shared by all instances of k. When sum executes, it requires *two* pointers to access local and object-level data. Local data, as usual, resides in a frame on the runtime stack. Data values for a particular instance of k are accessed through an **object pointer** (called the this pointer in Java, C++, and C♯). When obj.sum() is called, it is given an extra implicit parameter that is a pointer to obj. This is illustrated in Figure 12.5. When a+b is computed, b, a local variable, is accessed directly through the frame pointer. a, a member of object obj, is accessed indirectly through the object pointer that is stored in the frame (as all parameters to a method are).

C♯, C++, and Java also allow inheritance via *subclassing*. That is, a new class can extend an existing class, adding new fields and adding or redefining methods. A subclass D, of class C, may be be used in contexts expecting an object of class C (e.g., in method calls). This is supported rather easily; instances of class D always contain an instance of C within them. That is, if C has a field F within it, so does D. The fields D declares are merely appended at the end of the allocations for C. As a result, access to fields declared in C within an instance of D works perfectly. In Java, class Object is often used as a placeholder when an object of unknown type is expected. This works because all Java classes are

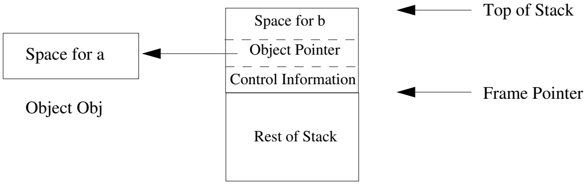


Figure 12.5: Accessing Local and Member data in Java

subclasses of `Object`.

Of course, the converse cannot be allowed. An instance of `C` may not be used where an instance of `D` is expected, since `D`'s fields are not present in `C`.

12.2.4 Handling Multiple Scopes

Older languages like Ada, Pascal, and Algol 60, as well as currently popular languages like Python, ML, and Scheme, allow subprogram declarations to nest. Java and C♯ allow classes to nest (see Exercise 7). Subprogram nesting can be very useful, allowing, for example, a private utility subprogram to directly access another routine's locals and parameters. However, runtime data structures are complicated because multiple frames, corresponding to nested subprogram declarations, may need to be accessed. To see the problem, assume that functions *can* nest in Java or C, and consider the following code fragment:

```
int p(int a){
    int q(int b){
         if (b < 0)
               q(-b);
         else  return a+b;
    }
    return q(-10);
}
```

When `q` executes, it can access not only its own frame, but also that of `p`, in which it is nested. If the depth of nesting is unlimited, so is the number of frames that must be made accessible. In practice, the level of procedure nesting actually seen is modest, usually no greater than two or three.

Two approaches are commonly used to support access to multiple frames. One approach generalizes the idea of dynamic links introduced earlier. Along with a dynamic link, we will also include a **static link** in the frame's control

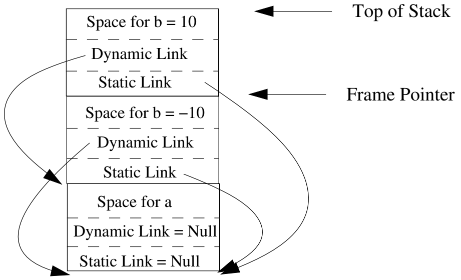


Figure 12.6: An Example of Static Links

information area. The static link will point to the frame of the procedure that statically encloses the current procedure. If a procedure is not nested within any other procedure, its static link is `null`. This approach is illustrated in Figure 12.6.

As usual, dynamic links always point to the next frame down in the stack. Static links always point downward in the stack, but they may skip past many frames. Static links always point to the most recent frame of the subprogram that statically encloses the current routine. Thus, in our example, the static links of both of `q`'s frames point to `p`, since it is `p` that encloses `q`'s definition. In evaluating the expression `a+b` that `q` returns, `b`, being local to `q`, is accessed directly through the frame pointer. Variable `a` is local to `p`, but also visible to `q` because `q` nests within `p`. `a` is accessed by extracting `q`'s static link, and then using that address (plus the appropriate offset) to access `a`.

An alternative to using static links to access frames of enclosing routines is the use of a **display**. A display generalizes our use of a frame pointer. Rather than maintaining a single register, we maintain a *set* of registers which comprise the display. If procedure definitions nest n deep (this can be easily determined by examining a program's **abstract syntax tree** (AST)), we will need $n + 1$ display registers. Each procedure definition is tagged with a *nesting level*. Procedures that are not nested within any other routine are at nesting level 0, while procedures that are nested within a procedure at level n are at level $n + 1$. Frames for routines at level n are always accessed using display register Dn. Thus, whenever a procedure `r` is executing, we have direct access to `r`'s frame plus the frames of all routines that enclose `r`. Each of these routines must be at a different nesting level, and hence will use a different display register. Consider Figure 12.7, which illustrates our earlier example, now using display registers rather than static links.

Since `q` is at nesting level 1, its frame is pointed to by $D1$. All of `q`'s local variables, including `b`, are at a fixed offset relative to $D1$. Similarly, since `p` is at nesting level 0, its frame and local variables are accessed via $D0$. Note that each

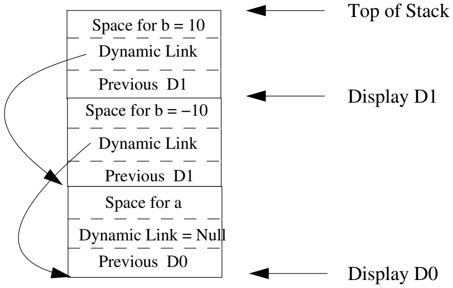


Figure 12.7: An Example of Display Registers

frame's control information area contains a slot for the previous value of the frame's display register. This value is saved when a call begins and is restored when the call ends. The dynamic link is still needed, because previous display values do not always point to the caller's frame.

Both static links and displays are used in real compilers, and each technique presents different tradeoffs. Displays allow direct access to all frames, and thus make access to all visible variables very efficient. However, if nesting is deep, several valuable registers may need to be reserved. Static links are very flexible, allowing unlimited nesting of procedures. However, access to non-local procedure variables can be slowed by the need to extract and follow static links.

Fortunately, the code generated using the two techniques can be readily improved. Static links are just address-valued expressions computed to access variables (much like address calculations involving pointer variables). A careful compiler can notice that an expression is being needlessly recomputed, and reuse a previous computation, often directly from a register. Similarly, a display can be allocated statically in memory. If a particular display value is used frequently, a register allocator will place the display value in a register to avoid repeated loads (just as it would for any other heavily used variable).

12.2.5 Block-Level Allocation

Java, C, C++, and C♯, and most other programming languages, allow declaration of local variables within blocks as well as within subprograms. Often a block will contain only one or two variables, along with statements that use them. Do we allocate an entire frame for each such block?

We could, by considering a block with local variables to be a call of an in-line subprogram without parameters. The call causes the allocation of a new frame. This implementation technique would require a display or static

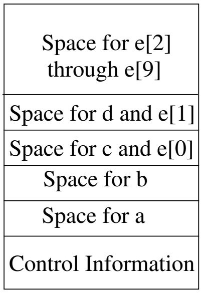


Figure 12.8: An Example of a Procedure-Level Frame

links, even in Java or C, because blocks can nest. Further, execution of a block would become a bit costly, since frames would need to be pushed and popped, display registers or static links updated, and so forth.

To avoid this overhead, we can choose to use frames only for true subprograms, even if blocks within a subprogram have local declarations. This technique is called **procedure-level frame allocation**, as contrasted with **block-level frame allocation**, which allocates a frame for each block that has local declarations.

The central idea of procedure-level frame allocation is that the relative location of variables in individual blocks within a procedure can be computed and fixed at compile-time. This works because blocks are entered and exited in a strictly textual order. Consider the following procedure:

```
void p(int a) {
     int b;
     if (a > 0)
           {float c, d;
           // Body of block 1 }
     else  {int e[10];
           // Body of block 2 }
}
```

Parameter a and local variable b are visible throughout the procedure. However, the then and else parts of the if statement are mutually exclusive. Thus, variables in block 1 and block 2 can *overlay* each other. That is, c and d are allocated just beyond b as is array e. This overlay is safe because variables in both blocks can never be accessed at the same time. The layout of the frame is illustrated in Figure 12.8.

Offsets for variables within a block are assigned just after the last variable in the enclosing scope within the procedure. Thus, both c and e[] are placed after b because both block 1 and block 2 are enclosed by the block comprising p's body. As blocks are compiled, a "high-water mark" is maintained that represents the maximum offset used by any local variable. This high-water mark determines the size of the overall frame. Thus, a[9] occupies the maximum offset within the frame, so its location determines the size of p's frame.

The process of assigning local variables procedure-level offsets is sometimes done using **scope flattening**. That is, local declarations are mapped to equivalent procedure-level declarations. This process is particularly effective if procedure-level register allocation is part of the compilation process (see Section 13.3.2 on page 508).

12.2.6 More About Frames

We now consider briefly a number of language and hardware issues that affect the design and use of frames at runtime.

Closures In C it is possible to create a pointer to a function. Since a function's frame is created only when it is called, a function pointer is implemented as the function's entry point address. In C++, pointers to member functions of a class are allowed. When the pointer is used, a particular instance of the class must be provided by the user program. That is, two pointers are needed, one to the function itself and a second to the class instance in which it resides. This second pointer allows the member function to correctly access local data belonging to the class.

Other languages, particularly functional languages like Lisp, Scheme, and ML, are much more general in their treatment of functions. Functions are *first-class objects*. They can be stored in variables and data structures, constructed during execution, and returned as function results.

Runtime creation and manipulation of functions can be extremely useful. For example, it is sometimes the case that the computation of f(x) takes a significant amount of time. Once f(x) is known, a common optimization, called **memoizing**, records the pair (x,f(x)) so that subsequent calls to f with argument x can use the known value of f(x) rather than recompute it. In ML it is possible to write a function memo that takes a function f and an argument arg. memo computes f(arg) and also returns a "smarter" version of f that has the value of f(arg) "built into" it. This smarter version of f can be used instead of f in all subsequent computations:

```
fun memo(fct,parm)= let val ans = fct(parm) in
 (ans, fn x=> if x=parm then ans else fct(x)) end;
```

When the version of fct returned by memo is called, it will access the values of parm, fct, and ans, which are used in its definition. After memo returns, its frame must be preserved since that frame contains parm, fct and ans within it.

In general, when a function is created or manipulated, we must maintain a pair of pointers. One is to the machine instructions that implement the function, and the other is to the frame (or frames) that represent the function's *execution environment*. This pair of pointers is called a **closure**. Note also that when functions are first-class objects, a frame corresponding to a call may be accessed *after* the call terminates. This means frames can not be routinely allocated on the runtime stack. Instead, they may be allocated in the heap and garbage-collected, just like user-created data. Intuitively this may seem to be inefficient, but Appel [App96] has shown that in some circumstances heap allocation of frames can be *faster* than stack allocation.

Cactus Stacks Many programming languages allow the concurrent execution of more than one computation in the same program. Units of concurrent execution are sometimes called *tasks*, *processes*, or *threads*. In some cases a new system-level process is created (as in the case of fork in C). Because significant operating system overhead is involved, such processes are called **heavyweight processes**. A less expensive alternative is to execute several threads of control in a single system-level process. Because much less state is involved, computations that execute concurrently in a single system process are called **lightweight processes**.

A good example of lightweight processes are *threads* in Java. As illustrated below, a Java program may initiate several calls to methods that will execute simultaneously:

```
public static void main (String args[]) {
        new AudioThread("Audio").start();
        new VideoThread("Video").start();
}
```

Here, two instances of Thread subclasses are started, and each executes concurrently with the other. One thread might implement the audio portion of an application, while the other implements the video portion.

Since each thread can initiate its own sequence of calls (and possibly start more threads), all the resulting frames cannot be pushed on a single runtime stack (the exact order in which threads execute is unpredictable). Instead, each thread gets its own stack segment in which frames it creates may be pushed. This stack structure is sometimes called a **cactus stack**, since it is reminiscent of the saguaro cactus, which sends out arms from the main trunk and from other arms. It is important that the thread handler be designed so that segments are

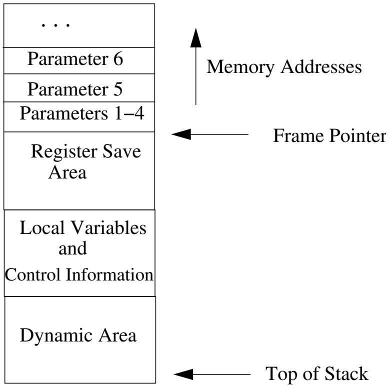


Figure 12.9: Layout for MIPS R3000

properly deallocated when their thread is terminated. Since Java guarantees that all temporaries and locals are contained within a method's frame, stack management is limited to proper allocation and deallocation of frames.

A Detailed Frame Layout The layout of a frame is normally standardized for a given architecture. This is necessary to support calls to subprograms translated by different compilers. Since languages and compilers vary in the features they support, the frame layout chosen as standard must be very general and inclusive. As an example, consider Figure 12.9, which illustrates the frame layout used by the MIPS architecture.

By convention, the first four parameters, if they are scalars or pointers to structures or arrays, are passed in registers. Additional parameters are passed through the stack. Parameters that cannot fit in registers, such as structures or arrays passed by value, are also passed on the stack. The slots for parameters 1–4 can be used to save parameter registers when a call is made from within a subprogram. The register save area is used at two different times. Registers are commonly partitioned into **caller-save registers** (for which a caller is responsible) and **callee-save registers** (for which a called subprogram is responsible). When execution of a subprogram begins, callee-save registers used by the subroutine itself are saved in the register save area. When a call is made from within the subprogram, caller-save registers that are in use are saved in the register save area. At different call sites, different registers may be in use. The register save area must be large enough to handle all calls within a particular subroutine. Often a fixed-size register save area, big enough to accommodate all caller-save and callee-save registers, is used. This may waste a bit of space, but only registers actually in use are saved.

The local variables and control information area contains space for all local variables. It also contains space for the return address register, and the value of the caller's frame pointer. The value of a static link or display register may be saved here if they are needed. The stack top may be reset, upon return, by adding the size of the parameter area to the frame pointer. (On the MIPS, as well as on many other computers, the stack grows *downward*, from high to low addresses.)

The details of subroutine calls are explored more thoroughly in Section 11.1 on page 418 (at the bytecode level) and Section 13.1.3 on page 496 (at the machine code level).

Because the **Java Virtual Machine** (JVM) is designed to run on a wide variety of architectures and only interacts with external code through a well-defined native interface, the exact details of its runtime frame layout are unspecified. A particular implementation (such as the JVM running on a MIPS processor), chooses a particular layout, similar to that shown in Figure 12.9.

Some languages allow the size of a frame to be expanded during execution. In C, for example, `alloca` allocates space on demand on the stack. Space is pushed beyond the end of the frame. Upon return, this space is automatically freed when the frame is popped.

Some languages allow the creation of **dynamic arrays** whose bounds are set at runtime when a frame is pushed (e.g., `int data[max(a,b)]`). At the start of a subprogram's execution, array bounds are evaluated and necessary space is pushed in the dynamic area of the frame.

C and C++ allow subroutines like `printf` and `scanf` to have a *variable* number of arguments. The MIPS frame design supports such routines, since parameter values are placed, in order, just above the frame pointer.

Non-scalar return values can be handled by treating the return value as the "zero-th parameter." As an optimization, calls to functions that return a non-scalar result sometimes pass an address as the zero-th parameter. This represents a place where the return value can be stored prior to return. Otherwise, the return value is left on the stack by the function.

12.3 Arrays

12.3.1 Static One-Dimensional Arrays

One of the most fundamental data structures in programming languages is the array. The simplest kind of array is one that has a single index and constant bounds. An example (in C or C++) is:

```
int a[100];
```

An array can allocated in memory as a sequence of N identical data objects, where N is determined by the declared size of the array. Hence in the above example 100 consecutive integers are allocated.

An array, like all other data structures, has a size and possibly an alignment requirement. An array's size is easily computed as:

$$size(array) = NumberOfElements * size(Element)$$

If the bounds of an array are included within its memory allocation (as is the case for Java and C♯), the array's memory requirement must be increased accordingly.

Many processors impose an **alignment restriction** on data. For example, integers, which are usually a word (four bytes) in size, often must be placed at memory addresses that are a multiple of four. An array's alignment restriction is that of its components. Thus an integer array must be word-aligned if integers must be word-aligned.

Sometimes **padding** is needed to guarantee alignment of all array elements. For example, given the C declaration:

```
struct s {int a; char b;} ar[100];
```

each element of array `ar` (a struct named `s`) must be padded to a size of 8 bytes. This is necessary to guarantee that `ar[i].a`, an integer field, is always word-aligned.

When arrays are copied, size information is used to determine how many bytes to copy. Either a series of load/store instructions or a copy loop can be used, depending on the size of the array.

In C, C++, Java and C♯, all arrays are **zero-based** (the first element of an array is always at position 0). This rule leads to a very simple formula for the address of an array element:

$$address(A[i]) = address(A) + i * size(Element)$$

For example, using the declaration of `ar` as an array of struct `s` given above:

$$\begin{aligned} address(ar[5]) &= address(ar) + 5 * size(s) \\ &= address(ar) + 5 * 8 = address(ar) + 40 \end{aligned}$$

Computing the address of a field within an array of structures is easy too. As discussed in Section 12.2.1:

$$address(struct.field) = address(struct) + offset(field)$$

Thus

$$\begin{aligned}\mathit{address}(\mathit{struct}[i].\mathit{field}) &= \mathit{address}(\mathit{struct}[i]) + \mathit{offset}(\mathit{field}) \\ &= \mathit{address}(\mathit{struct}) + i * \mathit{size}(\mathit{struct}) + \mathit{offset}(\mathit{field})\end{aligned}$$

For example,

$$\begin{aligned}\mathit{address}(\mathit{ar}[5].b) &= \mathit{address}(\mathit{ar}[5]) + \mathit{offset}(b) \\ &= \mathit{address}(\mathit{ar}) + 40 + 4 \\ &= \mathit{address}(\mathit{ar}) + 44\end{aligned}$$

In Java and C♯, arrays are allocated as objects; all the elements of the array are allocated within the object. Exactly how the objects are allocated is unspecified by Java's definition, but a sequential contiguous allocation, just like C and C++, is the most natural and efficient implementation.

Array Bounds Checking

An array reference is legal only if the index used is in bounds. References outside the bounds of an array are undefined and dangerous, as data unrelated to the array may be read or written. Java, with its attention to security, checks that an array index is in range when an array load or array store instruction is executed. An illegal index forces an `ArrayIndexOutOfBoundsException`. Since the size of an array object is stored within the object, checking the validity of an index is easy, though it does slow access to arrays.

In C and C++ array indices out of bounds are also illegal. Most compilers do not implement bounds checking, and hence program errors involving access beyond array bounds are common.

Why is bounds checking so often ignored? Certainly speed is an issue. Checking an index involves two checks (lower and upper bounds) and each check involves several instructions (to load a bound, compare it with the index, and conditionally branch to an error routine). Using unsigned arithmetic, bounds checking can be reduced to a single comparison (since a negative index, considered unsigned, looks like a very large positive value). Using the techniques of Chapter 14, redundant bounds checks can often be optimized away. Still, array indexing is a very common operation, and bounds checking adds a real cost (though buggy programs are costly too!).

A less obvious impediment to bounds checking in C and C++ is the fact that array names are often treated as equivalent to a pointer to the array's first element. That is, an `int[]` and a `*int` are often considered synonymous. When an array pointer is used to index an array, we do not know what the upper bound of the array is. Moreover, many C and C++ programs intentionally violate array bounds rules, initializing a pointer one position before the start of an array or allowing a pointer to progress one position past the end of an array.

We can support array bounds checking by including a ”size” parameter with every array passed as a parameter and every pointer that steps through an array. This size value serves as an upper bound, indicating the extent of access allowed. Nevertheless, it is clear that bounds checking is a difficult issue in languages where the difference between pointers and array addresses is blurred.

Array parameters often require information beyond a pointer to the array's data values. This includes information on the array's size (to implement assignment) and information on the array's bounds (to allow subscript checking). An array descriptor (sometimes called a **dope vector**), containing this information, can be passed for array parameters instead of just a data pointer.

Non-Zero Lower Bounds

In C, C++ and Java, arrays always have a lower bound of zero. This simplifies array indexing. Still, a single fixed lower bound can lead to clumsy code sequences. Consider an array indexed by years. Having learned in 1999 not to represent years as just two digits, we may prefer to use a full four-digit year as an index. Assuming we really want to only use years of the twentieth and early twenty-first centuries, an array starting at 0 is very clumsy. But so is explicitly subtracting 1900 from each index before using it.

A few languages, like Pascal and Ada, have already solved this problem. An array of the form `A[low..high]` may be declared, where all indices in the range `low`, ..., `high` are allowed. With this array form, we can easily declare an array indexed by four digit years: `data[1900..2020]`.

With a non-zero lower bound, our formula for the size of an array must be generalized a bit:

$$size(array) = (UpperBound - LowerBound + 1) * size(Element)$$

How much does this generalization complicate array indexing? Actually, surprisingly little. If we take the Java approach, we just include the lower bound as part of the array object we allocate. If we compute an element address in the code we generate, the address formula introduced above needs to be changed a little:

$$address(A[i]) = address(A) + (i - low) * size(Element)$$

We subtract the array's lower bound (*low*) before we multiply by the element size. Now it is clear why a lower bound of zero simplifies indexing—a subtraction of zero from the array index can be skipped. But the above formula can be rearranged to:

$$\begin{aligned} address(A[i]) &= address(A) + (i * size(Element)) - (low * size(Element)) \\ &= address(A) - (low * size(Element)) + (i * size(Element)) \end{aligned}$$

We now note that *low* and *size*(*Element*) are normally compile-time constants, so the expression (*low* ∗ *size*(*Element*)) can be reduced to a single value, *bias*. Now we have:

$$address(A[i]) = address(A) - bias + (i * size(Element))$$

The address of an array is normally a static address (a global) or an offset relative to a frame pointer (a local). In either case, the *bias* value can be folded into the array's address, forming a new address or frame offset reduced by the value of bias.

For example, if we declare an array `int data[1900..2020]`, and assign data an address of 10000, we get a bias value of $1900 * size(int) = 7600$. In computing the address of `data[i]` we compute $2400 + i * 4$. This is exactly the same form that we used for zero-based arrays.

Even if we allocate arrays in the heap (using `new` or `malloc`), we can use the same approach. Rather than storing a pointer to the array's first element, we store a pointer to its virtual "zero-th" element (which is what subtracting *bias* from the array address gives us). We do need to be careful when we assign such arrays though; we must copy data beginning with first valid position in the array. Still, indexing is far more common than copying, so this approach is a very reasonable one.

Dynamic and Flex Arrays

Some languages, including Algol 60, Ada, Java, and C♯ support **dynamic arrays** whose bounds and size are determined at runtime. When the scope of a dynamic array is entered, its bounds and size are evaluated and fixed. Space for the array is then allocated. The bounds of a dynamic array may include parameters, variables, and expressions. For example, if C were extended to allow dynamic arrays, a subprogram `P` might include the declaration:

```
  int examScore[numOfStudents()];
```

Because the size of a dynamic array is not known at compile-time, we cannot allocate space for it statically or in a frame. Instead, we must allocate space either on the stack (just past the current frame) or in the heap (Java does this). A pointer to the location of the array is stored in the scope in which the array is declared. The size of the array (and perhaps its bounds) are also stored. Using our above example, we would allocate space for `examScores` as shown

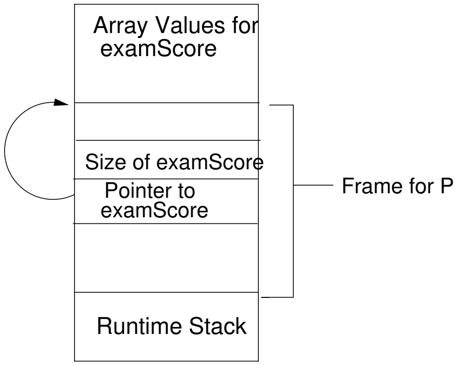


Figure 12.10: Allocation of a Dynamic Array

in Figure 12.10. Within P's frame we allocate space for a pointer to examScore's values as well as it size.

Accessing a dynamic array requires an extra step. First the location of the array is loaded from a global address or from an offset within a frame. Then, as usual, an offset within the array is computed and added to the array's starting location.

A variant of the dynamic array is the **flex array**, which can *expand* its bounds during execution. (The Java Vector class implements a flex array.) When a flex array is created, it is given a default size. If during execution an index beyond the array's current size is used, the array is expanded to make the index legal. Since the ultimate size of a flex array is not known when the array is initially allocated, we store the flex array in the heap, and point to it. Whenever a flex array is indexed, the array's current size is checked. If it is too small, another larger array is allocated, values from the old allocation are copied to the new allocation, and the array's pointer is reset to point to the new allocation.

When a dynamic or flex array is passed as a parameter, it is necessary to pass an array descriptor that includes a pointer to the array's data values as well as information on the array's bounds and size. This information is needed to support indexing and assignment.

12.3.2 Multidimensional Arrays

In most programming languages **multidimensional arrays** may be treated as *arrays of arrays*. In Java, for example, the declaration:

```
int matrix[][] = new int[5][10];
```

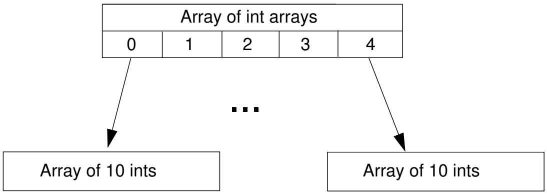
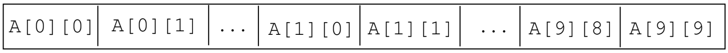


Figure 12.11: A Multidimensional Array in Java



A[0][0]	A[0][1]	...	A[1][0]	A[1][1]	...	A[9][8]	A[9][9]

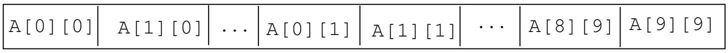
Figure 12.12: Array `A[10][10]` Allocated in Row-Major Order

first assigns to `matrix` an array object containing five references to integer arrays. Then, in sequence, five integer arrays (each of size ten) are created, and assigned to the array matrix references (see Figure 12.11).

Other languages, like C and C++, allocate one block of memory, sufficient to contain all the elements of the array. The array is arranged in **row-major order**, with values in each row contiguous and individual rows placed sequentially (see Figure 12.12). In row-major form, multidimensional arrays really are arrays of arrays, since in an array reference like `A[i][j]`, the first index (`i`) selects the i-th row, and the second index (`j`) chooses an element within the selected row.

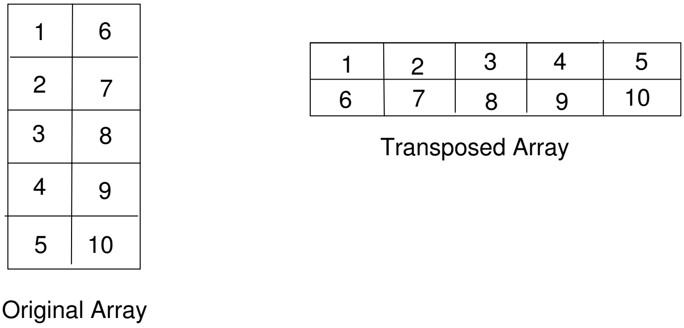
An alternative to row-major order is **column-major order**, which is used in Fortran and related languages. In column-major order values in individual columns are contiguous, and columns are placed adjacent to each other (see Figure 12.13). Again, the whole array is allocated as a single block of memory.

How are elements of multidimensional arrays accessed? For arrays allocated in row-major order (the most common allocation choice), we can exploit the fact that multidimensional arrays can be treated as arrays of arrays. In particular, to compute the address of `A[i][j]`, we first compute the address of `A[i]`, treating `A` as a one-dimensional array of values that happen to be arrays. Once we have the address of `A[i]`, we then compute the address of



A[0][0]	A[1][0]	...	A[0][1]	A[1][1]	...	A[8][9]	A[9][9]

Figure 12.13: Array `A[10][10]` Allocated in Column-Major Order



1	6
2	7
3	8
4	9
5	10

Original Array

1	2	3	4	5
6	7	8	9	10

Transposed Array

Figure 12.14: An Example of Array Transposition

X[j], where X is the starting address of A[i].

Let us look at the actual computations needed. Assume array A is declared to be an n by m array (e.g., it is declared as T A[n][m], where T is the type of the array's elements). We now know that:

$$\begin{aligned} \mathit{address}(A[i][j]) &= \mathit{address}(X[j]) \text{ where } X = \mathit{address}(A[i]) \\ \mathit{address}(A[i]) &= \mathit{address}(A) + i * \mathit{size}(T) * m \end{aligned}$$

Now:

$$\mathit{address}(X[j]) = \mathit{address}(X) + j * \mathit{size}(T)$$

Putting these together:

$$\begin{aligned} \mathit{address}(A[i][j]) &= \mathit{address}(A) + i * \mathit{size}(T) * m + j * \mathit{size}(T) \\ &= \mathit{address}(A) + (i * m + j) * \mathit{size}(T) \end{aligned}$$

Computation of the address of elements in a column-major array is a bit more involved, but we can make a useful observation. First, recall that *transposing* an array involves interchanging its rows and columns. That is, the first column of the array becomes the first row of the transposed array, the second column becomes the second row and so on (see Figure 12.14).

Now observe that a column-major ordering of elements in an array corresponds to a row-major ordering in the transposition of that array. Allocate an n-by-m array, A, in column-major order and consider any element A[i][j]. Now transpose A into AT, an m-by-n array, and allocate it in row-major order. Element AT[j][i] will always be the same as A[i][j].

What this means is that we have a clever way of computing the address of A[i][j] in a column-major array. We simply compute the address of AT[j][i], where `AT` is treated as a row-major array with the same address as `A`, but with interchanged row and column sizes (`A[n][m]` becomes `AT[m][n]`).

As an example, refer to Figure 12.14. The array on the left represents a 5-by-2 array of integers. In column-major order, the array's elements appear in the order 1 to 10. Similarly, the array on the right represents a 2-by-5 array of integers; in row-major order, the array's elements appear in the order 1 to 10. It is easy to see that a value in position `[i][j]` in the left array always corresponds to the value at `[j][i]` in the right (transposed) array.

12.4 Heap Management

The most flexible storage allocation mechanism is **heap allocation**. Any number of data objects can be allocated and freed at any time and in any order. A storage pool, usually called a **heap**, is used. Heap allocation is enormously popular. It is difficult to imagine a non-trivial Java or C program that does not use `new` or `malloc`.

Heap allocation and deallocation is far more complicated than is the case for static or stack allocation. Complex mechanisms may be needed to satisfy a request for space. Indeed, in some cases all of the heap (tens or hundreds of megabytes) may need to be examined. It takes great care to make heap management fast and efficient.

12.4.1 Allocation Mechanisms

A request for heap space may be *explicit* or *implicit*. An explicit request involves a call to a routine like `new` or `malloc`, with a request for a specific number of bytes. An explicit pointer or reference to the newly allocated space is returned (or a `null` pointer if the request could not be honored).

Some languages allow the creation of data objects of unknown size, which may involve an implicit heap allocation. Assume that in C++, as in Java, the `+` operator is overloaded to represent string concatenation. That is, the expression `Str1 + Str2` creates a new string representing the concatenation of strings `Str1` and `Str2`. There is no compile-time bound on the sizes of `Str1` and `Str2`, so heap space must be allocated to hold the newly created string.

Whether allocation is explicit or implicit, a **heap allocator** is needed. This routine takes a size parameter and examines unused heap space to find free space that satisfies the request. A **heap block** is returned. This block will be big enough to satisfy the space request, but it may well be bigger. Allocated heap blocks are almost always single- or double-word aligned to avoid alignment problems in heap-allocated arrays or class instances. Heap blocks contain a header field (usually a word) that contains the size of the block as well as auxiliary bookkeeping information. (The size information is necessary to properly "recycle" the block if it is later deallocated.) A minimum heap block size (commonly 16 bytes) is usually imposed to simplify bookkeeping and guarantee alignment.

The complexity of heap allocation depends in large measure on how **heap deallocation** is done. Initially, the heap is one large block of unallocated memory. Memory requests can be satisfied by simply modifying an "end of heap" pointer, very much as a stack is pushed by modifying a stack pointer. Heap allocation gets more involved when previously allocated heap objects are deallocated and reused. Some deallocation techniques *compact* the heap, moving all "in use" objects to one end of the heap. This means unused heap space is always contiguous, making allocation (via a heap pointer) almost trivial.

Some heap deallocation algorithms have the useful property that their speed depends not on the total number of heap objects allocated, but rather only on those objects *still in use*. If most heap objects "die" soon after their allocation (and often this does seem to be the case), deallocation of these objects is essentially free.

Unfortunately, many deallocation techniques do not perform compaction. Deallocated objects must be stored for future reuse. The most common approach is to create a **free space list**. A free space list is a linked (or doubly-linked) list that contains all the heap blocks known to not be in use. Initially it contains one immense block representing the entire heap. As heap blocks are allocated, this block shrinks. When heap blocks are returned, they are appended to the free space list.

The most common way of maintaining the free space list is to append blocks to the head of the list as they are deallocated. This simplifies deallocation a bit, but makes **coalescing of free space** difficult.

It often happens that two blocks that are physically adjacent in the heap are eventually deallocated. If we can recognize that the two blocks are now both free and adjacent, then they can be coalesced into one larger free block. One large block is preferable to two smaller blocks since the combined block can satisfy requests too large for either of the individual blocks.

The **boundary tags** approach [Knu73a] allows us to identify and coalesce adjacent free heap blocks. Each heap block, whether allocated or on the free space list, contains a tag word on both of its ends. This tag word contains a flag indicating "free" or "in use" and the size of the block. When a block is freed, the boundary tags of its neighbors are checked. If either or both neighbors are marked as free, then they are unlinked from the free space list and coalesced with the current free block.

A free space list may also be kept in **address order**; that is, sorted in order of increasing heap addresses. Boundary tags are no longer needed to identify adjacent free blocks, though maintenance of the list in sorted order is now more expensive.

When a request for n bytes of heap space is received, the heap allocator must search the free space list for a block of sufficient size. But how much of the free space list is to be searched? (It may contain many thousands of blocks.) What if no free block exactly matches the current request? There are many approaches that might be used. We will consider briefly a few of the most widely used techniques.

Best Fit The free space list is searched, perhaps exhaustively, for the free block that most closely matches the requested size. This minimizes wasted heap space, though it may create tiny fragments too small to be used very often. If the free space list is very long, a best fit search may be quite slow. Segregated free space lists (see below) may be preferable.

First Fit The first free heap block of sufficient size is used. Unused space within the block is split off and linked as a smaller free space block. This approach is fast, but may "clutter" the beginning of the free space list with a number of blocks too small to satisfy most requests.

Next Fit This is a variant of first fit in which succeeding searches of the free space list begin at the position where the last search ended rather than at the head of the list. The idea is to "cycle through" the entire free space list rather than always revisiting free blocks at the head of the list. This approach reduces **fragmentation** (in which blocks are split into small, difficult to use pieces). However, it also reduces locality (how densely packed active heap objects are). If we allocate heap objects that are widely distributed throughout the heap, we may increase cache misses and page faults, significantly impacting performance.

Segregated Free Space Lists There is no reason why we must have only *one* free space list. An alternative is to have several, indexed by the size of the free blocks they contain. Experiments have shown that programs frequently request only a few "magic sizes." If we divide the heap into segments, each holding only one size, we can maintain individual free space lists for each segment. Because heap object size is fixed, no headers are needed.

A variant of this approach is to maintain lists of objects of special "strategic" sizes (16, 32, 64, 128, etc.) When a request for size s is received, a block of the smallest size $\leq s$ is selected (with excess size unused within the allocated block).

Another variant is to maintain a number of free space lists, each containing a range of sizes. When a request for size *s* is received, the free space list covering *s*'s range is searched using a best fit strategy.

Fixed-Size Subheaps Rather than linking free objects onto lists according to their size, we can divide the heap into a number of **subheaps**, each allocating objects of a single fixed size. We can then use a **bitmap** to track an object's allocation status. That is, each object is mapped to a single bit in a large array. A 1 indicates the object is in use; a 0 indicates it is free. We need no explicit headers or free space lists. Moreover, since all objects are of the same size, any object whose status bit is 0 may be allocated. However, subheaps may be used unevenly, which can lead to poor memory utilization.

12.4.2 Deallocation Mechanisms

Allocating heap space is fairly straightforward. Requests for space are satisfied by adjusting an end-of-heap pointer, or by searching a free space list. But how do we deallocate heap memory no longer in use? Sometimes we may never need to deallocate. If heap objects are allocated infrequently or are very long-lived, deallocation is unnecessary. We simply fill heap space with "in use" objects.

Unfortunately, many (perhaps most) programs cannot simply ignore deallocation. Many programs allocate huge numbers of short-lived heap objects. If we "pollute" the heap with large numbers of **dead objects** (that are no longer accessible), locality can be severely impacted, with active objects spread throughout a large address range. Long-lived or continuously running programs can also be plagued by **memory leaks**, in which dead heap objects slowly accumulate until a program's memory needs exceed system limits.

User-Controlled Deallocation Deallocation can be manual or automatic. Manual deallocation involves explicit programmer-initiated calls to routines like `free(p)` or `delete(p)`. Pointer `p` identifies the heap object to be freed. The object's size is stored in its header. The object may be merged with adjacent unused heap objects (if boundary tags or an address-ordered free space list is used). It is then added to a free space list for subsequent reallocation.

It is the programmer's responsibility to free unneeded heap space by executing deallocation commands. The heap manager merely keeps track of freed space and makes it available for later reuse. The really hard decision (when space should be freed) is shifted to the programmer, possibly leading to catastrophic **dangling pointer** errors. Consider the following C program fragment:

```
q = p = malloc(1000);
free(p);
/* code containing a number of malloc's */
q[100] = 1234;
```

After p is freed, q is a dangling pointer. That is, q points to heap space that is no longer considered allocated. Calls to malloc may reassign the space pointed to by q. Assignment through q is illegal, but this error is almost never detected. Such an assignment may change data that is now part of another heap object, leading to very subtle errors. It may even change a header field or a free space link, causing the heap allocator itself to fail.

12.4.3 Automatic Garbage Collection

The alternative to manual deallocation of heap space is automatic deallocation, commonly called **garbage collection**. Compiler-generated code and support subroutines track pointer usage. After a heap object is no longer live, the object is automatically deallocated (collected) and made available for subsequent reuse.

Garbage collection techniques vary greatly in their speed, effectiveness, and complexity. We shall consider briefly some of the most important approaches. For a more thorough discussion see [Wil92, JL96].

Reference Counting One of the oldest and simplest garbage collection techniques is **reference counting**. A field is added to the header of each heap object. This field, the object's **reference count**, records how many references (pointers) to the heap object exist. When an object's reference count reaches zero, it is garbage and may be added to the free space list for future reuse.

The reference count field must be updated whenever a reference is created, copied, or destroyed. When a subprogram returns, the reference counts for all objects pointed to by local variables must be decremented. Similarly, when a reference count reaches zero and an object is collected, all pointers in the collected object must also be followed and corresponding reference counts decremented.

As shown in Figure 12.15, reference counting has particular difficulty with **circular heap structures**. If pointer p is set to null, the object's reference count is reduced to 1. Now both objects have a non-zero count, but neither is accessible through any external pointer. That is, the two objects are garbage, but will not be recognized as such.

If circular structures are rare, this deficiency will not be much of a problem. If they are common, then an auxiliary technique, like mark-sweep collection, will be needed to collect garbage that reference counting misses.

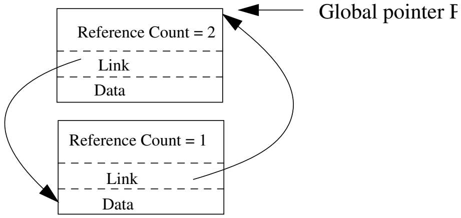


Figure 12.15: An Example of Circular Structures

An important aspect of reference counting is that it is *incremental*. That is, whenever a pointer is manipulated, a small amount of work is done to support garbage collection. This is both an advantage and a disadvantage. It is an advantage in that the cost of garbage collection is smoothly distributed throughout a computation. A program does not need to stop when heap space grows short and do a lengthy collection. This can be crucial when fast real-time response is required. (We do not want the controls of an aircraft to suddenly "freeze" for a second or two while a program collects garbage!)

The incremental nature of reference counting can also be a disadvantage when garbage collection is not really needed. If we have a complex data structure in which pointers are frequently updated, but in which few objects ever are discarded, reference counting always adjusts counts that rarely, if ever, go to zero.

How big should a reference count field be? Interestingly, experience has shown that it does not need to be particularly large. Often only a few bits suffice. The idea here is that if a count ever reaches the maximum representable count (perhaps 7 or 15 or 31), we "lock" the count at that value. Objects with a locked reference count will never ever be detected as garbage by their counts, but they can be collected using other techniques when circular structures are collected.

In summary, reference counting is a simple technique whose incremental nature is sometimes useful. Because of its inability to handle circular structures and its significant per-pointer operation cost, other garbage collection techniques, as described below, are often more attractive alternatives.

Mark-Sweep Collection Rather than incrementally collecting garbage as pointers are manipulated, we can take a batch approach. We do nothing until heap space is nearly exhausted. Then we execute a **marking phase**, which aims to identify all live (non-garbage) heap objects.

Starting with global pointers and pointers in stack frames, we mark reachable heap objects (perhaps setting a bit in the object's header). We then follow

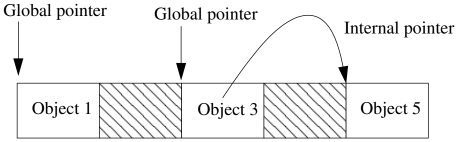


Figure 12.16: Mark-Sweep Garbage Collection

pointers in marked heap objects until all live heap objects are marked.

After the marking phase, we know that any object not marked is garbage that may be freed. We then "sweep" through the heap, collecting all unmarked objects and returning them to the free space list for later reuse. During the **sweep phase** we also clear all marks from heap objects found to still be in use.

Mark-sweep garbage collection is illustrated in Figure 12.16. Objects 1 and 3 are marked because they are pointed to by global pointers. Object 5 is marked because it is pointed to by object 3, which is marked. Shaded objects are not marked and will be added to the free space list.

In any mark-sweep collector it is vital that we mark *all* accessible heap objects. If we miss a pointer we may fail to mark a live heap object and later incorrectly free it. Finding all pointers is not too difficult in languages like Lisp and Scheme that have very uniform data structures, but it is a bit tricky in languages like Java, C, and C++, that have pointers mixed with other types within data structures, implicit pointers to temporaries, and so forth. Considerable information about data structures and frames must be available at runtime for this purpose. In cases where we cannot be sure if a value is a pointer or not, we may need to do conservative garbage collection (see below).

Mark-sweep garbage collection also has the problem that all heap objects must be swept. This can be costly if most objects are dead. Other collection schemes, like copying collectors, examine only live objects.

After the sweep phase, live heap objects are distributed throughout the heap space. This can lead to poor locality. If live objects span many memory pages, paging overhead may be increased. Cache locality may also be degraded.

We can add a **compaction phase** to mark-sweep garbage collection. After live objects are identified, they are placed together at one end of the heap. This involves another tracing phase in which global, local, and internal heap pointers are found and adjusted to reflect the object's new location. Pointers are adjusted by the total size of all garbage objects between the start of the heap and the current object. This is illustrated in Figure 12.17.

Compaction is attractive because all garbage objects are merged together into one large block of free heap space. Fragments are no longer a problem.

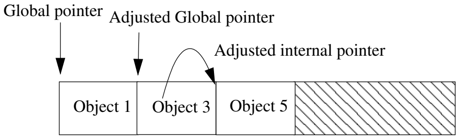


Figure 12.17: Mark-Sweep Garbage Collection with Compaction

Moreover, heap allocation is greatly simplified. The compacting collector maintains an "end of heap" pointer. Whenever it receives an allocation request, it adjusts the end of heap pointer, making heap allocation no more complex than stack allocation.

However, because pointers must be adjusted, compaction may not be suitable for languages like C and C++, in which it is difficult to unambiguously identify pointers.

Copying Collectors Compaction provides many valuable benefits. Heap allocation is simple and efficient. There is no fragmentation problem, and because live objects are adjacent, paging and cache behavior is improved. An entire family of garbage collection techniques, called **copying collectors**, have been designed to integrate copying with recognition of live heap objects. These copying collectors are very popular and are widely used, especially with functional languages like ML.

The following is a simple copying collector that uses **semispaces**. We start with the heap divided into two halves: the **from space** and **to space**. Initially, we allocate heap requests from the *from space*, using a simple "end of heap" pointer. When the *from space* is exhausted, we stop and do garbage collection.

Actually, though, we *do not* collect garbage. What we do is collect live heap objects; garbage is never touched. As was the case for mark-sweep collectors, we trace through global and local pointers to find live objects. As each object is found, it is moved from its current position in the *from space* to the next available position in the *to space*. The pointer is updated to reflect the object's new location. A "forwarding pointer" is left in the object's old location in case there are multiple pointers to the same object. (We want all original pointers properly updated to the new object location.)

This is illustrated in Figure 12.18. The *from space* is completely filled. We trace global and local pointers, moving live objects to the *to space* and updating pointers. This is illustrated in Figure 12.19 (dashed arrows are forwarding pointers). To handle pointers that are internal to copied heap objects, all copied heap objects are traversed. Objects referenced are copied and internal pointers are updated. Finally, the *to* and *from spaces* are interchanged, and

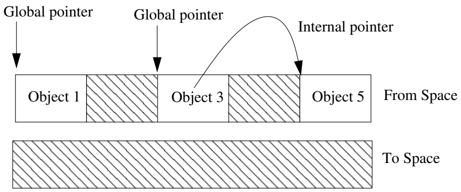


Figure 12.18: Copying Garbage Collection (a)

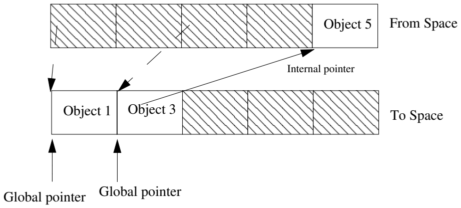


Figure 12.19: Copying Garbage Collection (b)

heap allocation resumes just beyond the last copied object. This is illustrated in Figure 12.20.

The biggest advantage of copying collectors is their speed. Only live objects are copied; deallocation of dead objects is essentially free. In fact, garbage collection can be made, on average, as fast as you wish simply by making the heap bigger. As the heap gets bigger, the time between collections increases, reducing the number of times a live object must be copied. In the limit, objects are never copied, so garbage collection becomes free!

Of course, we cannot increase the size of heap memory to infinity. In fact, we do not want to make the heap so large that paging is required, since swapping pages to disk is dreadfully slow. If we can make the heap large

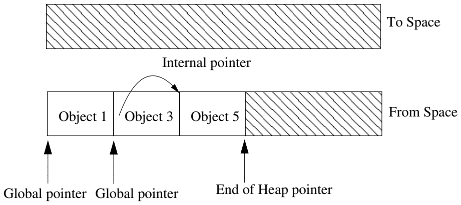


Figure 12.20: Copying Garbage Collection (c)

enough that the lifetime of most heap objects is less than the time between collections, then deallocation of short-lived objects will appear to be free, though longer-lived objects will still exact a cost.

It may seem that copying collectors are terribly wasteful. After all, at most only half of the heap space is actually used. The reason for this apparent inefficiency is that any garbage collector that does compaction must have an area to which live objects may be copied. Since in the worst case all heap objects could be live, the target area must be as large as the heap itself. To avoid copying objects more than once, copying collectors reserve a *to space* as big as the *from space*. This is essentially a space-time trade-off, making such collectors very fast at the expense of possibly wasted space.

If we have reason to believe that the time between garbage collections will be greater than the average lifetime of most heap objects, then we can improve our use of heap space. Assume that 50% or more of the heap will be garbage when the collector is called. We can then divide the heap into 3 segments, which we will call *A*, *B*, and *C*. Initially, *A* and *B* will be used as the *from space*, utilizing two-thirds of the heap. When we copy live objects, we will copy them into segment *C*, which will be big enough if half or more of the heap objects are garbage. Then we treat *C* and *A* as the *from space*, using *B* as the *to space* for the next collection. If we are unlucky and more than half of the heap contains live objects, we can still get by. Excess objects are copied onto an auxiliary data space (perhaps the stack), then copied into *A* after all live objects in *A* have been moved. This slows collection down, but only rarely (if our estimate of 50% garbage per collection is sound). Of course, this idea generalizes to more than 3 segments. Thus if two-thirds of the heap were garbage (on average), we could use 3 of 4 segments as *from space* and the last segment as *to space*.

Generational Garbage Collection The great strength of copying collectors is that they do no work for objects that are born and die between collections. However, not all heap objects are so short lived. In fact, some heap objects are very long lived. For example, many programs create a dynamic data structure at their start, and utilize that structure throughout the program. Copying collectors handle long-lived objects poorly. They are repeatedly traced and moved between semispaces without any real benefit.

Generational garbage collection techniques [Ung84] were developed to better handle objects with varying lifetimes. The heap is divided into two or more *generations*, each with its own *to* and *from space*. New objects are allocated in the youngest generation, which is collected most frequently. If an object survives across one or more collections of the youngest generation, it is "promoted" to the next-older generation, which is collected less often. Objects that survive one or more collections of this generation are then moved to the next older generation. This continues until very long-lived objects reach the oldest generation, which is collected very infrequently (perhaps even never).

The advantage of this approach is that long-lived objects are filtered out, greatly reducing the cost of repeatedly processing them. Of course, some long-lived objects will become unreachable and these will be caught when their generation is eventually collected.

An unfortunate complication of generational techniques is that although we collect older generations infrequently, we must still trace their pointers in case they reference an object in a newer generation. If this is not done, we may mistake a live object for a dead one. When an object is promoted to an older generation, we can check to see if it contains a pointer into a younger generation. If it does, we record its address so that we can trace and update its pointer. We must also detect when an existing pointer inside an object is changed. Sometimes we can do this by checking "dirty bits" on heap pages to see which have been updated. We then trace all objects on a page that is dirty. Otherwise, whenever we assign to a pointer that already has a value, we record the address of the pointer that is changed. This information then allows us to only trace those objects in older generations that might point to younger objects.

Experience shows that carefully designed generational garbage collectors can be very effective. They focus on objects most likely to become garbage, and spend little overhead on long-lived objects. Generational garbage collectors are widely used in practice.

Conservative Garbage Collection The garbage collection techniques we have studied all require that we identify pointers to heap objects accurately. In strongly typed languages like Java or ML, this can readily be done. We can table the addresses of all global pointers. We can include a code value in a frame (or use the return address stored in a frame) to determine the routine to which a frame corresponds. This allows us to then determine what offsets in the frame contain pointers. When heap objects are allocated, we can include a type code in the object's header, again allowing us to identify pointers internal to the object.

Languages like C and C++ are weakly typed, and this makes identification of pointers much harder. Pointers may be typecast into integers and then back into pointers. Pointer arithmetic allows pointers into the middle of an object. Pointers in frames and heap objects need not be initialized, and may contain random values. Pointers may overlay integers in unions, making the current type a dynamic property.

As a result of these complications, C and C++ have the reputation of being incompatible with garbage collection. Surprisingly, this belief is false. Using **conservative garbage collection**, C and C++ programs can be effectively garbage collected.

The basic idea is simple. If we cannot be sure whether a value is a pointer or not, we will be *conservative* and assume it is a pointer. If what we think is a pointer is not, we may retain an object that is really dead, but we will find all valid pointers, and never incorrectly collect a live object. We may mistake an integer (or a floating value, or even a string) as a pointer, so compaction in any form cannot be done. However, mark-sweep collection will work.

Garbage collectors that work with ordinary C programs have been developed [BW88]. User programs need not be modified. They simply are linked to different library routines, so that `malloc` and `free` properly support the garbage collector. When new heap space is required, dead heap objects may be automatically collected, rather than relying entirely on explicit `free` commands (though `free`s *are* allowed, they sometimes simplify or speed heap reuse).

With garbage collection available, C programmers need not worry about explicit heap management. This reduces programming effort and eliminates errors in which objects are prematurely freed, or perhaps never freed. In fact, experiments have shown [ZG92] that conservative garbage collection is very competitive in performance with application-specific manual heap management.

12.5 Region-Based Memory Management

Stack allocation is straightforward to implement and exhibits predictable, minimal overheads. However, it is inflexible when compared to heap allocation; all of the objects in a stack frame exist for the lifetime of a particular procedure activation. Heap allocation is more flexible than stack allocation, but requires careful implementation. Heap management also depends on garbage collection, which may introduce unpredictable latencies in program execution, or intricate manual storage management, which can lead to subtle and devastating bugs.

Region-based memory management aims to combine the predictable performance of stack allocation with the flexibility of heap allocation. Region-based memory management is an automated technique and, like garbage collection, is immune to dangling pointer errors. Unlike heap allocation with garbage collection, though, region-based memory management does not need to stop the program to free unused memory. As such, it can be particularly suitable for real-time applications with latency requirements.

A *region*, like a heap, is an area of memory in which new objects can be allocated as they are needed. The major difference between regions and heaps is that it is impossible to deallocate an individual object from a region. Rather, an entire region is deallocated at once. Therefore, a region may grow in size, but it will never shrink.

Region-based approaches to memory management require that programs are annotated with region creation and deletion operations, and that allocation statements (like `new` or `malloc`) specify in which region an object is to be allocated. Typically, regions are organized in a stack and have lexically scoped lifetimes. In this respect, region-based memory management can be similar to stack allocation. However, regions are more flexible than frames because it is possible to allocate an object into a region other than the region on top of the stack.

As an example, consider three procedures, *A*, *B*, and *C*. Each procedure begins by creating a new region and ends by destroying that region. *A* calls *B* and *B* calls *C*. It is possible for *C* to allocate an object in the region created by *A*. As a consequence, this object will be available during the lifetimes of *C*, *B*, and *A*, but it will be deallocated just before *A* returns, when *A* destroys its region.

Because the maximum lifetime of an object is essentially decided at compile-time based on the region in which it is allocated, region-based memory management need not stop the program to identify garbage objects. In addition, because entire regions are deallocated at once, region-based memory management can perform many individual deallocations in essentially constant time and limit the sort of heap fragmentation possible under other manual or automatic heap management approaches. However, region-based memory management requires that the input program is annotated with region creation and destruction operations, and that object allocations include a region parameter.

It is trivial to generate region annotations that are merely correct; one need only create a single region at the beginning of the program, allocate every object in this region, and destroy this region before the program terminates. However, such a program would hardly use memory efficiently. Statically identifying region annotations that are correct and waste as little memory as possible is a much trickier problem. Effective programming with explicit regions would thus be quite difficult (not to mention tedious). Furthermore, leaving region management to the programmer would enable him or her to introduce dangling pointer errors by deallocating a region when an object contained within it is still live.

Fortunately, systems that support region-based memory management typically generate region annotations automatically. In order to do this, compilers include an analysis called **region inference**, which determines where to place region creation and destruction operations as well as which regions should hold particular objects. Like all sound static analyses, region inference is *conservative* in that it may not identify the region annotations that correspond to the best possible use of memory. However, region inference is guaranteed to be safe, in that the region annotations it identifies will never result in dangling pointers or other runtime memory errors.

Region inference analyses and region-based memory managers typically require languages with strong type systems that are relatively easy to analyze precisely. Therefore, languages like C typically do not admit effective region inference algorithms, but support for regions is included in some real-time Java systems, many ML compilers, and in the Cyclone language, which is a C-like language that includes many interesting features from ML-family languages. Improving the precision of region inference analyses is an active and fruitful field of research. Tofte and Talpin [TT97] present an early overview of region-based memory management; Henglein, Makholm, and Niss [HMN05] present a contemporary and comprehensive summary of the field.

Exercises

1. Show the frame layout corresponding to the following C function:

```
int f(int a, char *b){
    char c;
    double d[10];
    float e;
    ...
}
```

   Assume control information requires 3 words and that f's return value is left on the stack. Be sure to show the offset of each local variable in the frame and be sure to provide for proper alignment (integers and floats on word boundaries and doubles on doubleword boundaries).

2. Local variables are normally allocated within a frame, providing for automatic allocation and deallocation when a frame is pushed and popped. Under what circumstance must a local variable be dynamically allocated? Are there any advantages to allocating a local variable statically (i.e., giving it a single fixed address)? Under what circumstances is static allocation for a local permissible?

3. Using the code below, show the sequence of frames, with dynamic links, on the stack when r(3) is executed assuming we start execution (as usual) with a call to main().

```
r(flag){
  printf("Here !!!\n"); }

q(flag){
  p(flag+1); }

p(int flag){
  switch(flag){
    case 1: q(flag);
    case 2: q(flag);
    case 3: r(flag); }

main(){
  p(1); }
```

4. Consider the following C-like program that allows subprograms to nest. Show the sequence of frames, with static links, on the stack when `r(16)` is executed assuming we start execution (as usual) with a call to `main()`. Explain how the values of `a`, `b`, and `c` are accessed in `r`'s print statement.

```
p(int a){
  q(int b){
     r(int c){
       print(a+b+c);
     }
    r(b+3);
 }

  s(int d){
    q(d+2);
  }

  s(a+1);

}

main(){
  p(10);
}
```

5. Reconsider the C-like program shown in Exercise 4, this time assuming display registers are used to access frames (rather than static links). Explain how the values of `a`, `b`, and `c` are accessed in `r`'s print statement.

6. Consider the following C function. Show the content and structure of `f`'s frame. Explain how the offsets of `f`'s local variables are determined.

```
int f(int a, int b[]){
   int i = 0, sum = 0;
   while (i < 100){
      int val = b[i]+a;
      if (b[i]>b[i+1]) {
         int swap = b[i];
         b[i] = b[i+1];
         b[i+1] = swap;
      } else {
         int avg = (b[i]+b[i+1])/2;
         b[i] = b[i+1] = avg; }
      sum += val;
      i++;
   }

   return sum;
}
```

7. Although the first release of Java did not allow classes to nest, subsequent releases did. This introduced problems of nested access to objects, similar to those found when subprograms are allowed to nest. Consider the following Java class definition:

   ```
   class Test {
      class Local {
           int b;
           int v(){return a+b;}
           Local(int val){b=val;}
      }

      int a = 456;

      void m(){
         Local temp = new Local(123);
         int c = temp.v();
      }
   }
   ```

   Note that method `v()` of class `Local` has access to field `a` of class `Test` as well as field `b` of class `Local`. However, when `temp.v()` is called, it is given a direct reference only to `temp`. Suggest a variant of static links that can be used to implement nested classes so that access to all visible objects is provided.

8. Consider the following C/C++ structure declarations:

   ```
   struct {int a; float b; int c[10];} s;

   struct {int a; float b; } t[10];
   ```

   Choose your favorite computer architecture. Show the code that would be generated for `s.c[5]` assuming `s` is statically allocated at address 1000. What code would be generated for `t[3].b` if `t` is allocated within a frame at offset 200?

9. Assume that in C we have the declaration `int a[5][10][20]`, where `a` is allocated at address 1000. What is the address of `a[i][j][k]` assuming `a` is allocated in row-major order? What is the address of `a[i][j][k]` assuming a is allocated in column-major order?

10. Most programming languages (including Pascal, Ada, C, and C++) allocate global aggregates (records, arrays, structs, and classes) statically, while local aggregates are allocated within a frame. Java, on the other hand, allocates all aggregates in the heap. Access to them is via object references allocated statically or within a frame. Is it less efficient to access an aggregate in Java because of its mandatory heap allocation? Are there any advantages to forcing all aggregates to be uniformly allocated in the heap?

11. In Java, subscript validity checking is mandatory. Explain what changes would be needed in C or C++ (your choice) to implement subscript validity checking. Be sure to address the fact that pointers are routinely used to access array elements. Thus you should be able to checks array accesses that are done through pointers, including pointers that have been incremented or decremented.

12. Assume we add a new option to C++ arrays that are heap-allocated, the *flex option*. A flex array is automatically expanded in size if an index beyond the array's current upper limit is accessed. Thus we might see:

    ```
    ar = new flex int[10]; ar[20] = 10;
    ```

    The assignment to position 20 in `ar` forces an expansion of `ar`'s heap allocation. Explain what changes would be needed in array accessing to implement flex arrays. What should happen if an array position beyond an array's current upper limit is read rather than written?

13. Fortran library subprograms are often called from other programming languages. Fortran assumes that multidimensional arrays are stored in column-major order; most other languages assume row-major order. What must be done if a C program (which uses row-major order) passes a multidimensional array to a Fortran subprogram. What if a Java method, which stores multidimensional arrays as arrays of array object references, passes such an array to a Fortran subprogram?

14. Recall that offsets within a record or struct must sometimes be adjusted upward due to alignment restrictions. Thus in the following two C structs, S1 requires 6 bytes whereas S2 requires only 4 bytes.

```
struct {                        struct {
  char  c1;                       char  c1;
  short s;                        char  c2;
  char  c2;                       short s;
} S1;                           } S2;
```

    Assume we have a list of the fields in a record or struct. Each is characterized by its size and alignment restriction. (A field with an alignment restriction *r* must be assigned an offset that is a multiple of *r*.)

    Give an algorithm that determines an ordering of fields and minimizes the overall size of a record or struct while maintaining all alignment restrictions. How does the execution time of your algorithm (as measured in number of execution steps) grow as the number of fields increases?

15. Assume we organize a heap using reference counts. What operations must be done when a pointer to a heap object is assigned? What operations must be done when a scope is opened and closed?

16. Some languages, including C and C++, contain an operation that creates a pointer to a data object. That is, `p = &x` takes the address of object `x`, whose type is `t`, and assigns it to `p`, whose type is `t*`.

    How is management of the runtime stack complicated if it is possible to create pointers to arbitrary data objects in frames? What restrictions on the creation and copying of pointers to data objects suffice to guarantee the integrity of the runtime stack?

17. Consider a heap allocation strategy we shall term *worst fit*. Unlike best fit, which allocates a heap request from the free space block that is closest to the requested size, worst fit allocates a heap request from the largest available free space block. What are the advantages and disadvantages of worst fit as compared with the best fit, first fit, and next fit heap allocation strategies?

18. The performance of complex algorithms is often evaluated by simulating their behavior. Create a program that simulates a random sequence of heap allocations and deallocations. Use it to compare the average number of iterations that the best fit, first fit, and next fit heap allocation techniques require to find and allocate space for a heap object.

19. In a strongly typed language such as Java, all variables and fields have a fixed type known at compile-time. What runtime data structures are needed in Java to implement the mark phase of a mark-sweep garbage collector in which all accessible ("live") heap objects are marked?

20. The second phase of a mark-sweep garbage collector is the sweep phase, in which all unmarked heap objects are returned to the free space list.

    Detail the actions needed to step through the heap, examining each object and identifying those that have not been marked (and hence are garbage).

21. In a language like C or C++ (without unions), the marking phase of a mark-sweep garbage collector is complicated by the fact that pointers to active heap objects may reference data within an object rather than the object itself. For example, the sole pointer to an array may be to an internal element, or the sole pointer to a class object may be a pointer to one of the object's fields.

    How must your solution to Exercise 19 be modified if pointers to data within an object are allowed?

22. One of the attractive aspects of conservative garbage collection is its simplicity. We need not store detailed information on what global, local, and heap variables are pointers. Rather, any word that *might* be a heap pointer is treated as if *is* a pointer.

    What criteria would you use to decide if a given word in memory is possibly a pointer? How would you adapt your answer to Exercise 21 to handle what appear to be pointers to data within a heap object?

23. One of the most attractive aspects of copying garbage collectors is that collecting garbage actually costs nothing since only live data objects are identified and moved. Assuming that the total amount of heap space live at any point is constant, show that the average cost of garbage collection (per heap object allocated) can be made arbitrarily cheap simply by increasing the memory size allocated to the heap.

24. Copying garbage collection can be improved by identifying long-lived heap objects and allocating them in an area of the heap that is not collected.

    What compile-time analyses can be done to identify heap objects that will be long lived? At runtime, how can we efficiently estimate the "age" of a heap object (so that long-lived heap objects can be specially treated)?

25. An unattractive aspect of both mark-sweep and copying garbage collection is that they are batch-oriented. That is, they assume that periodically a computation can be stopped while garbage is identified and collected. In interactive or real-time programs, pauses can be quite undesirable. An attractive alternative is **concurrent garbage collection** in which a garbage collection process runs concurrently with a program.

    Consider both mark-sweep and copying garbage collectors. What phases of each can be run concurrently while a program is executing (that is, while the program is changing pointers and allocating heap objects)? What changes to garbage collection algorithms can facilitate concurrent garbage collection?

13

Target Code Generation

Ultimately, each compiler must focus its translation on the capabilities of a particular machine architecture. In some cases, such as the **Java Virtual Machine** (JVM) and **Microsoft Intermediate Language** (MSIL), the architecture is **virtual**. Virtual machines allow program execution on a wide variety of computing platforms at the cost of an additional layer of software—the virtual machine simulator.

More traditionally, a compiler targets its translation to a particular machine architecture implemented in an actual physical microprocessor. Examples include the Intel® x86 processor series as well as the Sparc™, MIPS®, and PowerPC® processors. In all cases, whether the architecture is virtual or real, the code generator must decide how to map program code and data into a processor's memory. Fast and flexible data access is essential. Moreover, the capabilities of each processor should be effectively exploited, allowing fast and reliable program execution.

In this chapter we explore how intermediate forms, such as JVM bytecodes and **abstract syntax tree** (AST) subtrees, are translated into executable form. Collectively, this process is called **code generation**, though code generation actually involves a number of individual tasks that must be handled.

At this point in the translation process, the compiler has produced an intermediate form, such as the JVM bytecodes discussed in Chapter 10. This is accomplished by the code-generation visitors discussed in Chapters 2, 7, and 11. The bytecodes may be interpreted by a bytecode interpreter. Alternatively, we may wish to further the translation process and produce machine instructions native to a particular computer of interest.

The first problem we will face is **instruction selection**. Instruction selection is highly target-machine dependent; it involves choosing a particular instruction sequence to realize a portion of the intermediate representation. Even for one simple bytecode instruction we may have a choice of possible implementations. For example, the `iinc` instruction, which adds a constant to a local variable, might be implemented by loading the variable into a register, loading the constant into a second register, doing a register-to-register add, and storing the result back into the variable. Alternatively, we might choose to keep the variable in a register, allowing implementation using only a single add-immediate instruction.

Besides instruction selection, we must also deal with **register allocation** and **code scheduling**. Register allocation aims to use registers effectively by minimizing register spilling (storing a value held in a register and reloading the register with something else). Because memory transactions take considerably more time than arithmetic instructions on most processors, even a few unnecessary loads and stores can significantly reduce the speed of an instruction sequence. Code scheduling is concerned with the order in which generated instructions are executed. Not all valid instruction orderings are equally good—some incur unnecessary delays.

We shall first consider how bytecodes may be efficiently translated to machine-level instructions. Techniques that optimize the code we generate will be considered next, particularly the translation of expressions in tree forms. A variety of techniques that support efficient use of registers, especially graph coloring will be discussed. Approaches to code scheduling will next be studied. Techniques that allow us to easily and automatically retarget a code generator to a new computer will then be discussed. Finally, a form of optimization that is particularly useful at the code-generation level, peephole optimization, will be studied.

13.1 Translating Bytecodes

We will first consider how to translate the bytecodes produced by the techniques discussed in Chapter 11 into conventional machine code. Each computer architecture has its own machine instruction set. Examples include the Intel x86 architecture, the Sparc, the Alpha, the PowerPC, and the MIPS.

In this chapter we use the MIPS R3000 instruction set. This architecture is clean, easy to use, and a good representative of modern **reduced instruction set computer** (RISC) architectures. The MIPS R3000 is also supported by `SPIM` [Lar90], a widely available MIPS interpreter written in C.

```
iload 2       ; Push int b onto stack
iload 3       ; Push int c onto stack
iadd          ; Add top two stack values
iload 4       ; Push int d onto stack
isub          ; Subtract top two stack values
istore 1      ; Store top stack value into a
```

Figure 13.1: Bytecodes for `a = b + c - d;`

Most bytecodes map directly into one or two MIPS instructions. Thus an `iadd` instruction corresponds directly to the MIPS add instruction. The biggest difference in the design of bytecodes and the MIPS (or any other modern architecture) is that bytecodes are *stack-oriented* whereas the MIPS is *register-oriented*.

The most obvious approach to handling stack-based operands is to load top-of-stack values into registers when they are used, and to push registers onto the stack when values are computed. This unfortunately is also one of the *worst* approaches. The problem is that explicit pop and push operations imply memory load and store instructions which can be slow and bulky.

Instead, we will make a few simple, but important, observations on how stack operands are used. First, note that no operands are left on the stack between source-level statements. If they were, a statement placed in a loop could cause stack overflow. Thus, the stack is used only to hold operands while parts of a statement are executed. Moreover, each stack operand is "touched" twice—when it is created (pushed) and when it is used (popped).

These observations allow us to map stack operands directly into registers; no pushes or pops of a machine's runtime stack are really needed. We can imagine the JVM operand stack as containing register names rather than values. When a particular value is at the top of the stack, we will use the corresponding "top register" as the source of our operand. This may seem complex, but it really is quite simple. Consider the Java$^{\text{TM}}$ assignment statement `a = b + c - d;` (where `a`, `b`, `c`, and `d` are integers, indexed as locals 1 to 4). The corresponding bytecodes are shown in Figure 13.1 ("`;`" begins a one-line comment in JVM code).

Whenever a value is pushed, we will create a temporary location to hold it. This temporary location (usually just called a temporary) will normally be allocated to a register. We will track the names of the temporaries associated with stack locations as bytecodes are processed. At any point, we will know exactly what temporaries are logically on the stack. We say logically because these values are not pushed and popped at run time. Rather, values are directly accessed from the registers that hold them.

```
lw      $t0,16($fp)       # Load b, at 16+$fp, into $t0
lw      $t1,20($fp)       # Load c, at 20+$fp, into $t1
add     $t2,$t0,$t1       # Add $t0 and $t1 into $t2
lw      $t3,24($fp)       # Load d, at 24+$fp, into $t3
sub     $t4,$t2,$t3       # Subtract $t3 from $t2 into $t4
sw      $t4,12($fp)       # Store result into a, at 12+$fp
```

Figure 13.2: MIPS code for `a = b + c - d;`

Continuing with our example, assume `a`, `b`, `c`, and `d` are assigned frame offsets 12, 16, 20, and 24 respectively (we will discuss memory allocation for locals and fields in Section 13.1.1). These four variables are given offsets because local variables in a procedure or method are allocated as part of a **frame**—a block of memory allocated on the runtime stack whenever a call is made. Thus, rather than push or pop individual data values, as bytecodes do, we prefer to push a single large block of memory once per call.

Let us assume the temporaries we allocate are MIPS registers, denoted `$t0`, `$t1`, ... Each time we generate code for a bytecode instruction that pushes a value onto the stack, we will call GETREG (discussed in Section 13.3.1) to allocate a result register. Whenever we generate MIPS code for a bytecode that accesses stack values, we will use the registers already allocated to hold stack values. The net effect is to use registers rather than stack locations to hold operands, which yields fast and compact instruction sequences. For our above example we might generate the MIPS code shown in Figure 13.2 ("#" begins a one line comment in MIPS source code).

The `lw` instruction loads a word of memory into a register. Addresses of locals are computed relative to `$fp`, the frame pointer, which always points to the currently active frame. Similarly, `sw` stores a register into a word of memory. The `add` and `sub` instructions add and subtract two registers, respectively, putting the result into a third register.

Bytecodes that push constants, like `bipush n`, can be implemented as an immediate load of a literal into a register. As an optimization, we can delay loading the constant value into the register until we are sure it is actually needed there. To achieve this effect, we note that a particular stack location associated with a MIPS register will hold a known literal value. When that register is used, we determine if the constant value must be its own register, or if an **immediate instruction** can be used instead. For example, we may replace a register-to-register add with an add-immediate instruction.

13.1.1 Allocating memory addresses

As we learned in Section 12.2 on page 447, local variables and parameters are allocated in the frame associated with a procedure or method. Therefore we must map each JVM local variable into a unique frame offset, used to address the variable in load and store instructions. Since a frame contains some fixed-size control information followed by local data, a simple formula like *offset* = *const* + *size* ∗ *index* suffices, where *index* is the JVM index assigned to a variable, *size* is the size (in bytes) of each stack value, *const* is the size (in bytes) of the fixed-size control area in the frame, and *offset* is the frame offset used in generated MIPS code.

Static fields of classes are assigned fixed, static addresses when a class is compiled. These addresses are used whenever a static field is referenced. Instance fields are accessed as an offset relative to the beginning of a particular object. The compiler must provision for the instance fields of all superclasses of the compiled class. In Java, the `Object` class has no instance fields. Thus, if we had a class `Complex` defined as

```
class Complex extends Object { float re; float im;}
```

then the two fields `re` and `im`, each one word in size, can be given offsets of `0` and `4`, respectively, within instances of the class. The JVM instruction `getfield Complex/im F` fetches field `im` of the `Complex` object referenced by the top-of-stack. The `F` in the instruction denotes the type of the field (as discussed in Section 10.2.2 on page 399). Translation is straightforward. We first look up the offset of field `im` in class `Complex`, which is `4`. A pointer to the referenced object is in the register corresponding to top-of-stack, say `$t0`. We could add `4` to `$t0`, but since the MIPS has an indexed addressing mode that adds a constant to a register automatically (denoted `const($reg)`), we need generate no code. We simply generate `lw $t1,4($t0)`, which loads the field into register `$t1`, and which now corresponds to the top-of-stack.

13.1.2 Allocating Arrays and Objects

In Java, and hence in the JVM, all instantiated objects are allocated on the heap. To translate a `new` or `newarray` bytecode, we will need to call a heap allocation subroutine, like `malloc` in C. We pass the size of the object required and receive a pointer to a newly allocated block of heap memory. For a `new` bytecode, the size required is determined by the number and size of the fields in the object. In addition, a fixed-size header (to store the size and type of the object) is required. The overall memory size needed can be computed when the class definition of the object is compiled. In our earlier example of a `Complex` object, the size required would be 8 bytes plus header information, commonly 2 or 4 words.

For `newarray`, we determine the allocation size by multiplying the number of elements requested (in the register corresponding to the top-of-stack) by the size required for individual array elements (stored in the symbol table entry for the requested class). Again, space for a fixed-size object header must be included. The object header includes size and type information of the allocated array for runtime reference, as well as the object's monitor for synchronization. Default initialization of fields within objects must also be performed by clearing or copying appropriate bit patterns (based on type declarations).

In languages like C and C++, objects and arrays can be allocated **inline** in the current frame if their size is known at compile-time. We can do a similar allocation within the current frame if we know that no reference to the allocated object **escapes**. That is, if no reference to the allocated object or array is assigned to a field or returned as a function value, then the object or array is no longer accessible after the current method (and its corresponding frame) are terminated.

As a further optimization, string objects (which are immutable) are often defined to be a string literal. Space for such a string may be allocated statically, and accessed via a fixed static address.

In the JVM, array elements are accessed or updated using a single bytecode (e.g., `iaload` and `iastore` for integer arrays). In conventional architectures, like the MIPS, several instructions are needed to access an array, especially if array bounds checking is included. Details of array indexing are discussed in Section 12.3 on page 460. Here we will just show the kind of code that is needed to implement a JVM array load or store instruction.

An `iaload` instruction expects an array index at the top of the stack, and an array reference at the top − 1 position. In our implementation, both of these values will be loaded or evaluated into MIPS registers. Let us call these registers `$index` and `$array`. We will need to generate code to check that `$index` is a legal index, and then to actually fetch the desired integer value from the array. Since arrays are just objects, the size of the array, and the array elements, are at fixed offsets relative to the start of the array object. Assume the array size is at offset `SIZE` and that elements are at offset `OFFSET`. Then the MIPS code shown if Figure 13.3 can be used to implement `iaload`, leaving the array value in register `$val`. For simplicity and efficiency, we will assume a null reference is represented by an invalid address that will force a memory fault.

The register `$temp` is a work register, used within the code sequence to hold intermediate values (see Section 13.3.1). The `iastore` instruction is very similar. A value at top − 2 (in `$val`) is stored in the array referenced at top − 1

```
bltz    $index,badIndex      # Branch to badIndex if $index<0
lw      $temp,SIZE($array)   # Load size of array into $temp
slt     $temp,$index,$temp   # $temp = $index < size of array
beqz    $temp,badIndex       # Branch to badIndex if
                             # $index >= size of array
sll     $temp,$index,2       # multiply $index by 4 (size of
                             # an int) using a left shift
add     $temp,$temp,$array   # Compute $array + 4*$index
lw      $val,OFFSET($temp)   # Load word at
                             # $array + 4*$index + OFFSET
```

Figure 13.3: MIPS code for `iaload` bytecode

```
bltz    $index,badIndex      # Branch to badIndex if $index<0
lw      $temp,SIZE($array)   # Load size of array into $temp
slt     $temp,$index,$temp   # $temp = $index < size of array
beqz    $temp,badIndex       # Branch to badIndex if
                             # $index >= size of array
sll     $temp,$index,2       # multiply $index by 4 (size of
                             # an int) using a left shift
add     $temp,$temp,$array   # Compute $array + 4*$index
sw      $val,OFFSET($temp)   # Load $val into word at
                             #  $array + 4*$index + OFFSET
```

Figure 13.4: MIPS code for `iastore` bytecode

(`$array`) using the index at the top of the stack (`$index`). We can use the code shown in Figure 13.4 to implement an `iastore` instruction:

The MIPS code we have chosen for array indexing looks rather complex and expensive, especially since arrays are a very commonly used data structure. Part of this complexity is due to the fact that we have included array bounds checking, as required in Java. In C and C++, array bounds are rarely checked at runtime, allowing for faster (but less secure) code.

In many cases it is possible to optimize or entirely eliminate array bounds checks. On architectures that support unsigned arithmetic, the check for an index too large and the check for an index too small (less than zero) can be combined. The trick is to do an unsigned comparison between the array index and the array size. A negative index will be equivalent to a very large unsigned value (since its leftmost bit will be one), making it greater than the array size.

In a `for` loop, it is often possible to determine that a loop index is bounded by known lower and upper bounds. With this information, arrays indexed by the loop index may be known to be "in range," eliminating any need for explicit checking. Similarly, once an array bound is checked, subsequent checks of the same bound are unnecessary until the index is changed. Thus, in `a[i] = 100 - a[i]`, `a[i]` needs to be checked only once.

If array bounds checks are optimized away, or simply suppressed, array indexing is much more efficient, typically three (or fewer) instructions (a shift or multiply, an add, and a load or store). In the case where the array index is a compile-time constant (e.g., `a[100]`), we can reduce this to a single instruction by doing the computation of $size * index + offset$ at compile-time and using it directly in a load or store instruction. If the array index is a loop index variable, then **reduction in strength** (Section 14.1.1 on page 549) can increment the index variable by the appropriate amount each time around the loop, thus eliminating the multiply instruction.

13.1.3 Method Calls

The JVM makes method calls very simple. Many of the details of a call are hidden. In implementing an `invokestatic` or `invokevirtual` bytecode, we must make such hidden details explicit.

Let us look at `invokestatic` first. In the bytecode version of the call, parameters are pushed onto the stack, and a static method, qualified by its class and type (to support **overloading**) is accessed. In our MIPS translation, the parameters will be in registers, which is fine since that is how most current architectures pass scalar (word-sized) parameters.

We will need to guarantee that parameters are placed in the correct registers. On the MIPS, the first four scalar parameters are passed in registers `$a0` to `$a3`; remaining parameters and non-scalar parameters are pushed onto the runtime stack. In our translation, we can generate explicit register copy instructions to move argument values (already in registers) to the correct registers. If we wish to avoid these extra copy instructions, we can compute the parameters directly into the correct registers. This is called **register targeting**. Essentially, when the parameter is computed, we mark the target register into which the parameter will be computed to be the appropriate argument register. Graph coloring (as discussed in Section 13.3.2) makes targeting fairly easy to do. For parameters that are to be passed on the runtime stack, we simply extend the stack (by adjusting the top-of-stack register, `$sp`) and then store excess parameters into the locations just added.

To transfer control to the subprogram, we issue a `jal` (jump and link) instruction. This instruction transfers control to the start of the method to be called, using an address recorded when the subprogram was translated *and* stores a return address in the return address register `$ra`.

```
move    $a0,$t0                 # Copy $t0 to parm register 1
li      $a1,2                   # Load 2 into parm register 2
sw      $t0,32($fp)             # Store $t0 across call
jal     f                       # Call function f
                                # Function value is in $v0
lw      $t0,32($fp)             # Restore $t0
sw      $v0,a                   # Store function value in a
```

Figure 13.5: MIPS code for the function call `a = f(i,2);`

Additional details must be handled to complete our translation of an `invokestatic` instruction. Since variable and expression values may be held in registers at the point of call, these registers must be saved prior to execution of the method. All registers that hold values that may be destroyed during the call (by the instructions in the called method's body) are saved on the stack and restored after the method completes execution. Registers may be saved by the caller (these are **caller-save registers**) or by the method to be called (these are **callee-save registers**). It does not matter if the caller or callee does the saving (often both save selected registers), but any register holding a program value needed after the call must be protected.

If a non-local or global variable is held in a register, it must be saved prior to a call in its assigned memory location. This guarantees that the subprogram will see the correct value during the call. Upon return, registers holding non-local or global variables must be reloaded since the subprogram may have updated their values.

As an example, consider the function call `a = f(i,2);`, where `a` is a static field, `f` is a static method and `i` is a local variable held in register `$t0`, a caller-save register. Assume that a storage temporary, assigned frame offset `32`, is created to hold the value of `$t0` across the call. The MIPS code shown in Figure 13.5 is produced.

When a method is called, space for its frame must be pushed, and the frame and stack pointers must be properly updated. This is normally done in the **prologue** of the called method, just before its body is executed. Similarly, after a method is finished, its frame must be popped, and frame and stack pointers properly reset. This is done in the method's **epilogue**, just before branching back to the caller's return address. The exact code sequences vary according to hardware and operating system conventions. The MIPS instructions shown in Figure 13.6 can be used to push, and later pop, a method's frame. (The size of the frame, `frameSz`, is determined when the method is compiled and all its local declarations are processed; on the MIPS architecture, the runtime stack grows downward.)

```
subi   $sp,$sp,frameSz        # Push frame on stack
sw     $ra,0($sp)             # Save return address in frame
sw     $fp,4($sp)             # Save old frame pointer in frame
move   $fp,$sp                # Set $fp to access new frame
# Save callee-save registers (if any) here
# Body of method is here
# Restore callee-save registers (if any) here
lw     $ra,0($fp)             # Reload return address register
lw     $fp,4($fp)             # Reload old frame pointer
addi   $sp,$sp,frameSz        # Pop frame from stack
jr     $ra                    # Jump to return address
```

Figure 13.6: MIPS prologue and epilogue code

To translate an `invokevirtual` instruction, we must implement a **dynamic dispatching** mechanism. When a method `M` is called using `invokevirtual`, we are given, as the first parameter, a pointer to the object in which `M` is to execute. By semantic analysis we know this object's class, or a parent class, *must* contain a definition of `M`. But what if `M` is defined in both a parent class and a subclass (which is quite legal)? How do we know which version of `M` to execute?

To support garbage collection and heap management, each heap object has a **type code** as part of its header. This type code can be used to index into a **dispatch table** that contains the addresses of all methods the object contains. If we assign to each method a unique offset, we can use method `M`'s offset in the object's dispatch table to choose the correct method to execute.

Fortunately, it is often the case that a class `C` has no subclasses that redefine `M`. (e.g., if `C` or `M` is `private` or `final`). If dynamic resolution of `C` is not required, we can select `M`'s implementation at compile-time, and generate code to call it directly without any table-lookup overhead.

13.1.4 Example of Bytecode Translation

As an example of the overall bytecode translation process, let us consider the following simple method, `stringSum`. This method sums integers from 1 to its parameter, `limit`, and returns a string representation of the sum:

```
      iconst_0     ; Push 0
      istore_1     ; Store into variable #1 (sum)
      iconst_1     ; Push 1
      istore_2     ; Store into variable #2 (i)
      goto L2      ; Go to end of loop test
  L1: iload_1      ; Push var #1 (sum) onto stack
      iload_2      ; Push var #2 (i) onto stack
      iadd         ; Add sum + i
      istore_1     ; Store sum + i into var #1 (sum)
      iinc 2 1     ; Increment var #2 (i) by 1
  L2: iload_2      ; Push var #2 (i)
      iload_0      ; Push var #0 (limit)
      if_icmple L1 ; Goto L1 if i <= limit
      iload_1      ; Push var #1 (sum) onto stack
                   ; Call toString:
      invokestatic
            java/lang/Integer/toString(I)Ljava/lang/String;
      areturn      ; Return String reference to caller
```

Figure 13.7: Bytecodes for method `stringSum`

```
public static String stringSum(int limit){
   int sum = 0;
   for (int i = 1; i <= limit; i++)
      sum += i;
   return Integer.toString(sum);
}
```

The bytecodes listed in Figure 13.7 implement `stringSum`.

In analyzing `stringSum`, we see references to three local variables (including the `limit` parameter). Adding in two words of control information, we conclude that a frame size of 5 words (20 bytes) is required: `limit` will be placed at offset 8, `sum` at offset 12, and `i` at offset 16.

In the code shown in Figure 13.8, we will follow the MIPS convention that one word function values, including object references, will be returned in register `$v0`. We will also exploit the fact that register `$0` always contains a zero value. The code we generate will begin with a method prologue (to push `stringSum`'s frame) then a line-by-line translation of its bytecodes, followed by an epilogue to pop `stringSum`'s frame and return to its caller.

```
       subi    $sp,$sp,20     # Push frame on stack
       sw      $ra,0($sp)     # Save return address
       sw      $fp,4($sp)     # Save old frame pointer
       move    $fp,$sp        # Set $fp to access new frame
       sw      $a0,8($fp)     # Store limit in frame
       sw      $0,12($fp)     # Store 0 ($0) into sum
       li      $t0,1          # Load 1 into $t0
       sw      $t0,16($fp)    # Store 1 into i
       j       L2             # Go to end of loop test
L1:    lw      $t1,12($fp)    # Load sum into $t1
       lw      $t2,16($fp)    # Load i into $t2
       add     $t3,$t1,$t2    # Add sum + i into $t3
       sw      $t3,12($fp)    # Store sum + i into sum
       lw      $t4,16($fp)    # Load i into $t2
       addi    $t4,$t4,1      # Increment $t4 by 1
       sw      $t4,16($fp)    # Store $t4 into i
L2:    lw      $t5,16($fp)    # Load i into $t5
       lw      $t6,8($fp)     # Load limit into $t6
       sle     $t7,$t5,$t6    # set $t7 = i <= limit
       bnez    $t7,L1         # Goto L1 if i <= limit
       lw      $t8,12($fp)    # Load sum into $t8
       move    $a0,$t8        # Copy $t8 to parm register
       jal     String_toString_int_   # Call toString
                              # String ref now is in $v0
       lw      $ra,0($fp)     # Reload return address
       lw      $fp,4($fp)     # Reload old frame pointer
       addi    $sp,$sp,20     # Pop frame from stack
       jr      $ra            # Jump to return address
```

Figure 13.8: MIPS code for method stringSum

13.2 Translating Expression Trees

So far we have concentrated on generating code from ASTs. We now focus on generating code from **expression trees**. In an expression tree, interior nodes represent operators, and leaves represent variables and constants. Many ASTs use this format for expressions, so much of this discussion applies to ASTs as well.

An expression tree may be traversed and translated in many different orders. Normally, a left-to-right postorder traversal is used when translating expressions. A left-to-right postorder traversal always produces a valid translation. However, alternative traversals may lead to better code (if exceptions, which must be tested in source order, are not a concern).

Consider the expression `(a-b) + ((c+d)+(e*f))`. The most obvious translation would compute `(a-b)` first, leaving the subtraction result in one of the two registers that held `a` and `b`. Three other registers are then required to translate `(c+d)+(e*f)`. Two registers are required for `c+d`, with one holding the result of the sum; the register not holding the sum can be one of the two registers required to translate `c+d`, but this means an additional register is needed. Translation of `((c+d)+(e*f))` thus takes 3 registers. A total of four registers is used for the entire expression.

However, as we show below, if the right subexpression, `(c+d)+(e*f)`, is evaluated first, only three registers are needed, because once this subexpression is computed its value can be held in one register, using the other two registers to compute `(a-b)`.

We now consider an algorithm that determines the *minimum number* of registers needed to evaluate any expression or subexpression. We ignore for the moment any special properties of operators, such as associativity. The algorithm labels each node in a tree with the minimum number of registers needed to evaluate the subexpression rooted by that node. This labeling is called **Sethi-Ullman numbering** [SU70]. Once the minimum number of registers needed for each expression and subexpression is known, we traverse the tree in a manner that generates optimal code (that is, code that minimizes register use and hence register spilling).

As we did in previous sections, we assume a MIPS-like machine model which requires that all operands be register-resident. The algorithm works in postorder, first labeling leaves of the tree. All leaves are labeled with 1, since one register is needed to hold a variable or constant. Consider the following cases for an interior node n, which is assumed to be a binary operator (see Exercise 31):

- Suppose each of n's operands (subtrees) requires the same number of registers (say, r) for its computation. Once either subtree is evaluated, its result must be held in some register while the other subtree is evaluated.

```
procedure registerNeeds( $T$ )
    if $T.kind = Identifier$ or $T.kind = IntegerLiteral$
    then $T.regCount \leftarrow 1$
    else
        call registerNeeds( $T.leftChild$ )
        call registerNeeds( $T.rightChild$ )
        if $T.leftChild.regCount = T.rightChild.regCount$
        then $T.regCount \leftarrow T.rightChild.regCount + 1$
        else
            $T.regCount \leftarrow$ max( $T.leftChild.regCount, T.rightChild.regCount$ )
end
```

Figure 13.9: An Algorithm to Label Expression Trees with Register Needs

In the case considered here, that other subtree also requires r registers. Thus $r+1$ registers are required to cover the other subtree's computation as well as to hold the result from the already computed subtree.

Node n can therefore be evaluated using $r+1$ registers in this case.

- If n's subtrees require a different number of registers, say r_{left} and r_{right}, then the tree rooted at n can be evaluated as follows. Suppose $r_{right} > r_{left}$. We evaluate the right subtree first, with the result held in one register. Because the left subtree needs fewer registers than the right subtree, we can reuse the right subtree's registers for evaluation of the left subtree, except for the register holding the right subtree's result.

  Node n can therefore be evaluated using $\max(r_{left}, r_{right})$ registers in this case. A symmetric argument can be applied when $r_{right} < r_{left}$.

This analysis leads to the algorithm shown in Figure 13.9.

As an example of this algorithm, registerNeeds would label the expression tree for `(a-b) + ((c+d)+(e*f))` as shown in Figure 13.10 (*regCount* for each node is shown at its bottom).

We can use the *regCount* labeling to drive a simple, but optimal, code generator, treeCg, defined in Figure 13.12. treeCG takes a labeled expression tree and a list of registers it may use. It generates code to evaluate the tree, leaving the result of the expression in the first register on the list. If treeCG is given too few registers, it will spill registers, as necessary, into storage temporaries. (We use the standard list manipulation functions head and tail. head returns the first element of a list; tail returns all but the first list element. Neither changes the list parameter it uses.)

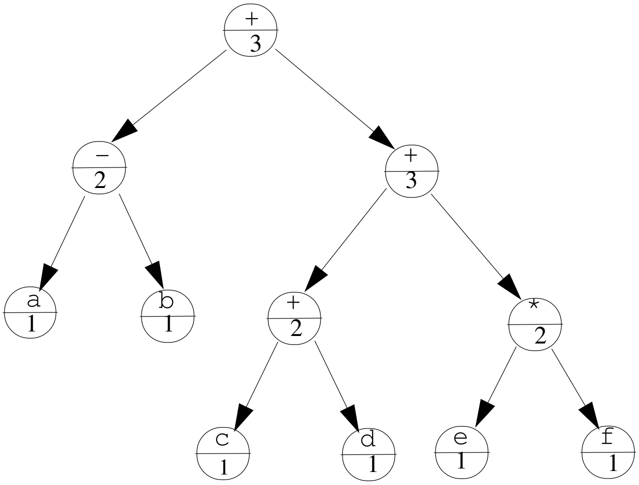


Figure 13.10: Expression Tree for `(a-b) + ((c+d)+(e*f))` with Register Needs.

```
lw   $10, c         # Load c into register 10
lw   $11, d         # Load d into register 11
add  $10, $10, $11  # Compute c + d into register 10
lw   $11, e         # Load e into register 11
lw   $12, f         # Load f into register 12
mul  $11, $11, $12  # Compute e * f into register 11
add  $10, $10, $11  # Compute (c + d) + (e * f) into reg 10
lw   $11, a         # Load a into register 11
lw   $12, b         # Load b into register 12
sub  $11, $11, $12  # Compute a - b into register 11
add  $10, $11, $10  # Compute (a-b)+((c+d)+(e*f)) into reg 10
```

Figure 13.11: MIPS code for `(a-b) + ((c+d)+(e*f))`

```
procedure treeCG( T, regList )
    r1 ← head( regList )
    r2 ← head( tail( regList ) )
    if T.kind = Identifier
    then
        /⋆  Load a variable.                                 ⋆/
        call generate( lw, r1, T.IdentifierName )
    else
        if T.kind = IntegerLiteral
        then
            /⋆  Load a literal.                              ⋆/
            call generate( li, r1, T.IntegerValue )
        else
            /⋆  T.kind must be a binary operator.            ⋆/
            left ← T.leftChild
            right ← T.rightChild
            if left.regCount ≥ length( regList ) and right.regCount ≥ length( regList )
            then
                /⋆  Must spill a register into memory.       ⋆/
                call treeCG( left, regList )
                /⋆  Get memory location.                     ⋆/
                temp ← getTemp( )
                call generate( sw, r1, temp )
                call treeCG( right, regList )
                call generate( lw, r2, temp )
                /⋆  Free memory location.                    ⋆/
                call freeTemp( temp )
                call generate( T.operation, r1, r2, r1 )
            else
                /⋆  There are enough registers; no spilling is needed.  ⋆/
                if left.regCount ≥ right.regCount
                then
                    call treeCG( left, regList )
                    call treeCG( right, tail( regList ) )
                    call generate( T.operation, r1, r1, r2 )
                else
                    call treeCG( right, regList )
                    call treeCG( left, tail( regList ) )
                    call generate( T.operation, r1, r2, r1 )
end
```

Figure 13.12: An Algorithm to Generate Optimal Code from Expression Trees

As an example, if we call treeCG with the labeled tree of Figure 13.10 and three registers, (`$10`, `$11` and `$12`), we obtain the code sequence shown in Figure 13.11.

treeCG illustrates nicely the principle of **register targeting**. Code is generated in such a way that the final result appears in the targeted register without any unnecessary moves.

Because our simple machine model requires that all operands be loaded into registers, **commutative operators** (for which *exp*1 *op exp*2 is identical to *exp*2 *op exp*1) cannot be exploited by the treeGC algorithm to reduce register usage. However, most computer architectures are not entirely symmetric. Thus, in the MIPS R3000 architecture, some operations (like add and subtract) allow the right operand to be immediate. **Immediate operands** are small literal values included directly into an instruction; they need not be explicitly loaded into registers (see Exercise 8). For commutative operators, a small literal used as a left operand can be treated as if it were a right operand.

Some operations, like addition and multiplication are **associative**. Operands of an associative operator may be processed in any order. Thus, mathematically, $(a + b) + c$ and $a + (b + c)$ are identical. Regrouping operands of associative operators can reduce the number of registers needed to evaluate an expression (see Exercise 9). For example, using registerNeeds we can establish that `(a+b)+(c+d)` requires three registers whereas `a+b+c+d` requires only two registers. Unfortunately, because of overflow and rounding issues, computer arithmetic is often *not* truly associative. For example, if `a` and `b` equal 1, `c` equals `maxint`, and `d` equals −10, `(a+b)+(c+d)` will evaluate correctly, whereas `a+b+c+d` may overflow. Most languages are careful to specify when such reordering is allowed, so that compilers can move operands only when it is absolutely safe to do so.

13.3 Register Allocation

Modern RISC architectures require that most operands reside in registers. The techniques explained in Section 13.2 use registers as necessary for translating expression trees, but each use of a variable name causes its value to be loaded into some register.

A machine's registers can be used to greater advantage if the association between a variable name and a register persists over several uses of that variable. For example, if register `$11` can be allocated to the variable `a` for a reasonable portion of a method's execution, then loads and stores for `a` could be satisfied by the relatively fast machine register instead of the relatively slower program memory. Reducing memory traffic in this manner can have a substantial impact on program performance.

An essential component of any code generator is therefore its register allocator. Machine registers are assigned to program variables and expressions. Since registers are limited in number, they must be reclaimed (reused) throughout a program.

A register allocator may be a simple **on-the-fly** algorithm that assigns and reclaims registers as code is generated. We will consider on-the-fly techniques first. More thorough register allocators, that consider the register needs of an entire subprogram or program will be considered next.

13.3.1 On-the-Fly Register Allocation

Most computers have distinct integer (general purpose) and floating register sets. In organizing our register allocator, we will divide each register set into a number of classes:

- Allocatable registers
- Reserved registers
- Work registers

Allocatable registers are explicitly allocated and freed by compile-time calls to register management routines. While allocated, registers are protected from use by any but the current "owner" of the register. It is therefore possible to guarantee that a register containing a data value will not be incorrectly changed by another use of the same register.

Requests for allocatable registers are usually generic; that is, requests are for any member of a register class, not for a particular register in that class. Usually any member of a register class will do. Further, generic requests eliminate the problem that arises if a particular requested register is already in use but many other registers in the same class are available.

A register, once allocated, must be freed by the compiler when its assignment to a particular task is completed. A register is usually freed in response to an explicit directive issued by a semantic routine. This directive also allows us to mark the last use of a register as dead. This is valuable information because better code may be possible if the contents of a register do not need to be preserved.

Reserved and work registers, on the other hand, are never explicitly allocated or freed. **Reserved registers** are assigned a fixed function throughout a program. Examples include display registers (Section 12.2.4 on page 453), stack-top registers, argument and return value registers, and return address registers. Since the function of reserved registers is set by the hardware or operating system, and they are in use for all of a program or procedure, it is unwise to use such registers for other than their designated purpose.

Work registers may be used at any time by any code-generation routine. Work registers may safely be used only in local code sequences, over which the code generator has complete control. That is, if we were generating code to do an indexing operation on an array (say, `a[i+j]`), it would be wrong to use a work register to hold the address of the array because computation of `i+j` might also use the same work register. An allocatable register would, of course, be protected. Work registers are useful in several circumstances:

- Sometimes we need a register for a very brief time. (For example, in compiling `a = b` we load `b` into a register and then immediately store the register into `a`.) Using a work register saves the overhead of allocating and then immediately freeing a register.
- Many instructions require that their operands be in registers. Since work registers are always free, we need not worry about there being no free registers. If necessary, we can load values from memory into work registers, execute an instruction or two, then save needed values back into memory.
- We can pretend we have more registers than we really do. Such registers, sometimes called **virtual registers** or **pseudo-registers**, can be simulated by allocating them in memory and placing their values into work registers when they are used in instructions.

In general, reserved registers are identified in advance by hardware and operating systems conventions. Sometimes work registers are also established in advance, and if they are not, we can choose three or four for this purpose. The remaining registers can be marked allocatable. They will hold temporary values, and may also be used to hold frequently accessed variables and constants.

The GETREG and FREEREG methods

To allocate and free registers, we will create two methods, GETREG and FREEREG. GETREG will allocate a single allocatable register and return its index. (If we have both integer and float registers, we will create GETREG and GETFLOATREG.) A register allocated by a call to GETREG remains allocated to the caller until it is returned.

What happens if no more registers are available for allocation? In simple compilers we can simply terminate compilation with a message that the program requires more registers than are available. Modern computers routinely have 20 or more allocatable registers. Unless registers are used aggressively to hold program variables and constants across a relatively large section of a program (Section 13.3.2), the chances are remote that a “real-life” program will exhaust the allocatable registers.

A more robust register allocator should not simply terminate when registers are exhausted. It can instead return **pseudo-registers** allocated in memory (in the frame of the procedure currently being translated). Pseudo-registers are encoded as integers greater than the indices of the real hardware registers. An array `regAddr[]` maps pseudo-registers into their memory addresses. Pseudo-registers are used exactly like real registers. Modern RISC architectures preclude memory-to-memory moves, insisting that values be loaded into and stored from architected registers. For such architectures, a load into a pseudo-register requires an architected work register to receive the value from memory. The pseudo-register then receives the value by a store from the work register. Stores into pseudo-registers similarly require a work register to accomplish the store.

In some situations we may need to allocate temporary values in memory rather than registers. This occurs when the temporary is too large to fit in a register (e.g., a `struct` returned by a function call) or when we need to be able to create a pointer to the temporary (most computers do not allow indirect references to registers). If we need storage-based temporaries, we can create getTemp and freeTemp functions that essentially parallel getReg and freeReg. Temporaries allocated for a procedure are placed in the procedure's frame (effectively they are anonymous local declarations). Temporaries used by the main program (and any other non-recursive procedure) may be allocated statically.

In some languages we may need to allocate temporaries whose size is not known at compile-time. For example, if we use the + operator to concatenate C-style strings, then the size of `str1 + str2` will not in general be known until runtime. The temporary used to hold the result of such an expression cannot be allocated statically or in a frame. Instead, we can allocate space for the temporary in the heap, reserving the space for a fixed-size pointer on the stack.

13.3.2 Register Allocation Using Graph Coloring

Using registers effectively is essential in generating efficient code for modern computers. We have already studied how to allocate registers in trees and "on-the-fly " as code is generated. In this section we address a greater challenge—how to allocate registers effectively through a relatively long-lived section of code. Since individual procedures (functions, methods, etc.) are the basic units of compilation in modern compilers, we have raised our sights from individual statements or expressions to entire procedure bodies.

Register allocation at the level of an entire procedure is called **procedure-level allocation**, in contrast to allocation at the level of a single expression or basic block, which is termed **local register allocation**. The term *global register allocation* is often used for procedure-level register allocation.

```
main() {
   a = f(x);     // Start of first live range
   print(a);     // End of first live range
   ....
   a = g(y)      // Start of second live range
   print(a);     // End of second live range
}
```

Figure 13.13: Example of Live Ranges

At the procedure level, a register allocator usually has many values that might profitably reside in registers: local and global variables, constants, temporaries containing available expressions, parameters, and return values, etc.). Each value that might profitably reside in a register is called a **register candidate**; typically there are many more register candidates than there are registers.

Procedure-level register allocators do not usually allocate a register to a single variable throughout the body of a subprogram. Rather, when possible, variables that do not interfere with each other are assigned to the same register. Thus, if variable a is used only at the top of a subprogram, and variable b is used only at the bottom of the subprogram, a and b may share the same register.

To enhance sharing, register candidates are divided into live ranges. A **live range** is the span of instructions in which a given value may be accessed, from its initial creation to its last use. For variables, a live range runs from its point of initialization or assignment to its last use. For expressions and constants, a live range spans from their first to final use. In Figure 13.13, variable a is broken into two separate and independent live ranges. Each is treated as a separate register candidate.

A live range can be readily computed using the **static single assignment** (SSA) form (described in Section 10.3 on page 410), since each use of a variable is tied to unique assignment. More generally, *live variables* can be computed (as described in Chapter 14) to determine the set of variable names or expressions that are live at any point in a program. Alternatively, one can avoid live range computation and simply treat each variable, parameter, or constant as a distinct register candidate.

The Interference Graph

One of the central problems in procedure-level register allocation is deciding which live ranges may share the same register and which may not. A live range l is said to *interfere* with another live range m if l's definition point (or

```
proc() {
   a = 100;
   b = 0;
   for (i=0;i<10;i++)
      b = b + i * i;
   print(a, b);
   c = 100;
   print(a*c);
}
```

Figure 13.14: A Simple Procedure with Candidates for Procedure-level Register Allocation.

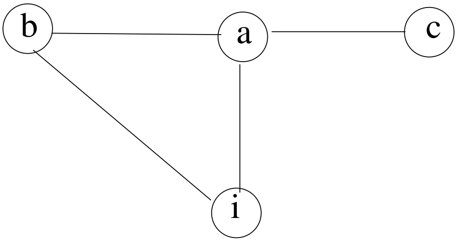


Figure 13.15: Interference Graph for procedure of Figure 13.14

beginning) is part of *m*'s range. In other words, *l* and *m* cannot share the same register if at the point *l* is first computed or loaded, *m* is also in use.

To represent all of the interferences in a subprogram (there normally are many), an **interference graph** is built. Nodes of the graph are the live ranges of the subprogram. An arc exists between live ranges *l* and *m* if *l* interferes with *m* or *m* interferes with *l* (the arc is undirected). Consider the simple procedure shown in Figure 13.14. It has four register candidates, a, b, c, and i. The live ranges for a, b, and i all interfere with each other; c's live range interferes only with a's. This interference information is concisely shown in the interference graph of Figure 13.15.

With an interference graph, the problem of allocating registers reduces to a well-known problem, that of **coloring** the nodes of a graph. In the **graph coloring problem**, we must determine whether *n* colors suffice to color a graph, given the rule that no two nodes connected by an arc may share the same color. This exactly models register allocation, where *n* is the number of registers we have available and each color represents a different register.

The problem of determining whether a graph is "*n*-colorable" is NP-complete [GJ79]. This means the best-known algorithms that solve this problem exactly have a time bound that is exponential in the size of the graph. As a result, register allocators based on graph coloring normally use *heuristics* to solve the coloring problem approximately.

We will first consider an approach to register allocation using coloring devised by Chaitin [CAC+81]. Initially, the algorithm assumes that all register candidates can be allocated registers. This is often an impossible goal, so the interference graph is tested to see if it is *n*-colorable (the NP-complete problem, which we discuss below), where *n* is the number of registers available for allocation. If the interference graph is *n*-colorable, a register allocation is produced from the colors assigned to the interference graph.

If the graph is not *n*-colorable, it is simplified. A node (corresponding to a live range) is selected and spilled. That is, the live range is denied a register. Instead, whenever it is assigned to or used, it is loaded from or stored into memory using work registers (similar to the pseudo-registers of Section 13.3.1).

Since the live range that was spilled is no longer a register candidate it is removed from the interference graph. The graph is simpler and may now be *n*-colorable. If it is, our register allocation is successful; all remaining candidates can be allocated registers. If the graph still is not *n*-colorable, we select and spill another candidate, further simplifying the graph. This process continues until an *n*-colorable graph is obtained.

Two questions arise. How do we decide if a graph is *n*-colorable? (Recall this is currently considered to be a very hard problem.) If a graph is not *n*-colorable, how do we choose the "right" register candidate to spill?

In testing for *n*-colorability, Chaitin made the following simple but powerful observation. If a node in the interference graph has fewer than *n* neighbors, that node can always be colored (just choose any color not assigned to any of its neighbors). Such nodes (termed **unconstrained nodes**) are removed from the interference graph. This simplifies the graph, often making further nodes unconstrained. Sometimes all nodes are removed, demonstrating that the graph is *n*-colorable.

When only nodes with *n* or more neighbors remain, a node is spilled to allow the graph to be simplified. Chaitin suggests that in choosing a node to spill, two criteria be considered. First, the *cost* of spilling a node should be considered. That is, we compute the extra loads and stores that will have to be executed should a live range be spilled, weighted by the loop nesting level. Each level of loop nesting is arbitrarily assumed to add a factor of 10 to costs. Thus a live range in a single loop has its loads and stores multiplied by 10, a doubly nested loop multiplies loads and stores by 100, etc.

The second criterion Chaitin used is the number of *neighbors* a node has. The greater the number of neighbors a node has, the greater the number of

```
procedure GCRegAlloc(proc, regCount)
    $ig \leftarrow$ buildInterferenceGraph(proc)
    $stack \leftarrow \emptyset$
    while $ig \neq \emptyset$ do
        if $\exists\, d \in ig \mid neighborCount(d) < regCount$
        then
            $ig \leftarrow ig - \{ d \}$
            call push(d)
        else
            $d \leftarrow$ findSpillNode(ig)
            $ig \leftarrow ig - \{ d \}$
            /⋆   Generate code to spill d's live range   ⋆/
    while $stack \neq \emptyset$ do
        $d \leftarrow$ pop( )
        $reg(d) \leftarrow$ any register not assigned to neighbors(d)
end
function findSpillNode(ig) returns Node
    $bestCost \leftarrow \infty$
    foreach $n \in ig$ do
        if $\frac{cost(n)}{neighborCount(n)} < bestCost$
        then
            $ans \leftarrow n$
            $bestCost \leftarrow \frac{cost(n)}{neighborCount(n)}$
    return (ans)
end
```

Figure 13.16: Chaitin's graph coloring register allocator.

interferences that will be removed by spilling the node. Chaitin suggests that the node with the smallest value of *cost*/*neighbors* is the best node to spill. That is, the ideal node to spill is one that has a low spill cost and many neighbors, yielding a very small *cost*/*neighbors* value.

Chaitin's algorithm is shown in Figure 13.16. As an example, consider the interference graph of Figure 13.15. Assume only two registers are available for allocation. Since `c` has only one neighbor, it is immediately removed from the graph and pushed on a stack for later register allocation. `a`, `b`, and `i` all have two neighbors. One will have to be spilled. `a` has a very low cost (3) because it is referenced only 3 times, all outside of the loop. `b` and `i` are used inside the loop and have much higher costs. Since all three nodes have the same number of neighbors, `a` is correctly chosen as the proper node to spill. After `a` is removed, `i` and `b` become unconstrained. When registers are assigned, `i` and `b` get different registers and `c` can be assigned either register. `a` gets no

```
int doubleSum(int initVal, int limit){
      int sum = initVal;
      for (int i=1; i <= limit; i++)
         sum += i;
      return 2*sum;    }
```

Figure 13.17: The subprogram doubleSum

register. Rather, whenever it is used, it is loaded from or stored into a memory location, just like an ordinary variable.

Improvements to Graph Coloring Register Allocators

Briggs et al. [BCT94] suggest a number of useful improvements to Chaitin's approach. They point out that nodes with the smallest number of neighbors ought to be removed first from the interference graph. This is because nodes with few neighbors are the easiest to color and hence they ought to be processed last during the phase in which stacked nodes are popped and colored.

Another improvement follows from the observation that as nodes are removed to simplify the interference graph, they need not be spilled immediately. Rather, removed nodes should be stacked just like unconstrained nodes. When nodes are colored, constrained nodes may be colorable (because they happen to have neighbors that share the same color or happen to have neighbors that are also marked to be spilled). Constrained nodes that cannot be colored are spilled only when we are sure they are uncolorable.

Register allocators need to handle two other problems. Assignments between register values are common. We would like to reduce register moves by assigning the source and target values in the assignment to the same register, making the assignment a trivial one. Moreover, architectural and operating system constraints sometimes force values to be assigned to specific registers. We would like our allocator to try to choose register assignments that anticipate and adhere to predetermined register conventions.

To see how coloring allocators can handle register moves and preallocated registers, consider the simple subprogram doubleSum shown in Figure 13.17. When doubleSum is translated, many short-lived temporary locations are created. Moreover, rules involving register allocation for parameters and return values are enforced. Prior to register allocation, doubleSum has the form shown in Figure 13.18.

The explicit use of register names in doubleSum represents live ranges that must be assigned to a particular register; such nodes are said to be **precolored**. If variables a and b are both allocated to registers, and we have the assignment

```
doubleSum(){
   initVal = $a0;    // First parm passed in $a0
   limit = $a1;      // Second parm passed in $a1
   sum = initVal;
   i = 1;
   temp1 = i <= limit;
   while (temp1) {
      temp2 = sum + i;
      sum = temp2;
      temp3 = i + 1;
      i = temp3;
      temp1 = i <= limit; }
   temp4 = 2 * sum;
   $v0 = temp4; // Return value register is $v0
}
```

Figure 13.18: Subprogram `doubleSum` with initial register assignments

`a = b`, an explicit register copy can be avoided if `a` and `b` are allocated to the same register. Values `a` and `b` will be automatically assigned to the same register if we **coalesce** their live ranges. That is, if we combine the nodes for `a` and `b` in the interference graph, then `a` and `b` must receive the same register.

When is coalescing `a` and `b` safe? At a minimum, they must not interfere. If they do interfere, then they are live at the same time and will need distinct registers. Even if `a` and `b` do not interfere, coalescing them may be problematic. The difficulty is that combining the live ranges of `a` and `b` will in general create a larger live range that is harder to color. We certainly do not want to spill a combined range when the individual ranges might have been individually colored.

To avoid problems in coloring coalesced interference graph nodes we can adopt a **conservative coalescing** approach. We will say a node in an interference graph has **significant degree** if it has n or more neighbors (where n is the number of colors available). A node of significant degree may have to be spilled. A node that has **insignificant degree** (i.e., is not significant) is always colorable. We can conservatively coalesce nodes a and b if the combined interference graph node has fewer than n significant neighbors. This is because insignificant neighbors are always removed since they are trivially colorable. If the combined node has fewer than n significant neighbors, then, after insignificant neighbors are removed, the combined node will have fewer than n neighbors, so it too will be trivially colorable.

```
doubleSum(){
   $v0 = 1;
   $t0 = $v0 <= $a1;
   while ($t0) {
      $a0 = $a0 + $v0;
      $v0 = $v0 + 1;
      $t0 = $v0 <= $a1;
   }
   $v0 = 2 * $a0;
}
```

Figure 13.19: Subprogram `doubleSum` after register allocation

In our above `doubleSum` example, we have three values that must be register-resident (the two parameter values at the start, and the return value at the end). We have eight local variables and temporaries (`initVal`, `limit`, `i`, `sum`, `temp1`, `temp2`, `temp3`, and `temp4`). We will aim for a 4-coloring (a register allocation that uses 4 registers). Temporary `temp1` interferes with `i`, `limit`, and `sum`, so we know that we cannot use fewer than 4 registers without spilling.

We can coalesce `temp4` and `$v0`, guaranteeing that `2*sum` is computed into the return value register. We can coalesce `$a0` and `initVal`, allowing `initVal` to be accessed directly from `$a0` throughout the subprogram. Even more interestingly, we can then coalesce `initVal` and `sum`, allowing `sum` to use `$a0` as well. Temporary `temp2` can also be coalesced with `sum`, allowing it to use `$a0` also. `limit` can be coalesced with `$a1`, allowing it to use `$a1` throughout the subprogram.

Temporary `temp3` can be coalesced with `i` since the combined node has fewer than 4 neighbors. Since neither `temp1` nor the combined `i` and `temp3` interfere with `$v0`, either of these can be assigned `$v0` to use. The other is assigned an unused register, for example `$t0`. The resulting register allocation, with register names replacing variables and temporaries, is shown in Figure 13.19. Note that all register-to-register copies have been removed, and that only one register, beyond the preassigned ones, is used.

It is sometimes possible to coalesce interference graph nodes that have more than *n* significant neighbors. This is done by iterating between interference graph simplification and node coalescing [GA96]. The resulting algorithm is very effective, and is one of the simplest and most effective register allocators in current use.

13.3.3 Priority-Based Register Allocation

Hennessy and Chow [CH90] and Larus and Hilfinger [LH86] suggest interesting alternatives to Chaitin's graph coloring approach. After unconstrained nodes (which are trivially colorable) are removed from the interference graph, a **priority** is computed for each remaining node. This priority is similar to Chaitin's cost estimate, except that it normalizes the cost using the size of the live range. That is, if two live ranges have the same cost, but one is smaller (in terms of the number of instructions it spans), the smaller live range ought to be given preference over the larger one. Thus, the smaller the live range is, the shorter the span of instructions in which it ties up a register. The priority function recommended is *cost*/*size*(*liverange*). The greater the priority of a live range, the more likely it is to receive a register.

Another important difference is that when a node cannot be colored (because its neighbors have been allocated all of the available colors), the node is *split* rather than being spilled. That is, if possible, the live range is divided into two smaller live ranges. Loads and stores are placed at the boundary of the split ranges, but each **split live range** may be allocated a (possibly different) register. Because split ranges usually have fewer interferences than the original range, split ranges are often colorable when the original range is not.

There are many ways a live range may be split into smaller ranges. The following simple heuristic is often used:

1. Remove the first instruction of the live range (usually a load or computation), putting it into a new live range, *NR*.
2. Move successors to instructions in *NR* from the original live range to *NR* as long as *NR* remains colorable.

The idea is to break off at least one instruction, and then add instructions as long as the split range appears colorable. Instructions not split off remain in what is left of the original live range, which may be split again. Single definitions or uses that cannot be colored are spilled.

A priority-based register allocator, PriorityRegAlloc, is shown in Figure 13.20. Reconsider the interference graph of Figure 13.15, assuming two registers, `$r1` and `$r2`. Variable `c` is unconstrained; it is trivial to color and will be handled after all other variables have been allocated registers. `a`, `b`, and `i` are all constrained. `i` has the highest priority for register allocation, since assigning it a register saves 51 loads and stores, and it spans only two statements. Assume it is assigned register `$r1`. Variable `b` has the next highest priority (22 loads and stores saved). It is given `$r2`. Variable `a` is the last candidate in *constrained*, but it cannot be colored. We split it into two smaller live ranges, `a1` and `a2`. `a1` is the single assignment at the top of the procedure. Range `a2` spans the two print statements. `a1` is effectively spilled since its range is a single instruction. `a2` interferes with `b` but not `i`. Hence it receives `$r1`. Finally, `c` receives `$r2`.

13.3.4 Interprocedural Register Allocation

The register allocators we have considered thus far are limited by the fact that they consider only one subprogram at a time; interprocedural interactions are ignored. Thus, when a subprogram is called, either the caller or callee must save and restore any registers that both might use. When registers are used aggressively to hold a large number of variables, constants, and expressions, saving and restoring common registers can make calls costly. Similarly, if a number of subprograms access the same global variable, each must load and later save the variable when it is used.

Interprocedural register allocation improves overall register allocation by identifying and removing register conflicts across calls. Wall [Wal86] considers interprocedural register allocation for architectures with a large number of registers. His goal is to assign registers so that caller and callee never use the same register. This guarantees that no saves or restores are needed during a call, making the call very inexpensive.

First, a priority estimate (similar to that of the previous section) is computed for each local variable or constant that might be kept in a register. These priorities are weighted by estimates of the execution frequency of each procedure. That is, variables used by frequently executed subroutines have a much higher priority than those of infrequently executed subroutines. This is reasonable, since we want to use registers most effectively in those subprograms that are executed most often. If procedure *a* calls *b*, the register allocator places the locals of *a* and *b* in different registers. Otherwise, a local of *a* and a local of *b* can share a common register. Groups of locals, one from each of a set of subprograms that can never be simultaneously active, are grouped together. The priority of a group is the sum of the priorities of all its member locals.

Registers are then **auctioned**. The group with the highest overall priority gets the first register. The next-highest-priority group gets the next register, and so forth. Global variables are handled by placing them in singleton groups, with a priority equal to the total savings that result in all subprograms by having the globals register-resident.

Wall found improvements from 10% to 28% in execution speed, while from 83% to 99% of all dynamic memory references to data were removed. Since Wall's scheme eliminates all saving and restoring, it works best when a large number of registers are available for allocation (52 in his tests). When fewer registers are available, saving and restoring must be included. Now the cost of giving a subprogram an extra register is compared to the benefit of having that register available for local use. If save-restore costs are less than the benefits, code is added to save and restore the registers.

```
procedure PriorityRegAlloc(proc, regCount)
    ig ← buildInterferenceGraph(proc)
    unconstrained ← $\{ n \in nodes(ig) \mid$ neighborCount($n$) $< regCount \}$
    constrained ← $\{ n \in nodes(ig) \mid$ neighborCount($n$) $\geq regCount \}$
    while $constrained \neq \emptyset$ do
        foreach $c \in constrained \mid \neg$colorable($c$) and canSplit($c$) do
            $c1, c2$ ← split($c$)
            constrained ← constrained $- \{ c \}$
            if neighborCount($c1$) $< regCount$
            then  unconstrained ← unconstrained $\cup \{ c1 \}$
            else  constrained ← constrained $\cup \{ c1 \}$
            if neighborCount($c2$) $< regCount$
            then  unconstrained ← unconstrained $\cup \{ c2 \}$
            else  constrained ← constrained $\cup \{ c2 \}$
            foreach  $d \in$ neighbors($c$) $\mid$ ( $d \in unconstrained$ and
                                                     neighborCount($d$) $\geq regCount$ )
            do
                unconstrained ← unconstrained $- \{ d \}$
                constrained ← constrained $\cup \{ d \}$
        /⋆  All nodes in constrained are colorable or cannot be split.  ⋆/
        $p$ ← GetMaxPriority(constrained)
        if colorable($p$)
        then Color $p$
        else  Spill $p$
    Color all nodes $\in$ unconstrained
end
function GetMaxPriority(nodeSet) returns Node
    bestPriority ← $-\infty$
    foreach $n \in nodeSet$ do
        if priority($n$) $> bestPriority$
        then
            ans ← $n$
            bestPriority ← priority($n$)
    return (ans)
end
```

Figure 13.20: A priority-based graph coloring register allocator

When we account for interprocedural effects, it is possible to assign registers and position save-restore code in such a way that optimal register allocation is obtained [KF96]. The improvements in execution speed that result can sometimes be dramatic.

Some architectures, most notably the Sparc, provide **register windows**. When a call is made, the callee is provided a set of architected registers that are physically distinct from the caller's architected registers. Each such set of registers is termed a *window* into the relatively large number of available physical registers. This reduces the cost of calls, as saving and restoring of registers is done automatically. Register windows are allowed to overlap partially to facilitate parameter-passing through registers. Some registers may remain common across calls to facilitate access to global values.

13.4 Code Scheduling

We have already discussed the issues of instruction selection and register allocation in code generation. Modern computer architectures have introduced a new problem—**code scheduling**. Most modern computers utilize a **pipelined architecture**. This means that instructions are processed in stages, with an instruction progressing from stage to stage until it is completed. A number of instructions can be in different stages of execution at the same time. This is very important since instruction execution overlaps, allowing much faster execution speeds.

What happens if one instruction being executed needs a value produced by an earlier instruction that has not yet completed execution? Normally this is not a problem; pipelines are designed to make results available as soon as possible. In a few cases however, a needed operand may not be available. Then we have a **stalled pipeline**, delaying execution of an instruction (and its successors) until the needed value is available.

Most current pipelined architectures are **delayed load**. This means that a register value fetched by a load instruction is not available in the very next cycle. Instead it is delayed for one or more execution cycles. For example, on a MIPS R3000 processor, loads are delayed by one instruction. This delay allows other instructions to be executed while the processor's cache is searched for the fetched value. However, if the instruction immediately following the load references the register, then the processor stalls while the cache is searched. Thus the following instruction sequence, though valid, would stall:

```
lw   $12, b          # Load b into register 12
add  $10, $11, $12   # Add reg 11 and reg 12 into reg 10
```

Stalls are not inevitable after a load. If another instruction can be placed between a load and the instruction that uses the loaded value, instruction execution proceeds without delay. Thus the following instructions would be delay-free:

```
lw  $12, b         # Load b into register 12
li  $11, 100       # Load 100 into register 11
add $10, $11, $12  # Add reg 11 and reg 12 into reg 10
```

The role of instruction scheduling is to order instructions so that stalls (and their delays) are minimized. The nature of processor stalls is generally architecture- and implementation-specific. For example, the MIPS stall described above might be avoided by superscalar processors that attempt out-of-order instruction execution.

Code scheduling is normally done at the basic block level. A **basic block** is a linear sequence of instructions that contains no branches except at its very end. Instructions within a basic block are always executed sequentially as a unit. During code scheduling, all the instructions within a basic block are analyzed to determine an execution order that produces correct computations with a minimum of interlocks or delays. We will consider a simple but effective **postpass code scheduler** devised by Gibbons and Muchnick [GM86].

Postpass code schedulers operate *after* code has been generated and registers have been chosen. They are very flexible and general because they can handle code generated by any compiler (or even hand-coded assembly language programs). However, because instructions and registers have already been selected, they cannot modify choices already made, even to avoid interlocks.

A code scheduler tries to move apart instructions that will interlock. However, instructions cannot be reordered haphazardly. Loads of a register must precede use of that register, and stores of a register must follow instructions that compute the register's value. We use a **dependency DAG** to represent dependencies between instructions. Nodes of the **directed acyclic graph** (DAG) are instructions that are to be scheduled. An arc exists from instruction i to instruction j if instruction i must be executed before instruction j. Thus, arcs are added between instructions that load or compute a register and instructions that use or store that register. Similarly, an arc is added between a load from memory location A and a subsequent store into location A. Also, an arc is added between a store into location B and any subsequent load or store involving location B. In the case of **aliasing**, where we cannot be certain at compile-time as to the location that is referenced by a load or store instruction, we make worst-case assumptions. Thus, a load through a pointer p must precede a store into any location p might alias, and a store through p must precede any load or store involving a location p might alias.

As an example, assume we generate the MIPS code shown in Figure 13.21

```
1. lw  $10,a          6.  add  $10,$10,$12
2. lw  $11,b          7.  mul  $11,$11,$10
3. mul $11,$10,$11    8.  mul  $12,$10,$12
4. lw  $10,c          9.  add  $12,$11,$12
5. lw  $12,d          10. sw   $12,a
```

Figure 13.21: MIPS code for `a=((a*b)*(c+d))+(d*(c+d))`

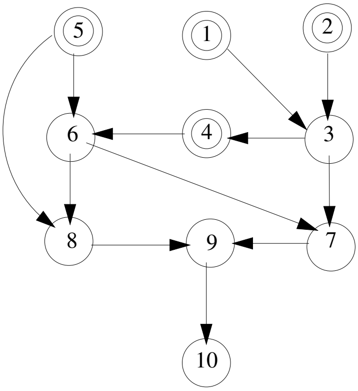


Figure 13.22: Dependency DAG for `a=((a*b)*(c+d))+(d*(c+d))`

for the expression `a=((a*b)*(c+d))+(d*(c+d))`. Figure 13.22 illustrates the corresponding dependency DAG. Double-circled nodes are loads, the critical nodes in this example because they can stall.

Dependency DAGs have the property that any **topological sort** of the nodes represents a valid execution order. That is, as long as an instruction is scheduled before any of its successors in the dependency DAG, it will execute properly. Any node that is a root of the dependency DAG may be scheduled immediately. It is then removed from the DAG, and again any resulting root may be scheduled. Our goal in scheduling instructions will be to choose roots that avoid stalls. In fact, the first rule in our scheduling algorithm is just that:

> When choosing a root to schedule, choose one that *will not* be stalled by the most recently scheduled node.

Sometimes we cannot find a root that does not stall its predecessor. Not all instruction sequences are stall-free.

If we find more than one root that does not stall its predecessor, secondary criteria apply. We try to select the "nastiest" root, the one most likely to cause future stalls or to complicate the scheduling process. Three criteria are considered, in decreasing order of importance:

1. Does the root stall any of its successors in the dependency DAG?

2. How many new roots will scheduling this root uncover?

3. What is the longest path from this root to a leaf of the dependency DAG?

If a root can stall a successor, we want to schedule it immediately so that other roots can be scheduled before the successor, avoiding a stall. If we schedule a root that exposes other new roots, we increase the range of choices available to the scheduler, simplifying its task. If we schedule a root with a long path to a leaf, we are attacking a "critical path," a long instruction sequence that allows the scheduler few choices in reordering instructions.

In our scheduling algorithm, SCHEDULEDAG, we will use an operation SELECT that takes a set of root nodes of the dependency DAG and a criterion. SELECT will choose the nodes in the root set that meet the criterion, as long as the set selected is non-empty. That is, if no node in the set meets the criterion, SELECT will return the entire input set. The reason for this is that a criterion that rejects all nodes is useless since our goal is to choose *some* node to schedule.

For example, SELECT(*nodeSet*, "Has the longest path to a leaf") selects the nodes in *nodeSet* with the greatest distance to a leaf (several nodes may be selected if they all share the same maximum distance to a leaf). However, SELECT(*nodeSet*, "Can stall some successor") would return all of *nodeSet* if no member of the set had a successor that it stalled. Once we have refined a *nodeSet* to a single node, further applications of SELECT are unnecessary as they will have no effect.

The complete definition of SCHEDULEDAG is shown in Figure 13.23. As an example, consider the dependency DAG of Figure 13.22. The code originally generated (Figure 13.21) contains two stalls (after instructions 2 and 5). The initial set of roots is 1, 2, and 5, all load instructions. All roots can stall a successor instruction and none expose a new root if scheduled. Both 1 and 2 have the longest path to a leaf, so 1 is arbitrarily chosen and scheduled. The root set is now 2 and 5. Instruction 2 is chosen because it exposes a new root, 3. Next, 5 is chosen because it can stall a successor. Instructions 3, 4, and 6 are chosen next, as they form, in turn, singleton root sets. Instructions 7 and 8 are the new root set; 7 is arbitrarily chosen, then 8, 9, and 10. The code we produce is shown in Figure 13.24.

```
procedure scheduleDAG(dependencyDAG)
    candidates $\leftarrow$ roots(dependencyDAG)
    while candidates $\neq \emptyset$ do
        call select(candidates, "Is not stalled by last instruction generated")
        call select(candidates, "Can stall some successor")
        call select(candidates, "Exposes the most new roots if generated")
        call select(candidates, "Has the longest path to a leaf")
        inst $\leftarrow$ Any node $\in$ candidates
        Schedule inst as next instruction to be executed
        dependencyDAG $\leftarrow$ dependencyDAG $-$ { inst }
        candidates $\leftarrow$ roots(dependencyDAG)
end
```

Figure 13.23: An Algorithm to Schedule Code from a Dependency DAG

```
1. lw  $10,a          6. add $10,$10,$12
2. lw  $11,b          7. mul $11,$11,$10
3. lw  $12,d          8. mul $12,$10,$12
4. mul $11,$10,$11    9. add $12,$11,$12
5. lw  $10,c          10 sw  $12,a
```

Figure 13.24: Scheduled MIPS code for `a=((a*b)*(c+d))+(d*(c+d))`

13.4.1 Improving Code Scheduling

The code shown in Figure 13.24 is not prefect. A stall still occurs after the fifth instruction. In fact, using just three registers a stall *cannot* be avoided. It is shown [KPF95] that sometimes an additional register is needed to avoid all stalls. One way to improve the code produced by scheduleDAG is to reallocate registers in the initial code sequence using an extra register beyond the original allocation.

To do this, we find instructions that stall and try to move them "up" in the instruction sequence. Sometimes we cannot move a stalling instruction earlier because it assigns to a register *r* used by the preceding instruction. In such cases, we reallocate register *r* so that it is unused by the preceding instruction. Because we have added an extra register, we can always find an unused register, and move the stalling instruction at least one position earlier in the execution sequence.

For example, reconsidering Figure 13.24, instruction 5 (a load) stalls because `$10` is used in instruction 6. We cannot move instruction 5 up because

```
 1. lw  $10,a         6.  add  $10,$13,$12
 2. lw  $11,b         7.  mul  $11,$11,$10
 3. lw  $12,d         8.  mul  $12,$10,$12
 4. lw  $13,c         9.  add  $12,$11,$12
 5. mul $11,$10,$11   10. sw   $12,a
```

Figure 13.25: Delay-free MIPS code for `a=((a*b)*(c+d))+(d*(c+d))`

instruction 4 uses a previous value of `$10`, loaded in instruction 1. If we add an additional register, `$13`, to our allocation, we can load it in instruction 5 (taking care to reference `$13` rather than `$10` in instruction 6). Now instruction 5 can be moved earlier in the sequence, avoiding a stall. The resulting delay-free code is shown in Figure 13.25.

It is evident that there is a tension between code scheduling, which tries to increase the number of registers used (to avoid stalls), and code generation, which seeks to reduce the number of registers used (to avoid spills and make registers available for other purposes). An alternative to postpass code scheduling is an *integrated* approach that intermixes register allocation and code scheduling.

The Goodman–Hsu [GH88] algorithm is a well-known **integrated register allocator and code scheduler**. As long as registers are available, it uses them to improve code scheduling by loading needed values into distinct registers. This allows loads to “float” to the beginning of the code sequence, eliminating stalls in later instructions that use the loaded values. When registers grow scarce, the algorithm switches emphasis and begins to schedule code to free registers. When sufficient registers are available, it resumes scheduling to avoid stalls. Experience has shown that this approach balances nicely the need to use registers sparingly and yet avoid stalls whenever possible.

13.4.2 Global and Dynamic Code Scheduling

Although we have focused on code scheduling at the basic block level, researchers have also studied **global code scheduling** [BR91]. Instructions may be moved upward, past the beginning of a basic block, to predecessor blocks in the control flow graph. We may need to move instructions out of a basic block because basic blocks are often very small, sometimes only an instruction or two in size. Moreover, certain instructions, like loads and floating point multiplies and divides, can incur long latencies. For example, a load that misses in the primary cache may stall for 10 or more cycles; a miss in the secondary cache (to main memory) can cost 100 or more cycles.

As a result, code schedulers often seek to move loads as early as possible in an instruction sequence. There are several complicating factors, however. To what predecessor block should we move an instruction? Ideally, to a predecessor that is **control equivalent**; that is, a predecessor that will be executed if, and only if, the current block is executed. An example of this is moving an instruction that follows an *if* statement to a position that precedes the *if* (and thereby past both arms of the *if*). An alternative is to move an instruction to a block that *dominates* it (that is, to a block that is a necessary predecessor). Now, however, the moved instruction may become a **speculative instruction—**it may be executed unnecessarily on some execution paths. Thus, if an instruction is moved from a then part to a position above the if, the instruction will be executed even when the else part is selected. Speculative instructions may waste computational resources by executing useless instructions. What is worse, if a speculative instruction faults (for example, a load through a null or illegal pointer), a false runtime error may be reported.

Even if we can move an instruction freely upward, how far should we move it? If we move the instruction too far forward it will "tie up" a register for an extended period, making register allocation harder and less effective. Some architectures, like the DEC alpha, provide a **prefetch instruction**. This instruction allows data to be loaded into the primary cache in advance, reducing the chance that a register load will miss. Again, placement of preloads is a tricky scheduling issue. We want to preload early enough to hide the delays incurred in loading the cache. But, if we preload too early, we may displace other useful cache data, causing cache misses when these data are used.

Many modern computer architectures (Intel Pentium®, PowerPC) include a sophisticated **dynamic scheduling** facility. These designs, sometimes called **out of order architectures** or *OOO*, delay instructions that are not ready to execute, and dynamically choose successor instructions that are ready to execute. These designs are far less sensitive to compiler-generated code schedules. In fact, dynamically scheduled architectures are particularly effective in executing old programs ("dusty decks") that were created before code scheduling was even invented.

Even with dynamically scheduled architectures, compiler-generated code scheduling is still an important issue. Loads, especially loads that frequently miss in the primary cache, must be moved early enough to hide the long delays a cache miss implies. Even the best current architectures cannot look dozens or hundreds of instructions ahead for a load that might miss in the cache. Rather, compilers must identify those instructions that might incur the greatest delays and move them earlier in the instruction sequence.

13.5 Automatic Instruction Selection

An important aspect of code generation is **instruction selection**. After a translation for a particular construct is determined, the machine-level instructions that implement the translation must be chosen. Thus, we may decide to implement a switch statement using a jump table. If so, instructions that index into the jump table and execute an indirect jump must be generated.

Often several different instruction sequences can implement a particular translation. Even something as simple as `a+1` can be implemented by loading 1 into a register and generating an add instruction, or by generating an increment or add-immediate instruction. We normally want the smallest or fastest instruction sequence. Thus, an add-immediate instruction is preferred because it avoids an explicit load.

In simple RISC architectures, the choice of potential instruction sequences is limited because almost all operands must be loaded into registers before they can be used (immediate operands being a notable exception). Further, the variety of addressing modes provided is also spartan; often only absolute and indexed addresses are allowed.

Older architectures, like the Motorola 680x0 and Intel x86, are much more elaborate. Many different operation codes are provided, and a wide variety of addressing modes are available. Operands do not always need to be loaded into registers; addressing modes can fetch operands indirectly and can increment and decrement registers. Different register classes (e.g., address registers and data registers) are used in different instructions (in a non-interchangeable manner) and particular registers are sometimes "wired into" certain instructions. The JVM also has a relatively elaborate instruction set because its design was based on achieving compact code sequences. As a result, there are several ways to increment the contents of a register, but one sequence of bytecode instructions achieves that effect most compactly.

For very complex architectures, a method of systematizing and automating instruction selection is vital. Even for simpler architectures, it may be necessary to "extend" a code generator when a successor architecture is introduced. Very ambitious compilers may aim to compile into more than one target architecture, mandating alternative instruction sequences for different target machines.

Instruction selection is often simplified by translating source language constructs into a very low-level tree-structured **intermediate representation** (IR). In this IR, leaves represent registers, memory locations, or literal values, and internal nodes represent basic operations on operand values. Detailed data access patterns and manipulations are exposed. Consider the statement `b[i]=a+1`, where `b` is an array of integers, `i` is a global integer variable, and `a` is a local variable accessed through the frame register, `$fp`. The statement's tree-structured IR is shown in Figure 13.26. This representation is very similar to

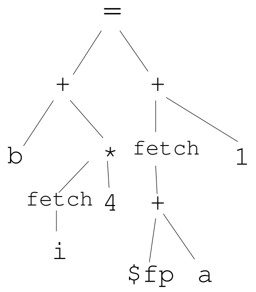


Figure 13.26: Low-Level IR Representation of `b[i]=a+1`

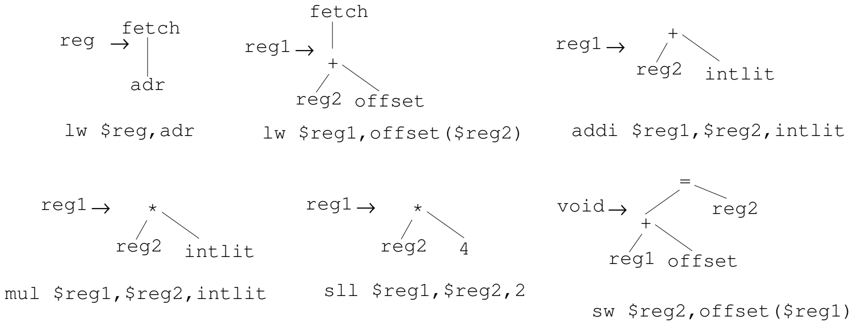


Figure 13.27: IR Tree Patterns for Various MIPS Instructions

the AST representation of a program (as discussed in Section 7.4 on page 250) except that memory traffic and address computations are explicit:

- Leaves corresponding to identifiers are their addresses (if globals) or offsets (if locals).
- Explicit memory fetches (using the fetch operator) are shown, as is the multiply by 4 needed to build a valid word address for an element of an array of integers.

A tree-structured IR may also be used to define the effect of each instruction of a computer. A tree defines the computation performed by the instruction as well as the kind of value it produces. This is illustrated in Figure 13.27 in which tree-structured patterns (or productions) are used to define valid IR trees.

Now instruction selection for a given IR tree becomes a matter of matching instruction patterns against the generated IR such that the IR tree is covered

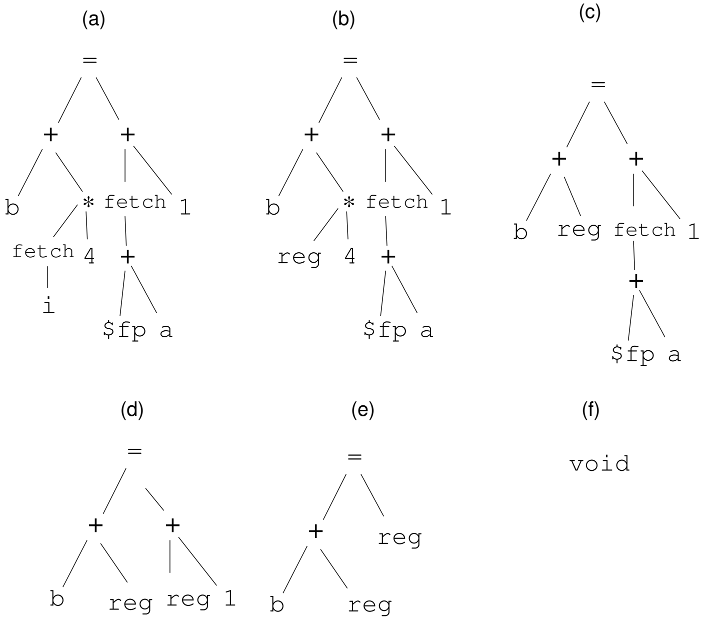


Figure 13.28: Instruction Selection Using Patterns

(parsed) with adjacent patterns. That is, we find a subtree in the IR translation that matches exactly the pattern for some instruction. That subtree is then replaced with the pattern's left-hand side. The process is repeated until the entire IR tree is reduced to a single node. This is very similar to ordinary bottom-up parsing (Chapter 6).

As instruction patterns are matched, their corresponding machine-language instructions are generated. Registers can be allocated "on the fly" using the techniques of Section 13.3.1. Alternatively, pseudo-registers can be allocated as code is generated, and then later mapped to real registers using the graph coloring techniques of Section 13.3.2.

As an example, reconsider the IR tree corresponding to `b[i]=a+1` shown in Figure 13.28 (tree a). We first match a load of `i` (tree b). Next, a multiply by 4 is matched (tree c). Then, an indexed load is generated for the local variable `a` (tree d). Finally, an add-immediate (tree e) and a store instruction (tree f) reduce the IR tree to void. The instructions generated (assuming calls to GETREG and FREEREG as code is generated) are shown in Figure 13.29.

```
lw   $t1,i
mul  $t1,$t1,4
lw   $t2,a($fp)
addi $t2,$t2,1
sw   $t2,b($t1)
```

Figure 13.29: MIPS code for `b[i]=a+1`

13.5.1 Instruction Selection Using BURS

It is often the case that more than one instruction sequence can implement the same construct. In terms of IR trees, different reductions of the same tree, yielding different instruction sequences, may be possible. How can we choose the instruction sequence to be generated?

A very elegant approach involves assigning costs to instruction patterns. The *cost* of an instruction is set when a code generator is built. This cost may be the size of an instruction, its execution speed, the number of memory references the instruction makes, or any criterion that measures how "good" an instruction is. When given a choice, we will prefer a cheaper instruction over a more expensive one.

Now matching of instruction patterns to an IR tree is generalized so that a **least-cost cover** is obtained. That is, the pattern matcher guarantees that the matches it selects have the lowest possible cost. Thus, using the measure of quality selected when the code generator was built, the best possible instruction sequence is generated.

To guarantee that a least-cost cover of an IR tree is found, we use **dynamic programming**. Starting at the leaves of the tree, we mark each leaf with the lowest cost possible to reduce the leaf to each of the nonterminals. (A **nonterminal**, as in context-free productions, is the symbol that appears on the left-hand side of an instruction pattern). Next, we consider the interior nodes just above the leaves. Each instruction pattern that correctly matches the interior node and has the correct number of children is considered. The cost of the pattern plus the costs of the node's children are considered. The node is marked with the cheapest cost possible to reduce the tree to each possible nonterminal. We continue traversing the IR tree, until the root node is reached. The lowest cost found to reduce the tree to any nonterminal is selected as the best cover.

IR trees for a large program or subroutine can easily comprise tens or hundreds of thousands of nodes. The extensive processing needed for each node would appear to make least-cost instruction selection using patterns a very slow process. Happily this is not the case.

An approach based on **bottom-up rewriting systems** (BURS) [PLG88] theory allows very fast instruction selectors (and code generators) to be built. Code generators built using BURS theory can be extremely fast because all dynamic programming is done in advance when a special BURS automaton is built. During compilation, it is only necessary to make two traversals of the IR tree: one bottom-up traversal to label each node with a *state* that encodes all optimal matches and a second top-down traversal that uses these states to select and generate code. It has been reported that careful encodings can produce an automaton that executes fewer than 90 RISC instructions per node to do both traversals.

The automaton that labels the tree is a simple finite state machine, similar to that used in shift-reduce parsers (Chapter 6). A bottom-up walk of the tree is performed, and the label for any given node is determined by a table lookup given the operator at the node and the states that label each of its children. The automaton that emits code is equally simple in design. The code to be emitted is determined by the state that labels a node and by the nonterminal to which that node should be reduced—another table lookup.

As an example, the instruction patterns of Figure 13.27 would all be given a cost of 1 except for `mul`, which would be given a cost of 3. This is because `mul` is actually implemented by the MIPS assembler using three hardware instructions, whereas all the other instructions are implemented using a single instruction. Returning to the example of Figure 13.26, all the leaves would be labeled with a state indicating that no reductions of individual leaf nodes are possible. Visiting `i`'s parent with its state, the fetch would be labeled with a state indicting that application of an `lw` pattern is possible, at a cost of 1. Going in turn to its parent (a * operator), the state reached would show that, although two reductions are possible (patterns for both `mul` and `sll` match), the two reductions are not of equal cost. `sll` is cheaper and will apply. That is, the instruction selector has recognized a well-known trick: multiplication by a power of two can often be implemented more efficiently by doing a left shift rather than an explicit multiply.

The automaton continues labeling the rest of the nodes; the remaining matches are identical to those illustrated in Figure 13.28. The state labeling the root tells us that the final instruction to be generated (to implement the assignment) will be an `sw`. The two subtrees are visited to generate the instructions needed to implement them. We therefore generate the root's instruction *after* returning from recursive visits to both children, guaranteeing that the store's operands are computed prior to its execution. We generate the code shown in Figure 13.30.

Two difficulties arise in creating a BURS-style code generator: efficiently generating the states and state transition tables (because *all* potential dynamic programming decisions are done at table-generation time, they must be done efficiently), and creating an efficient encoding of the automata for use in a com-

```
lw    $t1,i
sll   $t1,$t1,2
lw    $t2,a($fp)
addi  $t2,$t2,1
sw    $t2,b($t1)
```

Figure 13.30: Improved MIPS code for `b[i]=a+1`

piler. Fraser and Henry discuss a solution to the encoding problem in [FH91]. Proebsting created BURG [Pro91], a simple and efficient tool for generating BURS-style code generators. Using a very clean implementation and ingenious state-elimination techniques, least-cost code generators for a variety of architectures can be created very efficiently.

13.5.2 Instruction Selection Using Twig

Other code-generation systems based on tree pattern matching and dynamic programming have been developed. They differ primarily from BURS in how they do tree pattern matching and in the fact that they do dynamic programming as a compiler runs rather than when it is built.

Aho, Ganapathi, and Tjiang [AGT89] created a tree manipulation language and system called **Twig**. Given a specification of tree patterns and associated costs, Twig generates a top-down tree automaton that will find the least-cost cover of a subject tree. Twig uses fast top-down Hoffmann–O'Donnell [HO82] pattern matching in parallel with dynamic programming to find the least-cost cover.

Starting at the root of possible instruction trees, paths to each of the tree's children are traced. Whenever such a path is correctly traced, a counter is incremented. When the counter equals the number of children a pattern tree has, a potential match is recognized. Using costs and dynamic programming, the least-cost cover for an entire IR tree can be found.

The costs associated with patterns in Twig are more general than those afforded by any BURS system. Twig may compute the cost of a pattern dynamically, depending on semantic information available at compile-time. This flexibility further allows Twig to abort certain matches if semantic predicates are not satisfied. Thus, the applicability of Twig's patterns is context sensitive. BURS does not have this flexibility since all costs must be fixed prior to compilation to allow precomputation of dynamic programming decisions. The great advantage of BURS is its speed. All possible matches are anticipated in advance and tabulated. Twig must recognize partial matches and update counters as instruction selection proceeds. Given the huge IR trees that often need to be translated, even a little extra processing at each node can represent a significant slowdown.

13.5.3 Other Approaches

One of the first instruction selection techniques based on tree rewriting was that of Cattell [Cat80]. First, the effect of each instruction was described using a register-transfer notation. Then, a code generator "discovered" appropriate code sequences by matching instructions against IR trees. That is, the code generator explored ways to decompose an IR tree into combinations of special primitive trees, using backtracking if necessary. Because this process could be very slow, a catalog of the tree patterns that were implemented was precomputed. At compile-time, this catalog was searched to find available instruction sequences.

Glanville and Graham [GG78] observed that the problem of matching code templates against an IR tree is very similar to the problem of matching productions against a token sequence during parsing. They cleverly reformulated the template-matching problem in context-free parsing terms. Using standard shift-reduce parsers augmented to handle multiple template matches, instruction selection could be automated.

A limitation of the Graham-Glanville approach is that it is purely syntactic. It simply matches, in a context-free manner, sequences of symbols. Ganapathi and Fischer [GF85] suggested adding attributes to code templates. Attributes allow types, sizes, and values to influence instruction selection.

The **back-end generator** (BEG) [ESL89] finds a least-cost cover of the tree using dynamic programming techniques that are essentially identical to Twig's. Like Twig, BEG can guard patterns with semantic predicates. A BEG specification, in addition to having instruction patterns, includes a description of the register set of the target machine. This specification automatically generates the register allocator. Experiments show code quality and code-generation times to be comparable to handwritten code generators.

Fraser, Hanson, and Proebsting [FHP92] developed a code-generator generator based on naive pattern matching and dynamic programming. This system, **iburg**, maintains the same interface as BURG. Although iburg code generators are slower than those generated by BURG, iburg presents a simple and efficient framework for the development of pattern-based code generators.

13.6 Peephole Optimization

To produce high-quality code, it is necessary to recognize a multitude of special cases. For example, it is clear we would like to avoid generating code for an addition of zero to an operand. But where should we check for this special case? In each translation routine that might generate an add? In each code-generation routine that might emit an add instruction?

Rather than distribute knowledge of special cases throughout translation and code-generation routines, it is often preferable to utilize a distinct peephole optimization phase that looks for special cases and replaces them with improved code. **Peephole optimization** may be performed on ASTs, IR trees [TvSS82] or generated code [McK65]. As the term "peephole" suggests, a small window of two or three instructions or nodes is examined. If the instructions in the peephole match a particular pattern, they are replaced with a replacement sequence. After replacement, the new instructions are reconsidered for further optimization.

In general, we represent the collection of special cases that define a peephole optimizer as a list of pattern-replacement pairs. Thus,

$$\text{pattern} \Longrightarrow \text{replacement}$$

means that if an instruction sequence or tree matching the pattern is seen, it is replaced with the replacement sequence. If no pattern applies, the code sequence remains unchanged. Clearly, the number of special cases that might be included is unlimited. We will illustrate where peephole optimization can be employed and the kinds of optimizations that can be realized.

13.6.1 Levels of Peephole Optimization

In general there are three places where peephole optimization may be profitably employed. After parsing and type checking, a program is represented in AST form. Here, peephole optimization may be used to optimize the AST, recognizing special cases at the source level that are independent of how a construct is translated, or the code that is generated for it.

After translation, a program is represented in an IR or bytecode form. Here, peephole optimization can recognize optimizations that simplify or restructure an IR tree or bytecode sequence. These optimizations are independent of the actual target machine or the exact code sequences used to implement an IR tree or bytecodes.

Finally, after code generation, peephole optimization can replace pairs or triples of target machine instructions with shorter or simpler instruction sequences. At this level, the optimization is highly dependent on the details of a machine's instruction set.

AST-Level Optimizations

In Figure 13.31 we illustrate optimizations that can simplify or improve an AST representation of a program. In (a), an if statement whose condition

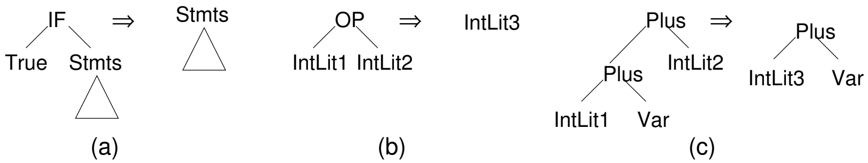


Figure 13.31: AST-Level Peephole Optimization

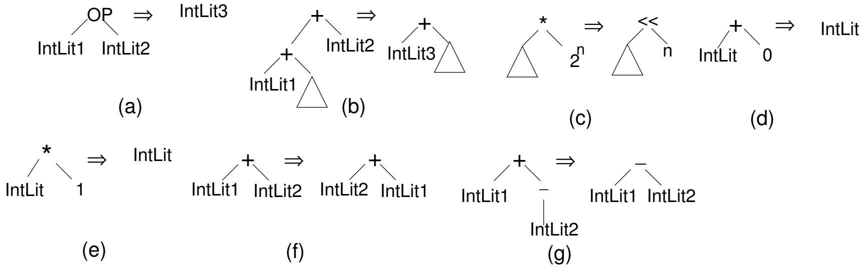


Figure 13.32: IR-Level Peephole Optimizations

is always true is replaced with the body of the conditional. In (b) and (c), expressions involving constant operands are **folded** (replaced with the value of the expression). This folding optimization can expose other optimizations (such as the conditional replacement optimization of (a)).

Optimizations at the AST level can be conveniently implemented using a tree rewriting tool like BURS. Source patterns are first recognized and labeled. Then, during the "processing" traversal, trees can be rewritten into the target form. If necessary, an AST can be traversed several times, so that rewritten ASTs can be matched and transformed several times.

IR-Level Optimizations

As illustrated in Figure 13.32, a variety of useful optimizations can be performed at the IR level. In (a) and (b), constant folding is specified. Since some arithmetic operations are exposed only after translation (e.g., indexing arithmetic), folding can be done at both the AST and IR levels. In (c), multiplication by a power of 2 is replaced with a left shift operation. In (d) and (e), identity operations are removed. In (f), the commutativity of addition is exposed, and in (g), addition of a negative value is transformed into subtraction. Transformations on IR trees can be conveniently implemented using a tool like BURS.

```
ldc IntLit1  ⇒  ldc IntLit3
ldc IntLit2
iadd
```

(a)

```
ldc IntLit1
{Bytecode          {Bytecode
 sequence      ⇒    sequence
 for operand}       for operand}
iadd               ldc IntLit3
ldc IntLit2        iadd
iadd
```

(b)

```
ldc 2^n       ldc n
imul    ⇒     ishl
```

(c)

```
ldc IntLit  ⇒  ldc IntLit
iconst_0
iadd
```

(d)

```
ldc IntLit  ⇒  ldc IntLit
iconst_1
imul
```

(e)

```
ldc IntLit1  ⇒  ldc IntLit2
ldc IntLit2     ldc IntLit1
iadd            iadd
```

(f)

```
ldc IntLit1  ⇒  ldc IntLit1
ldc IntLit2     ldc IntLit2
ineg            isub
iadd
```

(g)

Figure 13.33: Bytecode-Level Peephole Optimizations

As illustrated in Figure 13.33, optimizations corresponding to those of Figure 13.32 can be applied to a bytecode representation of a program. This level of optimization may be appropriate if bytecodes are later expanded into target machine code. Alternatively, the machine-level optimizations described in the next section may be applied to bytecodes, since bytecodes share much of the structure of conventional machine code.

Code-Level Optimizations

Figure 13.34 illustrates some simple peephole optimizations performed after code generation. In (a) a conditional branch around an unconditional branch is replaced with a single conditional branch (with the sense of the test inverted). In (b), a branch to the next instruction is removed (this is sometimes generated when a *then* or *else* part of an *if* is null). A branch to a second branch can be collapsed to a direct branch to the final target (c). In (d), a move from a register to itself is suppressed (this sometimes happens when a special register, such as a parameter register, is loaded with a value that already is in the correct register). In (e), a register is stored into a location and then that same register is immediately reloaded from the same location; the load is unnecessary and may be deleted.

More elaborate architectures present additional opportunities for peephole optimization. If a special increment or decrement instruction is available, it can replace an ordinary add-immediate instruction (which usually is longer and a bit slower). If auto-increment or auto-decrement addressing modes are available, these can subsume an explicit increment or decrement of an index. Some architectures have a special **loop control instruction** that decrements a

```
    beq $reg,$0,L1   ⇒  bneq $reg,$0,L2
    b L2
L1:                   L1:
```

(a)

```
    b L1   ⇒
L1:            L1:
```

(b)

```
    b L1   ⇒      b L2
L1: b L2       L1: b L2
```

(c)

```
move $reg,$reg   ⇒  (nothing)
```

(d)

```
sw $reg,loc   ⇒   sw $reg,loc
lw $reg,loc
```

(e)

Figure 13.34: Code-Level Peephole Optimizations

register and conditionally branches if it is zero.

Recognizing replacement patterns must be done quickly if peephole optimization is to be fast. Operator-operand combinations are hashed to applicable patterns. Also, the size of a peephole window is normally limited to two or three instructions. Using a carefully hashed implementation, speeds of several thousand instructions per second have been achieved [DF84].

The concept of analyzing physically adjacent instructions has been generalized to **logically adjacent instructions** [DF82]. Two instructions are logically adjacent if they are linked by flow of control or if they are unaffected by intervening instructions. (The "branch chain" of Figure 13.34 (c) is a good example of this.) By analyzing logically adjacent instructions it is possible to remove jump chains (jumps to jump instructions) and redundant computations (for example, unnecessarily setting a condition code). Detecting logical adjacency can be costly, so care is required to keep peephole optimization fast.

13.6.2 Automatic Generation of Peephole Optimizers

In [DF80], ways of automating the creation of peephole optimizers are discussed. The idea is to define the effect of target machine instructions at the **register-transfer level** (RTL). At this level, instructions are seen to modify primitive hardware locations, including memory (represented as a vector M), registers (represented as a vector R), the PC (program counter), various condition codes, and so on. A target machine instruction may have more than one effect and its definition at the RTL may include more than one assignment.

The peephole optimizer (PO) operates by considering pairs of instructions, expanding them to their RTL definitions, simplifying the combined definitions, and then searching for a single instruction that has the same effect as the combined pair. To be applicable, an instruction must perform all the register transfers of the combined instructions. It may also perform other register transfers as long as these are on dead registers (and therefore would have no effect on subsequent computations). Thus, an instruction may set a condition code, even if this is not wanted, as long as the updated condition code is not referenced by later instructions.

Instruction pairs that start with a conditional branch get special treatment. In particular, the second instruction is prefixed with a conditional representing the negation of the original condition (the only way the second instruction is executed is if the conditional branch fails). An unconditional branch is paired with its target instruction. This pairing often allows jump chains (a jump to another jump) to be collapsed. Note, however, that instruction pairs with the second instruction labeled are not optimized. This situation is needed to make jumps to such labels work correctly. However, if all references to a label are removed by the PO, then the label itself is also removed, possibly allowing new optimizations to be discovered.

The analysis and simplification of the instructions just described are not actually done during compilation because this would be far too slow. Rather, representative samples of actual programs are analyzed in advance, and the most common peephole optimizations are stored in a table. During compilation, this table is consulted to determine if the instructions currently in the peephole may be optimized.

Exercises

1. Consider the following Java method:

   ```
   public static int fact(int n){
      if (n == 0)
         return 1;
      else return n*fact(n-1); }
   ```

   Using your favorite Java compiler, show the JVM bytecodes that would be generated for this method. Explain what each of the generated bytecodes contributes to the execution of the methods.

   If the "`public static`" prefix is removed, the Java method becomes a valid C or C++ function. Compile it using your favorite compiler on your favorite processor using *no optimization*. List the machine instructions generated and show which machine instructions correspond to each generated JVM bytecode.

2. On many processors, certain designated registers must be used to hold a parameter to a subprogram or a return value from a function. Suggest how the techniques of Section 13.1 could be extended so that when bytecodes are translated, parameters and return values are computed directly into the designated register (without any unnecessary register-to-register moves).

3. Recall that a key to generating efficient target-machine code from bytecodes is to avoid explicit stack manipulations for bytecode operands. Rather, machine registers are used to hold "stacked" values.

   Assume we use the techniques of Section 13.3.1 to allocate registers on-the-fly. Explain how we could tag each bytecode, prior to code generation, with the machine registers the bytecode will use for its operands and result value. (These tags would then be used to "fill in" register names when bytecodes are expanded to machine code.)

4. A common subprogram optimization is *inlining*. At the point of call, the body of the called method is substituted for the call, with actual parameter values used to initialize local variables that represent formal parameters.

   Assume we have the bytecodes that represent the body of subprogram `P` that is marked `private` or `final` (and hence cannot be redefined in a subclass). Assume further that `P` takes n parameters and uses m local variables. Explain how we could substitute the bytecodes representing `P`'s body for a call to `P`, prior to machine code generation. What changes in the body must be made to guarantee that the substituted bytecodes do not "clash" with other bytecodes in the context of call?

5. Array bounds checks are mandatory in Java and C♯. They are very useful in catching errors, but are also fairly expensive, especially in loops. It is often the case that conditional branches provide information useful in optimizing or even eliminating unnecessary bounds checks. For example, in

   ```
   while (i < 10) {
     print(a[i++]);
   }
   ```

   we know `i` must be less than 10 whenever array `a` is indexed. Moreover, since `i` is never decreased in the loop, a single check that `i` is non-negative at loop entrance suffices.

   Suggest ways in which information provided by conditional branches (in conditionals and loops) can be exploited when code to index arrays is generated.

6. Show the expression tree, with registerNeeds labeling, that corresponds to the expression `a+(b+(c+((d+e)*(f/g))))`.

   Show the code that would be generated using the treeCG code generator.

7. Recall from Section 13.2 that registerNeeds gives the *minimum* number of registers needed to evaluate an expression without spilling registers to memory. Show that expressions of *unbounded* size exist that require only 2 registers for evaluation. Show that for any value of m, expressions exist that always require *at least m* registers.

8. Some computer architectures include an immediate operation of the form

   ```
   op $reg1,$reg2,val
   ```

   that computes `$reg1 = $reg2 op val`. In an immediate instruction, `val` does not need to be loaded into a register; it is extracted directly from the instruction's bit pattern.

   Explain how to extend REGISTERNEEDS and TREECG to accommodate architectures that include immediate operations.

9. Sometimes the code generated for an expression tree can be improved if the associative property of operators like + and * is exploited. For example, if the following expression is translated using TREECG, four registers will be needed:

   ```
   (a+b) * (c+d) * ((e+f) / (g-h))
   ```

   Even if the commutativity of + and * is exploited, four registers are still required. However, if the associativity of multiplication is exploited to evaluate multiplicands from right to left, then only three registers are needed. First `((e+f)/(g-h))` is evaluated, then `(c+d)*((e+f)/(g-h))`, and finally `(a+b)*(c+d)*((e+f)/(g-h))`.

   Write a routine ASSOCIATE that reorders the operands of associative operations to reduce register needs. (Hint: Allow associative operators to have more than two operands.)

10. In Section 13.4 we saw that many modern architectures are *delayed load*. That is, a value loaded into a register may not be used in the next instruction; a delay of one or more instructions is imposed (to allow time to access the cache).

    The TREECG routine of Section 13.2 is not designed to handle delayed loads. Hence, it almost always generates instruction sequences that stall at selected loads.

    Show that if an instruction sequence (of length 4 or more) generated by TREECG is given an additional register, it is possible to reorder the generated instructions to avoid all stalls for a processor with a one instruction load delay. (It will be necessary to reassign the register used by some operands to utilize the extra register.)

11. Following the example of `doubleSum` (page 515), convert the `stringSum` function (page 499) into a form that makes explicit register assignments for temporaries, live ranges, parameters, and return values. Then, create the interference graph for `stringSum`. Use this interference graph and GCRegAlloc to assign registers to `stringSum`, assuming three registers are available (including `$a0`, the parameter register, and `$vo` (the return value register)).

12. Assume we have the following method:

```
int f(int i) {
   g(1,i);
}
```

At the point where the second parameter of `g` is loaded, we have a conflict if we require that parameters be passed in registers. In particular, `i` is passed in using the first parameter register. But when the second parameter of `g` is loaded, the first parameter register is already loaded with the value 1, possibly making `i` inaccessible. How can a register allocator deal with the problem of reuse of dedicated parameter registers? That is, what rules should be followed in determining where a parameter value is to be allocated throughout a subprogram or method?

13. In GCRegAlloc, we spill a live range if we are unable to color it. An alternative to spilling a live range is to *split* it, as is done in PriorityRegAlloc. What changes are needed in GCRegAlloc if we split an uncolorable live range rather than spill it?

14. At the site of a method call, we may need to save registers currently in use (lest they be overwritten by the method about to be executed). Assume we allocate registers using GCRegAlloc. Explain how to determine which registers are in use at a particular method call.

15. Assume we have *n* registers available to allocate to a subprogram. Explain how, using either GCRegAlloc or PriorityRegAlloc, we can estimate the total cost of register spills within the subprogram. How could this cost estimate be used in deciding how many registers to allocate to a subprogram?

16. In performing on-the-fly register allocation, some implementations store freed registers on a stack. Thus, the most recently freed register will be the next register to be allocated. On the other hand, other implementations place freed registers at the back of a queue. Thus, the least-recently freed register will be the next to be allocated. (This is often called **round robin allocation**.)

    From the point of view of a postpass code scheduler, which of the register reallocation implementations (stack vs. queue) is preferable? Why?

17. The scheduleDAG code scheduler of Section 13.4 assumes that instructions that can stall have unit delay. That is, one instruction must separate an instruction that can stall from the first use of the value it produces. It may happen that some instructions have *n* cycle delays, meaning *n* instructions must separate the instruction from the first use of the value it produces.

    How must scheduleDAG be modified to handle instructions that have *n* cycle delays?

18. The scheduleDAG code scheduler is a **postpass code scheduler**. That is, it schedules instructions after registers have been allocated. It is possible to create a dependency DAG in terms of instructions that reference pseudo-registers. After instructions are scheduled, the pseudo-registers are mapped to real registers. Such a scheduler is a **prepass scheduler**, since it operates before register allocation.

    It is important to note that the order in which instructions are scheduled will affect the number of registers that are needed later. For example, scheduling all loads immediately will force each load to use a different register. Scheduling some loads after other operations may allow registers to be reused.

    If scheduleDAG is used as a prepass code scheduler, how should it be modified so that the number of pseudo-registers in use is a criterion in selecting the next instruction to schedule? That is, scheduling an instruction that increases the number of registers that will be needed should be discouraged unless it serves to avoid stalls in the code schedule.

19. It is sometimes the case that we need to schedule a small block of code that forms the body of a frequently executed loop. For example

```
for (i=2; a <1000000; i++)
    a[i] = a[i-1]*a[i-2]/1000.0;
```

Operations like floating point multiplication and division sometimes have significant delays (5 or more cycles). If a loop body is small, code scheduling cannot do much; there are not enough instructions to cover all the delays. In such a situation **loop unrolling** may help. The body of loop is replicated *n* times, with loop indices and loop limits suitably modified. For example, with *n* = 2, the above loop would become

```
for (i=2; a <999999; i+=2){
    a[i] =    a[i-1]*a[i-2]/1000.0;
    a[i+1] =  a[i]*a[i-1]/1000.0;}
```

A larger loop body gives a code scheduler more instructions that can be placed after instructions that may stall. How can we determine the value of *n* (the **loop unrolling factor**) necessary to cover all (or most) of the delays in a loop body? What factors limit how large *n* (or an unrolled loop body) should be allowed to become?

20. The scheduleDAG code scheduler is very optimistic with respect to loads. It schedules them assuming that they *always hit* in the primary cache. Real loads are not always so cooperative. Assume we can identify load instructions most likely to miss. How should scheduleDAG be modified to use "probability of cache miss" information in scheduling instructions?

21. Assume we extend the IR tree patterns defined in Figure 13.27 with the patterns for the MIPS add and load-immediate instructions shown in Figure 13.35.

Show how the IR tree of Figure 13.36, corresponding to

```
A[i+1] = (1+i)*1000,
```

would be matched. What MIPS instructions would be generated?

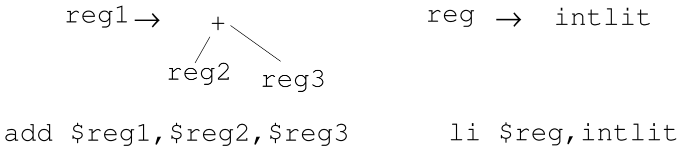


Figure 13.35: MIPS Instruction Patterns

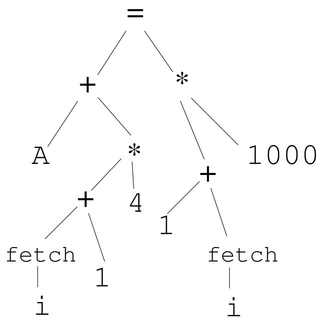


Figure 13.36: IR tree corresponding to `A[i+1] = (1+i)*1000`

22. Code generators that use IR tree pattern matching still have the problem of allocating registers for the generated code. Suggest how an on-the-fly register allocator can be integrated with pattern matching to form a complete code generator.

23. Instructions like the MIPS load-immediate instruction are complicated by the fact that the immediate operand may be too big to fit in a single instruction. In fact immediate operands that are too big force *two* instructions to be generated: a load-upper-immediate instruction that fills in the upper half of a word followed by an or-immediate instruction that fills in the lower half of a word.

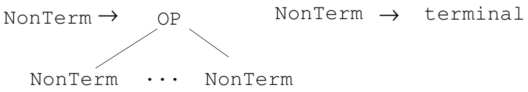


Figure 13.37: Simple Tree Patterns

How can costs and IR tree patterns be used to specify to an instruction selector that two alternative translations are possible depending on the size of an immediate operand?

24. Assume we have tree-structured instruction patterns limited to the two forms shown in Figure 13.37.

    That is, a nonterminal may generate a single terminal symbol or it may generate an operator, all of whose children are nonterminals.

    Give an algorithm that can walk any IR tree and determine whether it can be covered (matched) using a set of productions limited to the two forms described above.

25. Assume that we now add cost values (integer literals greater than or equal to 0) to instruction patterns limited to the two forms described in Exercise 24. Extend the algorithm you proposed in Exercise 24 so that it now finds a **least-cost cover**. That is, your algorithm should choose productions that minimize the overall cost of matching a given IR tree.

26. The following instruction sequence often appears in Java programs:

    ```
    a[i] = ...
    ... = a[i];
    ```

    That is, an element of an array is stored, then that same element is immediately reused. Suggest a peephole optimization rule, at the bytecode level, that would recognize this situation and optimize it using the dup bytecode.

27. Machines like the MIPS and Sparc have **delayed branch instructions**. That is, the instruction immediately following a branch is executed *prior* to transferring control to the target of the branch.

    Often, compilers simply generate a `nop` instruction after a branch, effectively hiding the effects of the delayed branch. Suggest a peephole optimization pattern for unconditional branches followed by a `nop` that swaps the instruction prior to the branch into the "delay slot" that follows it. Can this optimization always be done, or must some conditions be met to make the swap valid?

    Now consider a delayed *conditional* branch in which the value of a register is tested. If the condition is met, the instruction following the conditional branch is executed, and then the branch is taken. Otherwise, instructions following the conditional branch are executed (as usual); no branch is taken. Suggest a peephole optimization pattern that allows the instruction preceding a conditional branch to be moved after it as long as the swapped instruction does not affect the register tested by the conditional branch.

28. Many architectures include a *load-negative* instruction that loads the negation of a value into a register. That is, the value, while being loaded, is subtracted from zero, with the difference stored into the register. Suggest two instruction-level peephole optimization patterns that can make use of a load-negative instruction.

29. After a peephole optimization is performed, the optimized instruction that is substituted for the original instructions is reconsidered for further peephole optimizations. Give at least three examples of cases in which peephole optimizations may be profitably cascaded.

30. Assume we have a peephole optimizer that has n replacement patterns. The most obvious approach to implementing such an optimizer is to try each pattern in turn, leading to an optimizer whose speed is proportional to n.

    Suggest an alternative implementation, based on hashing, that is largely independent of n. That is, the number of patterns considered may be doubled without automatically doubling the optimizer's execution time.

31. The **Sethi-Ullman numbering** algorithm presented in Section 13.2 assumed that all expression tree nodes are *binary*. Develop a generalization of the Sethi-Ullman numbering algorithm given in Figure 13.9 that can be applied to trees whose nodes have an arbitrary number of children.

14

Program Optimization

This book has so far discussed the analysis and synthesis required to translate a programming language into interpretable or executable code. The analysis is concerned with making sure that the source program conforms to the definition of the programming language in which the program is written. After the compiler has verified that the source program conforms, synthesis takes over to translate the program. The target of this translation is typically an interpretable or executable instruction set. Thus, code generation consists of translating a portion of a program into a sequence of instructions that preserves the program's meaning.

As is true for most languages, there are multiple ways to say the same thing. In Chapter 13, **instruction selection** is presented as a mechanism for choosing an efficient sequence of instructions for the target machine. In this chapter, we examine more aggressive techniques for improving a program's performance. Section 14.1 introduces program optimization—its role in a compiler, its organization, and its potential for improving program performance. Section 14.2 covers some fundamental data structures and algorithms for representing a program's **control flow**. Section 14.3 presents **data flow analysis**—a technique for determining useful properties of a program at compile-time. The rest of this chapter considers advanced analysis and optimization techniques.

```
procedure MAIN( )
    /*  $A$, $B$, and $C$ are declared as $N \times N$ matrices              */
    $A = B \times C$
end

function $\times(Y, Z)$ returns $Matrix$
    if $Y.cols \neq Z.rows$                                                  ①
    then  /* Throw an exception */
    else
        for $i = 1$ to $Y.rows$ do
            for $j = 1$ to $Z.cols$ do
                $Result[i, j] \leftarrow 0$
                for $k = 1$ to $Y.cols$ do
                    $Result[i, j] \leftarrow Result[i, j] + Y[i, k] \times Z[k, j]$
    return ($Result$)
end

procedure $=(To, From)$
    if $To.cols \neq From.cols$ or $To.rows \neq From.rows$                  ②
    then  /* Throw an exception */
    else
        for $i = 1$ to $To.rows$ do
            for $j = 1$ to $To.cols$ do
                $To[i, j] \leftarrow From[i, j]$
end
```

Figure 14.1: Matrix multiplication using overloaded operators.

14.1 Overview

When compilers were first pioneered, they were considered successful if programs written in a high-level language attained performance that rivaled hand-coded efforts. By today's standards, the programming languages of those days may seem primitive. However, the technology that achieved the requisite performance is very impressive—these techniques are still used in compilers for modern programming languages. The scale of today's software projects would be impossible without the advent of advanced programming paradigms and languages. As a result, the goal of hand-coded performance has yielded to the goal of obtaining a *reasonable* fraction of a target machine's potential speed.

Meanwhile, the trend in **reduced instruction set computer** (RISC) architectures continues toward relatively low-level instruction sets. Such architectures feature shorter instruction times with correspondingly faster clock rates. Other developments include **liquid architectures**, whose operations, registers, and data paths can be reconfigured. Special-purpose instructions have also been introduced, such as the MMX instructions for Intel machines. These instructions facilitate graphics and numerical computations. Such architectural innovations cannot succeed unless compilers can make effective use of both RISC and the more specialized instructions. Thus, collaboration between computer architects, language designers, and compiler writers continues to be strong.

The programming language community has defined the **semantic gap** as a (subjective) measure of the distance between a compiler's source and target languages. As this gap continues to widen, the compiler community is challenged to build efficient bridges. These challenges come from many sources. Examples include object-orientation, mobile code, active network components, and distributed object systems. Compilers must produce excellent code quickly, quietly, and—of course—correctly.

14.1.1 Why Optimize?

Although its given name is a misnomer, it is certainly the goal of program optimization to improve a program's performance. Truly *optimal* performance cannot be achieved automatically, as the task subsumes problems that are known to be **undecidable** [Mar03]. It can be proven that there is no algorithm that can handle all instances of an undecidable problem. The main areas in which program optimizers strive for improvement are discussed in this section, beginning with the example shown in Figure 14.1. In this program, variables A, B, and C are of type *Matrix*. For simplicity, we assume all matrices are of size $N \times N$. The × and = operators are overloaded as shown to perform matrix multiplication and assignment, respectively, using the function and procedure provided in Figure 14.1.

High-Level Language Features

High-level languages contain features that offer flexibility and generality at the cost of some runtime efficiency. Optimizing compilers attempt to recover performance for such features in the following ways:

- Perhaps it can be shown that a given feature is not used by some portion of a program.

  In the example of Figure 14.1, suppose that the *Matrix* type is extended to *SymMatrix*—with definitions of the *Matrix* methods optimized for symmetric matrices. If A and B are actually of type *SymMatrix*, then languages that offer **virtual function dispatch** are obligated to call the most specialized method for an object's actual type. However, if the compiler can determine that × and = are not redefined in any subclass

```
for $i = 1$ to $N$ do                                                    ③
    for $j = 1$ to $N$ do                                                ④
        $Result[i, j] \leftarrow 0$
        for $k = 1$ to $N$ do
            $Result[i, j] \leftarrow Result[i, j] + B[i, k] \times C[k, j]$    ⑤
for $i = 1$ to $N$ do                                                    ⑥
    for $j = 1$ to $N$ do
        $A[i, j] \leftarrow Result[i, j]$
```

Figure 14.2: Inlining the overloaded operators.

of *Matrix*, then the result of a virtual function dispatch on these methods may be predictable at compile time.

Based on such analysis, **method inlining** expands the method definitions in Figure 14.1 at their call sites, substituting the appropriate parameter values. As shown in Figure 14.2, the resulting program avoids the overhead of function calls. Moreover, the code is now specially tailored for its arguments, whose row and column sizes are N. Program optimization can then eliminate the tests at Markers ① and ②.

- Perhaps it can be shown that a language-mandated operation is not necessary. For example, Java$^{\text{TM}}$ insists on subscript checks for array references and type-checks for **narrowing casts**. Such checks are unnecessary if an optimizing compiler can determine the outcome of the result at compile-time. When code is generated for Figure 14.2, **induction variable analysis** can show that i, j, and k all stay within the declared range of matrices A, B, and C. Thus, subscript tests can be eliminated for those arrays.

  Even if the outcome is uncertain at compile-time, a test can be eliminated if its outcome is already computed. Suppose the compiler is required to check the subscript expressions for the *Result* matrix at Marker ⑤. Most likely, the code generator would insert a test for each reference of *Result*$[i, j]$. An optimization pass could determine that the second test is redundant.

Modern software-construction practices dictate that large software systems should be comprised of small, easily written, readily reusable components. As a result, the size of a compiler's **compilation unit**—the text directly presented in one run of the compiler—has has been steadily dwindling. In response, optimizing compilers consider the problem of **whole-program optimization** (WPO), which requires analyzing the interaction of a program's compilation units. Method inlining (shown in Figure 14.2) is one example of

```
for $i = 1$ to $N$ do
    for $j = 1$ to $N$ do
        $A[i, j] \leftarrow 0$
        for $k = 1$ to $N$ do                                          ⑦
            $A[i, j] \leftarrow A[i, j] + B[i, k] \times C[k, j]$      ⑧
```

Figure 14.3: Fusing the loop nests.

the benefits of WPO. Even if a method cannot be inlined, WPO can generate a version of the invoked method that is customized to its calling context. In other words, the trend toward reusable code can result in systems that are general but not as efficient as they could be. Optimizing compilers try to regain some of the lost performance.

Target-Specific Optimization

Portability ranks high among the goals of today's programming languages. Ideally, once a program is written in a high-level language, it should be movable without modification to any computing system that supports the language. Architected interpretive languages such as the **Java Virtual Machine** (JVM) support portability nicely. Any computer that hosts a JVM interpreter can run any Java program. But the question remains—how fast? Although most modern computers are based on RISC principles, the details of their instruction sets vary widely. Moreover, various models for a given architecture can also differ with regard to their storage hierarchies, their instruction timings, and their degree of concurrency.

Continuing with our example, the program shown in Figure 14.2 is an improvement over the version shown in Figure 14.1, but it is possible to obtain still better performance. Consider the behavior of the matrix *Result*. The values of *Result* are computed in the loop nest at Markers ③ and ④—one element at a time. Each of these elements is then copied from *Result* to *A* by the loop nest at Marker ⑥. Poor performance can be expected on any **non-uniform memory access** (NUMA) system that cannot accommodate *Result* at its fastest storage level. Better performance can be obtained if the data is accumulated in a register and then stored directly in *A*. Optimizing compilers that feature **loop fusion** can identify that the outer two loop nests at Markers ③ and ⑥ are structurally equivalent. **Dependence analysis** can show that each element of *Result* is computed independently. The loops can then be fused to obtain the program shown in Figure 14.3 in which the *Result* matrix is eliminated.

```
$r_i \leftarrow 1$
while $r_i \leq N$ do
    $r_j \leftarrow 1$
    while $r_j \leq N$ do
        $r_A \leftarrow \star\,(Addr(A) + (((r_i - 1) \times N + (r_j - 1))) \times 4)$
        $\star(r_a) \leftarrow 0$
        $r_k \leftarrow 1$
        $r_{sum} \leftarrow -$
        while $r_k \leq N$ do
            $r_A \leftarrow \star\,(Addr(A) + (((r_i - 1) \times N + (r_j - 1))) \times 4)$        ⑨
            $r_B \leftarrow \star\,(Addr(B) + (((r_i - 1) \times N + (r_k - 1))) \times 4)$
            $r_C \leftarrow \star\,(Addr(C) + (((r_k - 1) \times N + (r_j - 1))) \times 4)$
            $r_{sum} \leftarrow r_A$
            $r_{prod} \leftarrow r_B \times r_C$
            $r_{sum} \leftarrow r_{sum} + r_{prod}$
            $r_A \leftarrow \star\,(Addr(A) + (((i - 1) \times N + (j - 1))) \times 4)$        ⑩
            $\star(r_a) \leftarrow r_{sum}$
            $r_k \leftarrow r_k + 1$
        $k \leftarrow r_k$                                                                   ⑪
        $r_j \leftarrow r_j + 1$
    $j \leftarrow r_j$                                                                       ⑫
    $r_i \leftarrow r_i + 1$
$i \leftarrow r_i$                                                                           ⑬
```

Figure 14.4: Low-level code sequence. The $\star$ operator denotes *indirection*: $\star x$ causes its operand x to be evaluated and then treated as an address.

Artifacts of Program Translation

In the process of translating a program from a source to a target language, compilers can introduce spurious computations. As discussed in Chapter 13, compilers try to keep frequently accessed variables in fast registers. For example, it is likely that the iteration variables in Figure 14.3 are kept in registers. Figure 14.4 shows the results of straightforward code generation for the loops in Figure 14.3.

The loops contain instructions at Markers ⑪, ⑫, and ⑬ that store the iteration variable register in the named variable. However, this particular program does not require a value for the iteration variables when the loops are finished. Thus, such stores are unnecessary. In Chapter 13, register allocation can avoid such saves if the iteration variable can be allocated a register without spilling.

Because code generation is mechanically invoked for each program ele-

```
$FourN \leftarrow 4 \times N$
for $i = 1$ to $N$ do
    for $j = 1$ to $N$ do
        $a \leftarrow \&(A[i, j])$
        $b \leftarrow \&(B[i, 1])$
        $c \leftarrow \&(C[1, j])$
        while $b < \&(B[i, 1]) + FourN$ do
            $\star a \leftarrow \star a + \star b \times \star c$
            $b \leftarrow b + 4$
            $c \leftarrow c + FourN$
```

Figure 14.5: Optimized matrix multiply.

ment, it is easy to generate redundant computations. For example, Markers ⑨ and ⑩ compute the address of $A[i, j]$. Only one such computation is necessary.

Conceivably, the code generator could be written to account for these conditions as it generates code. Superficially, this may seem desirable, but modern compiler design practices dictate otherwise:

- Such concerns can greatly complicate the task of writing the code generator. Issues such as instruction selection and register allocation are the primary concerns of the code generator. Generally, it is wiser to craft a compiler by combining simple, single-purpose transformations that are easily understood, programmed, and maintained. Each such transformation is often called a **pass** of the compiler.

- There are typically many portions of a compiler that can generate superfluous code. It is more efficient to write one compiler pass that is dedicated to the removal of unnecessary computations than to duplicate that functionality throughout the compiler.

For example, **dead code elimination** and **unreachable code elimination** remove unnecessary computations. Most compilers include these passes, if only to clean up after themselves.

Continuing with our example, consider code generation for Figure 14.3. Let us assume that each array element occupies 4 bytes and that subscripts are specified in the range of 1..N. The code for indexing the array element $A[i, j]$ becomes

$$Addr(A) + (((i - 1) \times N + (j - 1))) \times 4$$

which takes

4	integer "+" and "−"
2	integer "×"
6	integer operations

Since Marker ⑧ has 4 such array references, each execution of this statement takes

16	integer "+" and "−"
8	integer "×"
3	loads
1	floating "+"
1	floating "×"
1	store
30	instructions

Thus, the loop contains 2 floating-point instructions and 24 fixed-point instructions. On superscalar architectures that support concurrent fixed- and floating-point instructions, this loop can pose a severe bottleneck for the fixed-point unit.

The computation can be greatly improved by the following optimizations that are available in most compilers:

- **Loop-invariant detection** can determine that the (address) expression $A[i, j]$ does not change at Marker ⑧.
- **Reduction in strength** can replace the address computations for the matrices with simple increments of an index variable. The iteration variables themselves can disappear, with loop termination based on the subscript addresses.

The result of applying these optimizations on the innermost loop is shown in Figure 14.5. The inner loop now contains 2 floating-point and 2 fixed-point operations—a balance that greatly improves the loop's performance on modern processors.

A Series of Small Passes

It is expedient to organize an optimizing compiler as a series of *passes*. Each pass should have a narrowly defined goal, so that its scope is easily understood. This facilitates development of each pass as well as the integration of the separate passes into an optimizing compiler. For example, it should be possible to run the **dead code elimination** pass at any point to remove useless code. Certainly this pass would be run prior to code generation. However, the effectiveness of an individual pass as well as the overall speed of the optimizing compiler can be improved by sweeping away unnecessary code.

To facilitate development and interoperability, it is useful for each pass to accept and produce programs in the **intermediate language** (IL) of the compiler, as shown in Figure 10.3 on page 396.

Unfortunately, the passes of a compiler can interact in subtle ways. For example, **code motion** can rearrange a program's computations to make better use of a given architecture. However, as the distance between a variable's definition and its uses increases, so does the pressure on the **register allocator**. Thus, it is often the case that a compiler's optimizations are somewhat at odds with each other. Ideally, the passes of an optimizing compiler should be fairly independent, so that they can be reordered and retried as situations require [CGH$^+$05].

14.2 Control Flow Analysis

This section describes structures that represent a program's flow of control. A flow graph is essentially a finite-state automaton whose nodes represent various locations in a program and whose edges indicate possible transition among these locations. As a system of highways interconnects cities on a map, so do flow graphs serve as a roadmap for optimization. The nodes of a flow graph represent sites where optimization information can be generated or consumed; the graph's edges are conduits for combining and further propagating such information.

> **Definition 14.1** *A **flow graph** $\mathcal{G} = (\mathcal{N}, \mathcal{E}, root)$ is a directed graph: $\mathcal{N}$ is a set (of nodes) and $\mathcal{E}$ is a binary relation on $\mathcal{N}$. The root node is the distinguished entry node of the flow graph: $\forall\ X \in \mathcal{N}, (X, root) \notin \mathcal{E}$.*

Although flow graphs are quite general, we are concerned with representing the following levels of program behavior:

- A **control flow graph** $\mathcal{G}_{cf}$ represents potential execution paths within a procedure. Generally, each node in $\mathcal{N}_{cf}$ corresponds to a *straight-line* sequence of operations; each edge in $\mathcal{E}_{cf}$ represents potential transfer of execution from the end of one sequence to the beginning of another. Control flow graphs are used in **intraprocedural analyses**.

- A **procedure call graph** $\mathcal{G}_{pc}$ represents potential execution paths between the procedures of a program. Each node in $\mathcal{N}_{pc}$ corresponds to a procedure of the program; each edge in $\mathcal{E}_{pc}$ represents a potential procedure call. Procedure call graphs are used in **interprocedural analyses**.

The diversity of programming constructs such as loops, alternate entry points, procedure exits, recursion, and arbitrary `goto` statements could conceivably yield flow graphs of no discernible structure. However, trends in programming languages and software development practices tend to yield programs whose structure is relatively clear and whose flow graphs enjoy properties conducive to efficient analysis.

In practice, the behavior of a program is captured by multiple flow graphs, each of differing granularity, according to the expected benefits of optimization at various levels. For example, each node of the procedure call graph is a procedure that is itself represented by an intraprocedural control flow graph. Theory and practice alike suggest the benefits of this organization, and a compiler designer must consider the expense and expected benefits of representing program behavior at a given level.

14.2.1 Control Flow Graphs

From an optimization point of view, the nodes of a control flow graph represent a *sequence* of operations. Efficiency considerations aside, a sequence could correspond to a single machine instruction or to an entire program, because at some level, each can be viewed as a sequence of instructions whose execution modifies the state of a computation. However, the choice of node composition deserves careful attention, because the chosen level of representation substantially affects the precision and efficiency of analysis and optimization.

Suppose a variable x is always assigned the constant value 2 or 3 within a program. If the behavior of the entire program is represented by a single flow graph node, then x is not constant with respect to this level of representation. On the other hand, if a node represents a single machine instruction, then there are many more points at which a variable could be constant. However, such fine granularity is inefficient for most programs. In our example, x most likely retains the same value for a large number of instructions.

Thus, the representation level must be chosen with care: sufficiently fine to represent useful information yet sufficiently coarse to avoid excessive redundancy. Some popular approaches are as follows:

- Programmers tend to construct procedures whose statements accomplish meaningful changes in program state. Thus, a common strategy is to associate a node with each statement. Often, extra nodes are introduced to place the flow graph in a canonical form (see Figure 14.40 on page 597).

- Another approach especially suitable for language-independent optimization is to associate a node with each statement or instruction of the compiler's intermediate language (see Chapter 10).

- If a given level of granularity is too fine, then instructions can be grouped into maximal straight-line sequences, or **basic blocks**. A node then represents the longest sequence that is entered only at its first operation and terminates after any operation with more than one successor. Thus, in any trace of a program's execution, no instruction from a basic block can appear unless all instructions of that block appear, and the instructions of a basic block always appear in the same order.

```
procedure FOO$(d,x,y,z,f,g,h,c)$
    if $d = 15$
    then
        $x \leftarrow 0$
        $y \leftarrow x + y * z$
        call BAR$(x,y)$
        $z \leftarrow 2$
        if $f * g + h = 12$
        then
            $y \leftarrow 3$
            $x \leftarrow 4$
    else
        $z \leftarrow 5$
        while $c < 12$ do
            $x \leftarrow 6$
            $y \leftarrow 7$
        $y \leftarrow 8$
end
```

(a)

```
        procedure FOO$(d,x,y,z,f,g,h,c)$
            if $(d \neq 15)$ then goto L2        ⑭
            $x \leftarrow 0$                     ⑮
            $t1 \leftarrow y * z$
            $y \leftarrow x + t1$
            call BAR$(x,y)$
            $z \leftarrow 2$
            $t2 \leftarrow f * g$
            $t3 \leftarrow t2 + h$
            if $(t3 \neq 12)$ then goto L1
            $y \leftarrow 3$                     ⑯
            $x \leftarrow 4$
L1:         goto L5                              ⑰
L2:         $z \leftarrow 5$                     ⑱
L3:         if $(c \geq 12)$ then goto L4        ⑲
            $x \leftarrow 6$                     ⑳
            $y \leftarrow 7$
            goto L3
L4:         $y \leftarrow 8$                     ㉑
            goto L5
L5:     end                                      ㉒
```

(b)

Figure 14.6: (a) Source program; (b) Intermediate text and basic block decomposition of the source program.

Figure 14.6 shows a program and the basic block partition of its intermediate code. Each of the statements found at Marker ⑭ through Marker ㉒ begins a new basic block. Notice that the block at Marker ⑮ contains a procedure call. At this level of analysis, control flow within the procedure BAR is obscured, so the procedure call does not interrupt the basic block.

Although organization of statements into basic blocks may improve space efficiency, some reasons for avoiding basic blocks are as follows:

- With respect to large address spaces, sparse flow graphs consume very little space, even where program behavior is modeled at a relatively fine level.

- There is almost no time saved in organizing instructions into basic blocks, especially if these blocks must be “opened up” each time a node is visited.

- There are two levels of data flow analysis typically associated with a graph comprised of basic blocks: *local* data flow analysis establishes

```
procedure formBasicBlocks( )
    $leaders \leftarrow \{$ first instruction in stream $\}$
    foreach instruction $s$ in the stream do                          ㉓
        $targets \leftarrow \{$ distinct targets branched to from $s\}$      ㉔
        if $|targets| > 1$
        then
            foreach $t \in targets$ do  $leaders \leftarrow leaders \cup \{t\}$
    foreach $l \in leaders$ do                                         ㉕
        $block(l) \leftarrow \{l\}$
        $s \leftarrow$ next instruction after $l$
        while $s \notin leaders$ and $s \neq \perp$ do
            $block(l) \leftarrow Block(l) \cup \{s\}$
            $s \leftarrow$ Next instruction in stream
end
```

Figure 14.7: Partitioning of instructions into basic blocks. The ⊥ value in pseudocode means *undefined* and is typically denoted as **null** in most programming languages.

> solutions within a basic block, and *global* data flow analysis solves a problem over the flow graph. In terms of software development and maintenance, two levels of analysis are at least twice as costly as one level. A more general solution would allow arbitrary levels of analysis, but typical programs do not require such sophistication. In practice, a single, wisely chosen level of representation suffices for most applications.

Basic blocks can be constructed on-the-fly as intermediate code is generated. Figure 14.7 contains a two-pass algorithm that partitions a flat (nonstructured) stream of instructions into basic blocks. Marker ㉓ considers each instruction in the stream in turn, adding to *leaders* those instructions that begin a new basic block. Note that nonbranching and conditionally branching statements can implicitly branch to the next instruction in the stream. Also, branching instructions may repeatedly target the same instruction. To avoid spurious basic blocks, Marker ㉔ must find the number of *distinct* successors of instruction *s*. Marker ㉕ performs a second pass, creating a basic block from each leader and its ensuing instructions, up to the next statement that begins a new basic block. If the instruction stream is too large to store conveniently, then the *leaders* set can be organized (e.g., sorted) so that random access to instructions is unnecessary. Alternatively, the instruction stream can be considered in fixed-sized segments, where the first statement in each segment is a leader. Although this approach may introduce spurious basic blocks, these affect only the space efficiency of the representation, not the accuracy or overall time.

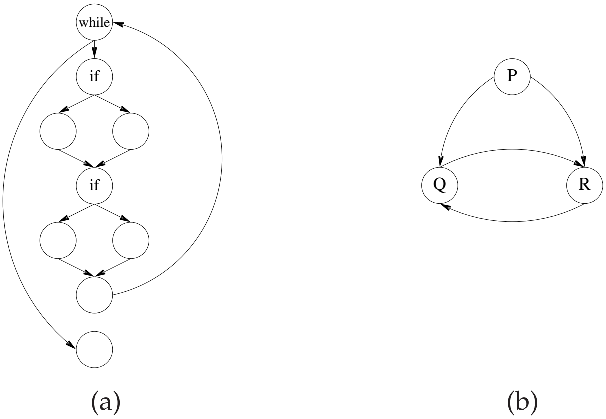


Figure 14.8: (a) Structured control flow graph; (b) Canonical irreducible graph.

Having shown how to construct basic blocks, and having stated reasons for avoiding their construction, we assume from this point forward that a flow graph node corresponds to a single statement of the input language.

14.2.2 Program and Control Flow Structure

We next describe language constructs that always result in control flow graphs with certain desirable properties. A more general treatment of this subject is possible after interval analysis (Section 14.2.9) has been discussed.

If a procedure's loops are written using `while-do` and `repeat-until` constructs, then entry to any loop in a procedure occurs through exactly one node called the **header node**. Control flow graphs with this property are called **reducible**, and such graphs are amenable to the exhaustive and incremental **interval!analysis** techniques described by Burke [Bur90]. The canonical example of an **irreducible flow graph** is shown in Figure 14.8(b).

If branching within a procedure is specified using only `if-then-else`, `while-do`, and `repeat-until` constructs, then the resulting control flow graphs are **structured**. An example is shown in Figure 14.8(a). Structured programs are generally associated with clear programming style, because the control flow within such programs is readily apparent by inspection. Not surprisingly, analysis and optimization for structured control flow graphs can be simpler than for nonstructured graphs.

14.2.3 Direct Procedure Call Graphs

This section describes the program call graph $\mathcal{G}_{pc}$, which is the interprocedural counterpart of the control flow graph. In general, construction of an accurate $\mathcal{G}_{pc}$ can be considerably more difficult than construction of a control flow graph. In some languages, methods are dispatched **virtually**, which can make it difficult to decide which method is actually called. For languages with higher-order functions or procedure variables, sophisticated data flow techniques are required even to *approximate* the procedure call graph [GC01]. Here, we limit our discussion to construction of *direct* procedure call graphs, which are useful for interprocedural analysis and straightforward to compute.

Each procedure corresponds to a node of $\mathcal{G}_{pc}$. An edge is placed between nodes P and Q if P can invoke Q by its name (i.e., procedure variables are not allowed). Any sequence of procedure invocations (without return) has a corresponding path in this flow graph. Because a flow graph can only simulate finite-state behavior, we cannot expect $\mathcal{G}_{pc}$ to model the stack-like behavior of procedure calls.

Although the graph of Figure 14.8(b) reflects how procedures might *call* each other, returns from procedure calls at not explicitly represented. For example, the graph does not show any transfer of control from R to P when R returns from a call by P. A *supergraph* [Mye81] could be created with an edge from R and Q to P, showing the behavior of `return` statements. However, such a graph cannot easily distinguish between P and R calling each other recursively and the situation where R returns control to P.

Optimizations and data flow analyses can be formulated over control flow graphs and procedure call graphs alike. Although the techniques for describing and solving these problems are similar, we expect more accurate solutions for intraprocedural problems because intraprocedural control flow analysis is generally more accurate.

14.2.4 Depth-First Spanning Tree

One abstraction of a flow graph is its **depth-first spanning tree** (DFST), which is useful for computing the dominator tree (Section 14.2.5) and interval partition (Section 14.2.9) of a flow graph. The DFST of a flow graph contains all of the flow graph's nodes and just enough of the flow graph's edges to constitute a tree.

The depth-first search algorithm shown in Figure 14.9 builds the DFST of a flow graph while computing its depth-first numbering. The algorithm accepts a flow graph $\mathcal{G}_f = (\mathcal{N}_f, \mathcal{E}_f)$ and takes $O(\mathcal{E}_f)$ time and $O(\mathcal{N}_f)$ space to produce the following data structures:

```
/* The variable $num$ is global between DFST and DFS. */
procedure DFST($\mathcal{G}_f$)
    $num \leftarrow 0$
    foreach $Z \in \mathcal{N}_f$ do
        $child(Z) \leftarrow$ null
        $dfn(Z) \leftarrow 0$
    $parent(root) \leftarrow$ null
    call DFS($root$)
    $NumNodes \leftarrow num$
end

procedure DFS($X$)
    $num \leftarrow num + 1$
    $dfn(X) \leftarrow num$
    $vertex(num) \leftarrow X$
    foreach $Y \in Succ(X)$ do                                  26
        if $dfn(Y) = 0$
        then
            $parent(Y) \leftarrow X$
            $sibling(Y) \leftarrow child(X)$                    27
            $child(X) \leftarrow Y$                             28
            call DFS($Y$)
    $progeny(X) \leftarrow num - dfn(X)$
end
```

Figure 14.9: Depth-first numbering and spanning tree. The algorithm's input is a flow graph $\mathcal{G}_f = (\mathcal{N}_f, \mathcal{E}_f, root)$.

Depth-first spanning tree

parent(*Z*)	parent of node *Z*
child(*Y*)	head of the list of *Y*'s children
sibling(*Z*)	next node in the list of *parent*(*Z*)'s children

Depth-first numbering

dfn(*Z*)	the depth-first number associated with node *Z*
vertex(*n*)	the node with depth-first number *n*
progeny(*Z*)	the number of proper descendants of node *Z* in the DFST
NumNodes	the number of nodes in $\mathcal{N}_f$ that are reachable from *root*

These data structures use a constant number of references per node to represent a tree, as described in Section 7.4.2 on page 251. Nodes in the flow graph shown in Figure 14.10 are already labeled with their depth-first numbering. The DFST

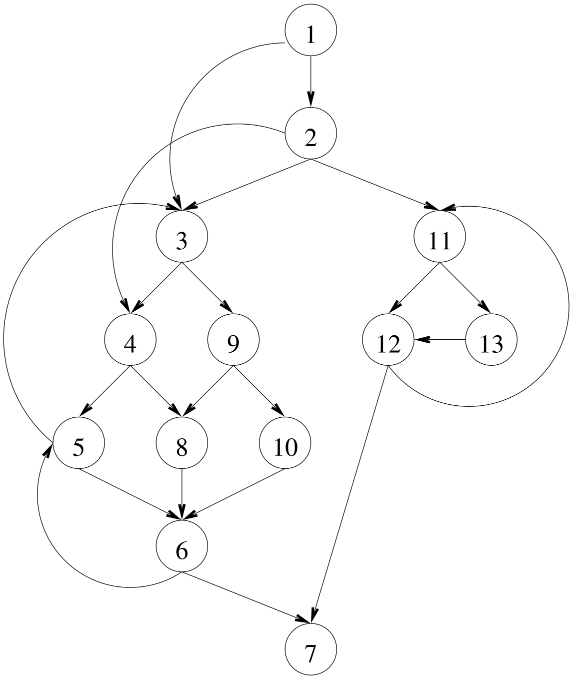


Figure 14.10: Sample flow graph.

computed for that graph is shown in in Figure 14.12.

The algorithm does not consider successors of node X in any particular order at Marker ㉖. A graph's DFST is therefore not necessarily unique. It is convenient to depict a DFST so that its children are drawn left-to-right in the order of their discovery by depth-first search, as shown in Figure 14.12. However, the algorithm collects the children of node X by insertion at the *head* of a linked list (Markers ㉗ and ㉘). The resulting list therefore contains children in the *reverse* order of their discovery:

- The last child of X discovered by depth-first search will appear as $child(X)$.
- The child of X discovered just before node Y will appear as $sibling(Y)$.

The traversal shown in Figure 14.11 therefore has the effect of traversing the DFST in **right-to-left preorder**, which is an order conducive to evaluation of data flow framework as discussed in Section 14.5.

A DFST is constructed using some of a graph's edges. For those graph edges not participating in the tree, some algorithms make use of their relationship to the structure of the DFST. Figure 14.13 shows all edges of Figure 14.10

```
procedure rightToLeftTraversal( root )
    call visitNode( root )
end

procedure visitNode( n )
    c ← child(n)
    while c ≠ null do
        /★   Preorder code goes here                    ★/ (29)
        call visitNode( c )
        /★   Postorder code goes here                   ★/ (30)
        c ← sibling(c)
end
```

Figure 14.11: Right-to-left DFST traversal.

superimposed on the tree of Figure 14.12. Recall that the words *ancestor*, *descendant*, and *child* refer to nodes in *tree* data structure (such as a DFST), while the words *successor* and *predecessor* refer to nodes of a *graph*. Each edge in $\mathcal{E}_f$ can then be uniquely described with respect to a given DFST for $\mathcal{G}_f$ as follows:

Tree: A **tree edge** appears in both $\mathcal{E}_f$ and the given DFST for $\mathcal{G}_f$. In Figure 14.13, the tree edges are shown with normal, solid lines. For example, 4 → 8 is a tree edge.

Back: A **back edge** connects a node Y with an ancestor X of Y. In Figure 14.13, the back edges are shown in bold: 6 → 5, 5 → 3, and 12 → 11.

Chord: A **chord edge** connects a node X with a proper descendant of X. In Figure 14.13, the dotted edges 2 → 4 and 1 → 3 are the only chord edges.

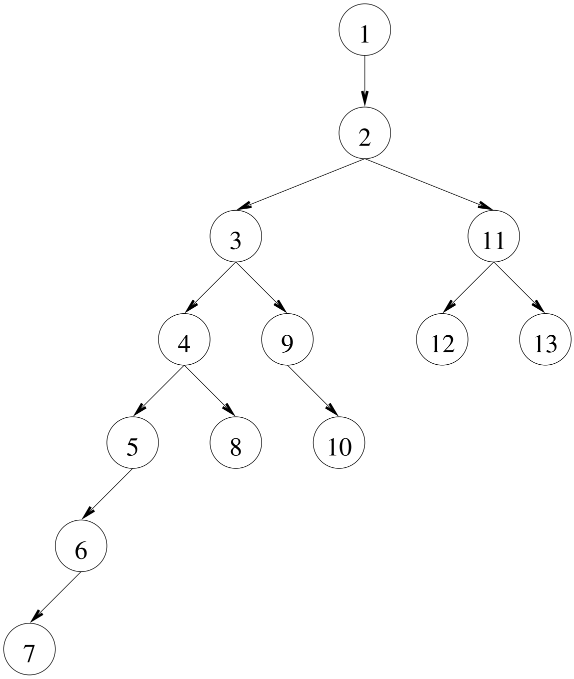
Cross: The remaining edges are **cross edges**. If a DFST is drawn so that a node's children appear left to right in order of their depth-first discovery, then a graph's cross edges are always oriented from right to left when superimposed on the DFST. In Figure 14.13, the cross edges are drawn with dashes: 8 → 6, 9 → 8, 10 → 6, 12 → 7, and 13 → 12.

The data structures computed by the algorithm of Figure 14.9 suffice to determine the type of any edge due to the following theorem:

Theorem 14.2 *Node Y is in the depth-first spanning subtree rooted at X, denoted $X \lhd Y$, if and only if*

$$dfn(X) \leq dfn(Y) \leq dfn(X) + progeny(X)$$

Proof: Left as Exercise 13. □



Y	*progeny*(Y)	*parent*(Y)	*child*(Y)	*sibling*(Y)
1	12	**null**	2	**null**
2	11	1	11	**null**
3	7	2	9	**null**
4	4	3	8	**null**
5	2	4	6	**null**
6	1	5	7	**null**
7	0	6	**null**	**null**
8	0	4	**null**	5
9	1	3	10	4
10	0	9	10	**null**
11	2	2	13	3
12	0	11	**null**	**null**
13	0	11	**null**	12

Figure 14.12: Depth-first spanning tree of the flow graph from Figure 14.10 and the tree's representation as a data structure.

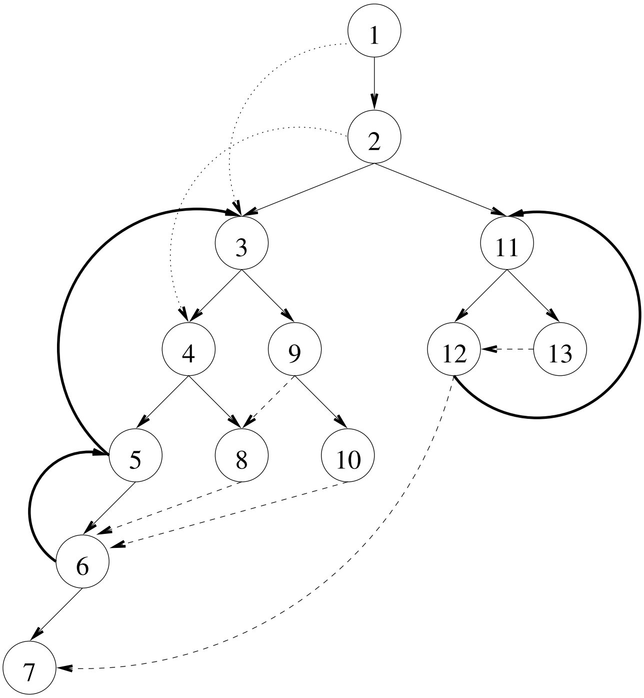


Figure 14.13: Superposition of edges from Figure 14.10 on the DFST of Figure 14.12. Back edges are bold, cross edges are dashed, and chord edges are dotted. All other edges are tree edges from the DFST.

Thus, once the depth-first numbering has been computed, a **constant-time test** can determine if X is a ancestor of Y. Figure 14.14 shows how to use the structures developed by the algorithm of Figure 14.9 to determine the type of an edge $X \rightarrow Y$ with respect to a DFST. The property satisfied by a flow graph edge or path is denoted below the edge or path symbol.

14.2.5 Dominance

Another useful set of abstractions for program analysis and optimization are the dominance data structures—in particular, the **dominator tree** of a flow graph. The dominators of node Z act like a sequence of gates in a flow graph: control flow can reach Z only by passing through Z's dominators. The various forms of dominance are defined with respect to a control flow graph $\mathcal{G}_f = (\mathcal{N}_f, \mathcal{E}_f, root)$ as follows:

Property	Test
$X \underset{tree}{\longrightarrow} Y$	$parent(Y) = X$
$X \underset{chord}{\longrightarrow} Y$	$dfn(X) < dfn(Y)$ and $parent(Y) \neq X$
$X \underset{back}{\longrightarrow} Y$	$Y \lhd X$
$X \underset{cross}{\longrightarrow} Y$	$dfn(X) \geq dfn(Y)$ and $Y \ntriangleleft X$

Figure 14.14: Determining the relationship of edge $X \rightarrow Y$.

Definition 14.3

- *Node Y* ***dominates*** *node Z, denoted* $Y \underline{\gg} Z$, *iff every path in* $\mathcal{G}_f$ *from root to Z includes node Y. A node always dominates itself.*
- *Node Y* ***strictly dominates*** *Z, denoted* $Y \gg Z$, *iff* $Y \underline{\gg} Z$ *and* $Y \neq Z$.
- *The* ***immediate dominator*** *of node Z, denoted idom(Z), is the closest strict dominator of Z:*

$$Y = idom(Z) \Longleftrightarrow (Y \gg Z \text{ and } \forall\, X \gg Z, X \underline{\gg} Y)$$

- *The* ***dominator tree*** *for* $\mathcal{G}_f$ *has nodes* $\mathcal{N}_f$*; Y is a parent of Z in this tree iff* $Y = idom(Z)$.

As a simple example, consider a sequence of nodes $X_1, X_2, \ldots, X_n$ where control flow enters the sequence only at X_1, exits only after X_n, and control transfers only from node X_i to node X_{i+1}. Such an example is shown in Figure 14.15(a) for $n = 3$. Recalling the discussion of Section 14.2.1, such a sequence is essentially a basic block and might be represented by a single node in the flow graph. However, if the nodes are distinct in the flow graph as shown in Figure 14.15(a), then each X_i dominates nodes in $\{ X_j \mid j \geq i \}$, strictly dominates nodes in $\{ X_j \mid j > i \}$, and immediately dominates node X_{i+1}.

As another example, consider the flow graph for an `if-then-else` statement as shown in Figure 14.15(b). In this graph, node A immediately dominates nodes B, C, and D. The control flow graph in Figure 14.15(c) models a loop where node F decides whether to continue or terminate the loop. Node E dominates all of the nodes; node F dominates nodes F, G, and H; nodes G and H dominate only themselves.

A visual interpretation of dominance is shown in Figure 14.16. Suppose the *root* node of a flow graph is a light source, and the edges of the flow graph

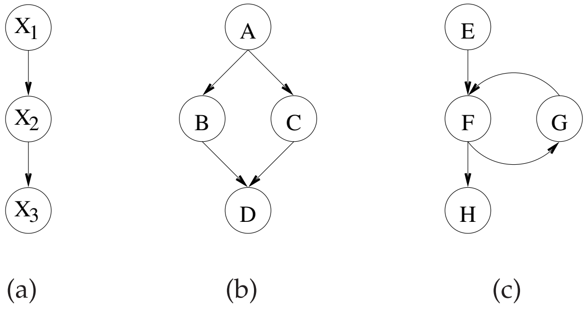


Figure 14.15: Flow graph examples for dominance.

act as fibers along which light can be transmitted. To find the nodes dominated by *X*, imagine that *X* is opaque: any light entering *X* is blocked and cannot be transmitted along edges leaving *X*. Nodes that fall in the resulting shadow cast by *X* are strictly dominated by *X*. With node 3 opaque in Figure 14.16, light cannot travel to nodes 9 or 10. Thus node 3 dominates nodes 3, 9, and 10. Light reaches node 4 from node 2, so 4 is not dominated by node 3.

14.2.6 Simple Dominance Algorithm

We first examine an algorithm that determines *all* dominators of each node in a flow graph $\mathcal{G}_f = (\mathcal{N}_f, \mathcal{E}_f)$. The algorithm is based on the observation that a node *Z* is dominated by itself and by any node that dominates *all* of *Z*'s predecessors. In other words, to reach *Z*, control flow must pass through one of *Z*'s flow graph predecessors. Any node *Y* that dominates each predecessor of *Z* must appear on every path from *root* to *Z*.

Equation 14.4 *The nodes that dominate Z can be determined as follows:*

$$dom(Z) = \{\, Z \,\} \cup \bigcap_{(Y,Z) \in \mathcal{E}_f} dom(Y)$$

Each node of the flow graph contributes such an equation, and we seek a solution that satisfies every equation while providing the best answer. This style of problem formulation is very similar to the formulation of the data flow problems we study in Section 14.4, so it is worthwhile to examine this method in some detail.

Consider applying Equation 14.4 to the graph shown in Figure 14.15(c). Applying Equation 14.4 to node *E* yields $dom(E) = \{\, E \,\}$. The equation written

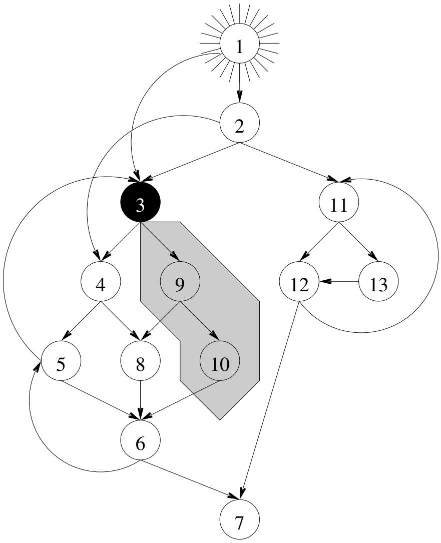


Figure 14.16: Visual interpretation of dominance. With node 1 as a light source, nodes 9 and 10 are in the shadow cast when node 3 is opaque. Node 3 therefore strictly dominates nodes 9 and 10.

for node F requires knowing the solution at node G:

$$dom(F) = \{F\} \cup (dom(E) \cap dom(G))$$

It is no easier to resolve node G since its solution depends on node F. Thus, before applying Equation 14.4 to node F, it seems we require an initial assumption about the solution at nodes such as G. A safe (but ultimately inferior) approach is to assume each solution is initially the empty set ($\emptyset$). From that assumption, the solutions develop as follows:

$$\begin{aligned}
dom(E) &= \{E\} \\
dom(F) &= \{F\} \cup (dom(E) \cap dom(G)) \\
&= \{F\} \cup (\{E\} \cap \emptyset) \\
&= \{F\} \cup \emptyset \\
&= \{F\} \\
dom(G) &= \{G\} \cup dom(F) \\
&= \{F, G\} \\
dom(H) &= \{H\} \cup dom(F) \\
&= \{F, H\}
\end{aligned}$$

```
procedure simpleDominators($\mathcal{G}_f$)
    foreach $X \in \mathcal{N}_f$ do  $dom(X) \leftarrow \mathcal{N}_f$                31
    $worklist \leftarrow$ root                                                          32
    while $worklist \neq \emptyset$ do
        $Y \leftarrow worklist$.pickAndRemove( )                                        33
        $newdom \leftarrow \{Y\} \cup \bigcap_{(X,Y)\in\mathcal{E}_f} dom(X)$           34
        if $newdom \neq dom(Y)$                                                         35
        then
            $dom(Y) \leftarrow newdom$
            foreach $(Y,Z) \in \mathcal{E}_f$ do  $worklist \leftarrow worklist \cup \{Z\}$   36
end
```

Figure 14.17: Dominators algorithm.

This solution is a **fixed point**:

- For each node Z under consideration, every node in $dom(Z)$ does dominate Z. For example, both F and H dominate node H.
- Further application of Equation 14.4 at any node does not change the solution at any node.

Although the system of equations is satisfied, we have obtained the **minimum** instead of **maximum fixed point** solution. In the above example, node E should dominate all nodes, but it appears only in $dom(E)$. For each node Z, we would like $dom(Z)$ to be as *large* as possible while satisfying Equation 14.4.

While establishing a fixed point solution of this system of equations, an intermediate solution at node Z is obtained by applying Equation 14.4 to the intermediate solutions at the predecessors of Z. The difference between successive solutions at node Z can only be attributed to the *intersection* ($\bigcap$) operation in Equation 14.4 acting on changes in solutions at the predecessors of Z.

If initially assuming $dom(Y) = \emptyset$ results in the minimum fixed point, then we might (correctly) guess that initializing $dom(Y) = \mathcal{N}_f$ results in the maximum fixed point. Intuitively, the intersection operator whittles away at the solution at each node in the flow graph, because set intersection can never produce a set larger than its inputs. To obtain the largest final solution at node Z, we must trust (or, better yet, *prove*) that repeated applications of Equation 14.4 throughout the flow graph eventually stabilize to a safe fixed point.

Most data flow algorithms maintain a list of nodes or computations that must be considered by the algorithm before it is done. The dominators algorithm shown in Figure 14.17 maintains a *worklist* of nodes whose dominators

Node Y	Old	*Pred*(Y)	New	New
picked by	*dom*(Y)		*dom*(Y)	*worklist*
Marker ㉝		Marker ㉞		Marker ㊱
⇒ 1	$\mathcal{N}_f$	{ }	{1}	{2,3}
⇒ 2	$\mathcal{N}_f$	{1}	{1,2}	{3,4,11}
3	$\mathcal{N}_f$	{1,2,5}	{1,3}	{4,11,9}
⇒ 4	$\mathcal{N}_f$	{2,3}	{1,4}	{11,9,5,8}
11	$\mathcal{N}_f$	{2,12}	{1,2,11}	{9,5,8,12,13}
⇒ 9	$\mathcal{N}_f$	{3}	{1,3,9}	{5,8,12,13,10}
5	$\mathcal{N}_f$	{4,6}	{1,4,5}	{8,12,13,10,6,3}
⇒ 8	$\mathcal{N}_f$	{4,9}	{1,8}	{12,13,10,6,3}
12	$\mathcal{N}_f$	{11,13}	{1,2,11,12}	{13,10,6,3,7,11}
⇒ 13	$\mathcal{N}_f$	{11}	{1,2,11,13}	{10,6,3,7,11,12}
⇒ 10	$\mathcal{N}_f$	{9}	{1,3,9,10}	{6,3,7,11,12}
6	$\mathcal{N}_f$	{5,8,10}	{1,6}	{3,7,11,12,5}
⇒ 3	{1,3}	{1,2}	*same*	{7,11,12,5}
⇒ 7	$\mathcal{N}_f$	{6,12}	{1,7}	{11,12,5}
⇒ 11	{1,2,11}	{2,12}	*same*	{9,5,8,12,13}
⇒ 12	{1,2,11,12}	{11,13}	*same*	{5}
⇒ 5	{1,4,5}	{4,6}	{1,5}	{6}
⇒ 6	{1,6}	{5,8,10}	*same*	{ }

Figure 14.18: Dominators computation. An arrow before a node marks that node's final computation of its dominators.

should be recomputed by applying Equation 14.4. The solution for all nodes is initialized to $\mathcal{N}_f$ and the *worklist* is initialized to *root* by Marker ㉜. The call to pickAndRemove at Marker ㉝ picks *any* element to be removed from the list and returned for assignment to Y. If the recomputation of a node Y's dominators at Marker ㉞ produces a different solution at node Y, then the successors of Y are placed on the *worklist* by Marker ㊱ because their solution may consequently change.

We now apply the dominators algorithm to the flow graph of Figure 14.10. The steps of the algorithm and the development of the dominator sets are shown in Figure 14.18. Node 1 (the root) has no predecessors, so its solution changes from $\mathcal{N}_f$ to just itself, which causes its successors 2 and 3 to be placed on the *worklist*. The first time 3 is taken from the *worklist*, its dominators change from $\mathcal{N}_f$ to $\{1,3\}$, which causes nodes 4 and 9 to be added to the *worklist*. Node 4 was already on the *worklist* due to the change at node 2, so the *worklist* expands only with node 9. The next time node 3 is taken from the *worklist*, no change results from recomputing *dom*(3), so the *worklist* does not grow.

The only node that changes on second computation is node 5. When $dom(5)$ was first computed, its predecessor node 6 had dominators $\mathcal{N}_f$, so that $dom(5) = \{1, 4, 5\}$. After node 4 is removed from $dom(6)$, node 5 is placed on the *worklist* and its recomputation establishes its dominators as $\{1, 5\}$.

14.2.7 Fast Dominance Algorithm

The simple dominance algorithm is easy to program and performs efficiently for most flow graphs. Nonetheless, most compilers use the "fast" algorithm, due to Lengauer and Tarjan [LT79], for the following reasons:

- The fast algorithm requires only $O(\mathcal{E}_f\, \alpha(\mathcal{E}_f, \mathcal{N}_f))$ time, where α is a function with *very* slow growth. Experimentally, the algorithm presented in this section is faster than the algorithm in Figure 14.17 on all but the smallest of flow graphs.

- The algorithm in Figure 14.17 computes the set $dom(Y) = \{ X \mid X \gg\!\!= Y \}$, but the optimization problems we study do not usually require computing *all* of Y's dominators. For example, the algorithms discussed in Section 14.7 require only immediate dominance—information found in a dominator tree. The fast algorithm directly computes the dominator tree, from which all dominators of a node can be easily obtained.

- The dominators algorithm uses $O(\mathcal{N}_f{}^2)$ space, while the faster algorithm requires only $O(\mathcal{N}_f)$ space.

The fast dominance algorithm is also a useful exercise in applying depth-first numbering and traversing the DFST. These structures will also be used in computing the *interval* partition of a flow graph in Section 14.2.9.

The term *fast* applies here because of the following result [LT79]. If a graph has m nodes and n edges, then the best implementation of the fast algorithm runs in $O(m\alpha(m, n))$ time, where $\alpha(m, n)$ is an extremely slow-growing functional inverse of the **Ackermann function**. In other words, the algorithm runs in **almost linear time** in the size of the flow graph. The implementation discussed here is a simpler formulation, also found in [LT79], which runs in $O(m \log n)$ time. A faster algorithm has been developed that runs in linear time [GT04], but that algorithm is more complex and it uses structures similar to the algorithm described here.

Observe that the dominators of each node Y in Figure 14.18 are ancestors of Y in the DFST shown in Figure 14.12: For example, the dominators of node 8 include all of its ancestors. However, not all of a node's ancestors dominate that node. For example, the dominators of node 9 do not include its ancestor (node 2).

Lemma 14.5 *If $X \underline{\gg} Y$ in $\mathcal{G}_f$, then X must be an ancestor of Y in the depth-first spanning tree of $\mathcal{G}_f$.*

Proof: Suppose by contradiction that X is not an ancestor of Y in the DFST. Then there exists a path (of tree edges) from *root* to Y that does not contain X, so X cannot dominate Y. □

It follows that the immediate dominator of Y, $idom(Y)$, is a proper ancestor of Y in the DFST.

Recall from Section 14.2.4 that edges of a flow graph are classified as tree, chord, back, or cross edges with respect to a DFST of the flow graph. Figure 14.13 shows such a classification for the edges of Figure 14.10 with respect to the DFST show in Figure 14.12. The fast algorithm uses these edge classifications and performs the following steps:

1. A new graph $\mathcal{G}_{f'}$ is created from $\mathcal{G}_f$ with the same nodes, tree edges, and *at least* the same chord edges as found in a DFST for $\mathcal{G}_f$. However, $\mathcal{G}_{f'}$ has neither cross nor back edges. With respect to dominance, the effects of all cross and back edges are represented by **forward** (tree or chord) edges in $\mathcal{G}_{f'}$.
2. Immediate dominators are computed for $\mathcal{G}_{f'}$, which contains sufficient chord edges so that $X = idom(Y)$ in $\mathcal{G}_{f'}$ iff $X = idom(Y)$ in $\mathcal{G}_f$.

In other words, the algorithm eliminates back and cross edges in favor of tree and (potentially new) chord edges. This reduces the original problem to the simpler problem of computing immediate dominance for graphs having only forward edges.

Elimination of Cross and Back Edges

From Lemma 14.5, we know that Y's immediate dominator is some DFST ancestor of Y.

Lemma 14.6 *Node s can be the immediate dominator of Y only if there is a flow graph path*

$$p = (s \xrightarrow{+} Y)$$

such that s and Y are the only DFST ancestors of Y in p.

Proof: Suppose that s dominates Y, but no such path p exists in the flow graph. The flow graph must then contain a path

$$s \xrightarrow{+} W \xrightarrow{+} Y$$

where s is a proper ancestor of W, which is a proper ancestor of Y, and there is no path p in the graph from s that reaches Y without including W. We then have the relation

$$s \gg W \gg Y$$

which precludes s from *immediately* dominating Y. □

Definition 14.7 *Consider nodes A, B, and C in a DFST. Node B is **between** A and C iff $A \lhd B$, $B \lhd C$, $A \neq B$, and $B \neq C$. In other words, A is a proper ancestor of B and B is a proper ancestor of C.*

*The **intervening ancestors** between A and C are defined as the set of all nodes that are **between** A and C.*

In Figure 14.19(b), nodes X and Y are the intervening ancestors between s and Z.

Based on Lemma 14.6, imagine that an ancestor s of Y tries to establish itself as Y's immediate dominator. It can do so by using a path that avoids all nodes in the DFST between itself and Y. Such a path can be as simple as an edge from s to Y.

Definition 14.8 *Node s is a **sneaky ancestor** of Y if s can reach Y using a path that avoids all ancestors between s and Y.*

The manner in which node s can be a sneaky ancestor of Y can be analyzed with respect to the flow graph edge classification given in Section 14.2.4:

($s \underset{tree}{\longrightarrow} Y$): The DFST parent of Y is vacuously sneaky. There are no intervening ancestors, and *parent*(Y) can certainly reach Y.

($s \underset{chord}{\longrightarrow} Y$): If $(s, Y) \in \mathcal{E}_f$, then s is sneaky by definition.

($s \overset{+}{\rightarrow} T \underset{cross}{\longrightarrow} Y$): If Y is the target of a cross edge, then Figure 14.19(a) shows how s could initiate a path that avoids intervening ancestors and concludes with the edge $T \rightarrow Y$. The effect is the same as if $\mathcal{E}_f$ contained the edge (s, Y), as shown by the dashed edge in Figure 14.19(a).

In this case, we say that s is sneaky using the cross edge $T \rightarrow Y$.

($s \overset{+}{\rightarrow} Z \underset{back}{\longrightarrow} Y$): If Y is the target of a back edge, then Figure 14.19(b) shows how s could initiate a path that avoids intervening ancestors, enters the depth-first spanning subtree rooted at Y, and concludes with the edge $Z \rightarrow Y$. The effect is the same as if $\mathcal{E}_f$ contained the edge (s, Y), as shown by the dashed edge in Figure 14.19(b).

In this case, we say that s is sneaky using the back edge $Z \rightarrow Y$.

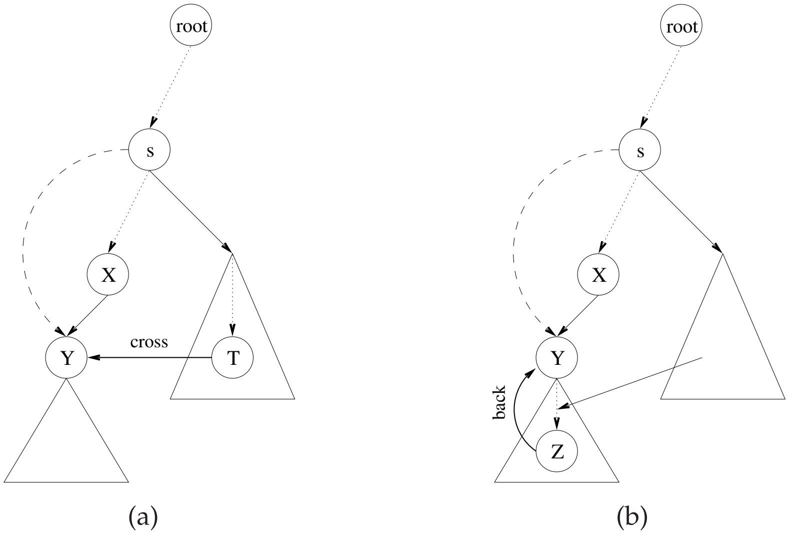


Figure 14.19: Sneaky behavior from cross and back edges can be summarized by chord edges. Node s reaches Y using (a) the cross edge $T \rightarrow Y$, and (b) the back edge $Z \rightarrow Y$. A dashed chord edge summarizes this behavior in each case. The triangles represent depth-first spanning subtrees.

Figure 14.19 shows how sneaky behavior reaching node Y (using cross or back edges) can be summarized by a suitable chord edge directly to Y. The algorithm shown in Figure 14.20 visits each node in the flow graph to determine the chord edges that can stand for the sneaky behavior due to cross and back edges. It constructs a graph $\mathcal{G}_{f'}$ with no back or cross edges, such that the immediate dominators of $\mathcal{G}_{f'}$ are the same as the immediate dominators of the input graph $\mathcal{G}_f$.

Although the algorithm in Figure 14.20 appears to be simple, the complexity of computing *sneaky*(X, Y) at Marker (38) has not been addressed. The efficient algorithm given in Figure 14.21 is based on the following observations:

- With respect to node Y, only the sneakiest of its ancestors need be remembered.

- The depth-first numbering provides an efficient means of computing $\mathcal{G}_{f'}$. In particular, considering nodes in descending order of their depth-first numbering allows an efficient implementation.

```
procedure eliminateCrossBackEdges( )
    $\mathcal{N}_{f'} \leftarrow \mathcal{N}_f$
    foreach $Y \in \mathcal{N}_f$ do                                                    ㊲
        foreach $X \in Preds(Y)$ do
            foreach $s \in sneaky(X, Y)$ do  $\mathcal{E}_{f'} \leftarrow \mathcal{E}_{f'} \cup \{ (s, Y) \}$       ㊳
end

function sneaky(X, Y) returns { nodes }
    return ({ $s$ | $s$ is a sneaky ancestor of $Y$ using edge $(X, Y)$ } )
end
```

Figure 14.20: Elimination of back and cross edges.

Suppose Y has multiple sneaky ancestors, s_1 and s_2, where s_1 is an ancestor of s_2 in the DFST. Since s_1 can reach Y without involving s_2, node s_2 cannot dominate Y. Introducing the edge (s_2, Y) into $\mathcal{G}_{f'}$ would be superfluous for computing *idom*(Y). In processing edges to Y, Marker ㊳ in Figure 14.20 need place only one edge in $\mathcal{G}_{f'}$—for the *sneakiest* ancestor of Y:

Definition 14.9 *The **semidominator**[1] of Y, denoted sdom(Y), is the depth-first number of the **sneakiest** DFST ancestor of Y. Because sdom(Y) is sneaky, there must be a flow graph path from sdom(Y) to Y that avoids all of Y's ancestors between itself and sdom(Y).*

Although a semidominator is formally defined as the depth-first number of a node, it is often convenient to refer to the node instead of its depth-first number. A node with depth-first number n can therefore be denoted as n, avoiding the *vertex*(n) notation, when this is clear in context.

The depth-first numbering of a graph allows a more formal definition of *sdom*(Y):

Definition 14.10 *Given nodes s and Y, $s \lhd Y$, let P be the set of all paths in $\mathcal{G}_f$ of the form*

$$s \rightarrow v_1 \rightarrow v_2 \rightarrow \ldots \rightarrow v_n \rightarrow Y$$

such that $\forall_i \; dfn(v_i) > dfn(Y)$. All such paths are sneaky, since no v_i can be a proper ancestor of Y. Then

$$sdom(Y) = \min_{s \xrightarrow{*} Y \in P} dfn(s)$$

[1] Literally *half*-dominator, the term *semidominator* is taken from the paper describing the fast dominance algorithm [LT79]. The use of this name becomes clearer when we compute immediate dominators from semidominators.

Marker ㊵ in Figure 14.21 invokes the EVAL function on each predecessor X of node Y. The EVAL method considers X and all of its ancestors that have already been visited thus far by Marker ㊴, returning the one that has the sneakiest semidominator. That ancestor is received as a at Marker ㊵. If $sdom(a)$ is sneakier than what has been found so far for Y, then Y's semindominator is updated at Marker ㊶. Each node Y contributes a single edge $(sdom(Y), Y)$ to $\mathcal{E}_{f'}$ at Marker ㊺.

Marker ㊲ in Figure 14.20 does not consider nodes in any particular order, but the fast algorithm in Figure 14.21 considers nodes in reverse order of their depth-first numbering. When visiting node Y, semidominators have already been computed for any cross or back edges to Y, because the source of each such edge is depth-first numbered greater than Y.

At Markers ㊷ and ㊸, the algorithm maintains a linked list of nodes with the same semidominator. This structure is used in Figure 14.24 to visit all nodes semidominated by a given node.

The computation of semidominators for the graph shown in Figure 14.13 is given in Figure 14.22. Figure 14.23 shows the graph $\mathcal{G}_{f'}$ created from the graph of Figure 14.13, with all cross and back edges eliminated in favor of tree and chord edges.

Dominators of a Graph with Tree and Chord Edges

The fast dominance algorithm is next concerned with determining immediate dominators for graphs with neither cross nor back edges, such as the one shown in Figure 14.23. If node n receives a curved edge, then the source of that edge is n's semidominator; otherwise, n's DFST parent is its semidominator. For example, a curved edge in Figure 14.23 shows that node 3 serves as node 5's semidominator, with node 4 as node 5's parent in the DFST. Node 9 receives no curved edges, so its semidominator is its DFST parent, node 3.

Unfortunately, a node may not be dominated by its semidominator. For example, node 5 is semidominated by node 3, but Figure 14.18 indicates that $dom(5) = \{1, 5\}$. Thus node 3 does not dominate node 5, so it certainly cannot serve as 5's immediate dominator. This situation happens when a node's semidominator is *bypassed* by one of its ancestors. In Figure 14.23, node 3 would have dominated node 5 if not for the edge $2 \rightarrow 4$, which bypasses 3. Node 2 would have then dominated node 5 except that it also is bypassed by the edge $1 \rightarrow 3$.

Lemma 14.11 *If $sdom(Y)$ dominates Y, then $sdom(Y)$ is the immediate dominator of Y.*

Proof: We need only show that no ancestor of Y between itself and $sdom(Y)$ can dominate Y. From Definition 14.10, there must be a path

```
procedure SEMIDOMINATORS($\mathcal{G}_f$)
    foreach $X \in \mathcal{N}_f$ do
        $sdom(X) \leftarrow dfn(X)$
        $s.head(X) \leftarrow ancestor(X) \leftarrow$ null
    for $n = NumNodes$ downto 2 do
        $Y \leftarrow vertex(n)$                                        (39)
        foreach $X \in Pred(Y)$ do
            $a \leftarrow$ EVAL($X$)                                    (40)
            if $sdom(a) < sdom(Y)$                                      (41)
            then  $sdom(Y) \leftarrow sdom(a)$
        $s.next(Y) \leftarrow s.head(vertex(sdom(Y)))$                  (42)
        $s.head(vertex(sdom(Y))) \leftarrow Y$                          (43)
        $ancestor(Y) \leftarrow parent(Y)$                              (44)
    $\mathcal{N}_{f'} \leftarrow \mathcal{N}_f$                        (45)
    $\mathcal{E}_{f'} \leftarrow \{ (vertex(sdom(Y)), Y) \mid Y \in \mathcal{N}_{f'} \}$
    $\mathcal{E}_{f'} \leftarrow \mathcal{E}_{f'} \cup \{$ all DFST tree edges $\}$
end

function EVAL($X$) returns node
    $sneakiest \leftarrow \infty$                                       (46)
    for ($p = X$) repeat ($p \leftarrow ancestor(p)$) do
        if $p = \perp$
        then  return ($accomplice$)
        else
            if $sdom(p) < sneakiest$
            then
                $accomplice \leftarrow p$
                $sneakiest \leftarrow sdom(p)$
end
```

Figure 14.21: Semidominator algorithm.

sdom(Y) $\overset{+}{\rightarrow}$ Y such that all intermediate nodes are depth-first numbered greater than Y. Such a path prevents any ancestor of Y between itself and *sdom*(Y) from dominating Y. □

Lemma 14.11's one-way implication helps explain the term **semidominator**: *sdom*(Y) is sometimes the immediate dominator of Y. However, it is possible that *sdom*(Y) does not even dominate Y, as shown by the examples above and by the illustration in Figure 14.25. Even in this case, *sdom*(Y) plays a role in computing Y's immediate dominator in the algorithm of Figure 14.24. This algorithm performs two passes (described below) to compute the immediate dominators for $\mathcal{N}_{f'}$. The dominators of $\mathcal{N}_{f'}$ are also the dominators of $\mathcal{N}_f$.

In the first pass, Marker (48) considers nodes in the reverse order of their

Node	Chord or Tree Edge from	Cross or Back Edge from	Semi Value
13	11		11
12	11		11
		13	11
11	2		2
		12	11
10	9		9
9	3		3
8	4		4
		9	3
7	6		6
		12	2
6	5		5
		8	3
		10	3
5	4		4
		6	3
4	3		3
	2		2
3	2		2
	1		1
		5	2
2	1		1
1	∅	∅	∅

Figure 14.22: Computing the semidominators for the graph shown in Figure 14.13.

depth-first numbering. For each node Y, all nodes semidominated by Y are examined at Marker (49). Recall that these nodes can be retrieved using the linked list managed at Markers (42) and (43) in Figure 14.21.

Lemma 14.12 *The node sdom(Z) dominates Z if and only if for all ancestors t of Z between itself and sdom(Z), sdom(t) < sdom(Z).*

Proof: Left as exercise Exercise 20. For intuition, consult the example in Figure 14.23. □

Each node Z that is semidominated by Y is tested against the condition specified in Lemma 14.12 by Marker (51). At Marker (50), semidominators have already been computed for all nodes. Therefore, the result returned EVAL(Z)

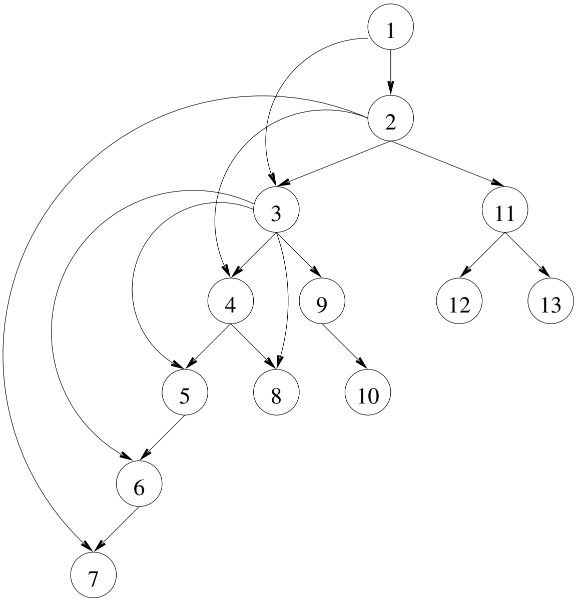


Figure 14.23: Dominator-equivalent flow graph from Figure 14.13 with only chord and DFST tree edges. The chord edges represent the semindominator relation (Figure 14.22) for nodes whose parent is not their semidominator.

at Marker 50 can determine if *sdom*(Z) dominates Z. If $Y = sdom(Z)$ happens to dominate Z, then Y is established as *idom*(Z) at Marker 52. Otherwise, we have the situation described previously for Figure 14.23 and shown more explicitly in Figure 14.25. Some ancestor s of Z *bypasses* Y by semidominating t. In this case, if s is the lowest-numbered node to bypass Y with respect to Z, then nodes t and Z have the same immediate dominator:

Lemma 14.13 *In the situation shown in Figure 14.25,* $dom(t) = dom(Z)$ *if* s *is the lowest-numbered semidominator of all ancestors of* Z *between* $sdom(Z)$ *and* Z*.*

Proof: Left as Exercise 21. □

Corollary 14.14 *In Lemma 14.13, if* $s = sdom(Z)$ *then* $idom(Z) = vertex(s)$*.*

Figure 14.26 shows a trace of the loop at Marker 48. Although the loop counts down by depth-first number, the actual node under consideration is node Z, semidominated by node Y, at Marker 49. Figure 14.26 shows the accomplice t found at Marker 50 by calling EVAL. Node s is set to t's semidominator and then compared to Z's semidominator at Marker 51.

```
procedure fastDominators($\mathcal{G}_{f'}$)
    foreach $X \in \mathcal{N}_{f'}$ do ancestor(X) ← null                 (47)
    for n = NumNodes downto 1 do                                        (48)
        Y ← vertex(n)
        foreach { Z | n = sdom(Z) } do                                  (49)
            t ← eval(Z)                                                 (50)
            s ← sdom(t)
            if s = n                                                    (51)
            then
                idom(Z) ← Y                                             (52)
            else
                idom(Z) ← null                                          (53)
                SameDomAs(Z) ← t
        foreach c ∈ Children(Y) do ancestor(c) ← Y                      (54)
    for n = 2 to NumNodes do                                            (55)
        Z ← vertex(n)
        if idom(Z) = null
        then idom(Z) ← idom(SameDomAs(Z))
    idom(root) ← null
end
```

Figure 14.24: Algorithm for computing immediate dominators of a graph with no cross or back edges. The function eval is taken from Figure 14.21, but the *ancestor* map is reset at Marker (47).

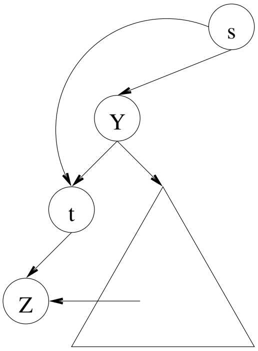


Figure 14.25: A node's semidominator does not necessarily dominate that node.

Node	Z at	t at	s at	$idom(Z)$	$SameDomAs(Z)$
Y	(49)	(50)	(51)	(52)	(53)
13					
12					
11	12	12	11	11	
	13	13	11	11	
10					
9	10	10	9	9	
8					
7					
6					
5					
4					
3	5	4	2		4
	6	4	2		4
	8	4	2		4
	9	9	3	3	
2	4	3	1		3
	7	3	1		3
	11	11	2	2	
1	2	2	1	1	
	3	3	1	1	

Figure 14.26: Trace of the loop at Marker (48).

Given the information as shown in Figure 14.26, the final pass processes nodes in *increasing* order of their depth-first number, starting with node 2. Each node either has its immediate dominator already computed in Figure 14.26, or else its immediate dominator is known to be the same as some node of lower depth-first number. Nodes 2 and 3 are known to have node 1 as their immediate dominator. Node 4 has the same immediate dominator as node 3, which can now be resolved as node 1. Nodes 5 and 6 have the same immediate dominator as node 4, which can now be resolved as node 1. This process continues until the dominator tree is obtained, as shown in Figure 14.27.

14.2.8 Dominance Frontiers

Sections 14.2.6 and 14.2.7 discuss dominators of a flow graph. If $X \in dom(Y)$, then X appears on all paths in the flow graph from *root* to Y. Figure 14.16 on page 568 provides a visual interpretation of dominance. If the *root* of the flow graph is a light source and light is transmitted along edges, then nodes strictly dominated by X are in the shadow cast by making X opaque, so that no light

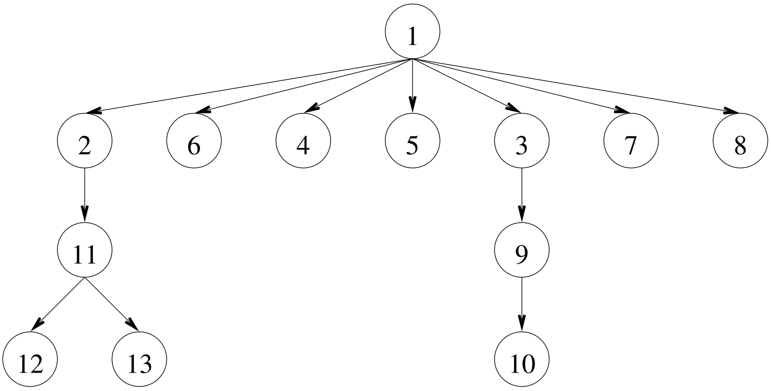


Figure 14.27: Dominator tree for the flow graph in Figure 14.10.

is transmitted on its outgoing edges.

Definition 14.15 *A node Y is in the* dominance frontier *of X, denoted DF*(*X*), *if Y is just outside of (i.e., one edge away from) the shadow cast by making node X opaque.*

More formally, DF(*X*) *is the set of nodes Z such that X dominates a predecessor Y of Z but does not strictly dominate Z:*

$$DF(X) = \{ Z \mid (\exists (Y, Z) \in \mathcal{E}_f)(X \underline{\gg} Y \textit{ and } X \not\gg Z) \}$$

In Figure 14.16, nodes 4, 8, and 6 are in *DF*(*X*). A node can be in its own dominance frontier. For example, node 11 has a predecessor (node 12) that is dominated by node 11, but node 11 cannot strictly dominate itself. Therefore, 11 ∈ *DF*(11).

Dominance frontiers are useful for computing **control dependence** (Exercise 11) and **static single assignment** (SSA) form (Section 14.7). Based on the above definition, a simple algorithm for computing dominance frontiers is given in Figure 14.28. A clue to the inefficiency of this algorithm is its use of the potentially quadratic *full* dominance information at Markers 56 and 58. A reasonable approach for improving this algorithm is to limit the search to the *immediate* dominators computed in Figure 14.24. Another improvement comes from visiting nodes in a favorable order as compared with Marker 57.

```
procedure simpleDominanceFrontiers($\mathcal{G}_f, dom$)
    $DomBy(X) \leftarrow \{ Z \mid X \in dom(Z) \}$                56
    foreach $X \in \mathcal{N}_f$ do                                57
        foreach $Y \in DomBy(X)$ do                                 58
            foreach $Z \in Succ(Y)$ do
                if $Z \notin (DomBy(X) - \{ X \})$
                then  $DF(X) \leftarrow DF(X) \cup \{ Z \}$
end
```

Figure 14.28: Simple dominance frontiers algorithm.

Consider the dominance frontiers of nodes 3, 9, and 10, as shown in the following table:

Node X	Nodes dominated by X	$DF(X)$
3	$\{3, 9, 10\}$	$\{4, 8, 6\}$
9	$\{9, 10\}$	$\{8, 6\}$
10	$\{10\}$	$\{6\}$

The second column shows nodes that fall in the shadow of X when X is opaque, and the third column shows the dominance frontier of X, which can be computed by inspection or by the above definition. The dominance frontier of node 3 contains node 4, along with the dominance frontiers computed for nodes 9 and 10. Notice the relationship between nodes 3, 9, and 10 in the dominator tree shown in Figure 14.27. If we express the dominance frontier for node X in terms of its children in the dominator tree, then a single pass over the tree suffices to compute the dominance frontiers of all nodes. We therefore express the dominance frontier of a given node as the contribution of two intermediate sets, DF_{local} and DF_{up}, as follows:

Equation 14.16

$$\begin{aligned} DF(X) &= DF_{local}(X) \cup \bigcup_{Z \mid X = idom(Z)} DF_{up}(Z) \\ DF_{local}(X) &\overset{\text{def}}{=} \{ Y \in Succ(X) \mid X \not\gg Y \} \\ DF_{up}(Z) &\overset{\text{def}}{=} \{ Y \in DF(Z) \mid idom(Z) \not\gg Y \} \end{aligned}$$

The *local* contribution comes from successors of X not strictly dominated by X. In our example, node 4 is in $DF_{local}(3)$. The *up* contribution comes from the dominance frontiers of nodes that are immediately dominated by X. In our example, nodes 6 and 8 are passed from node 9 to its immediate dominator, node 3.

```
procedure DOMINANCEFRONTIERS($\mathcal{G}_f$, $DomTree$)
    traverse tree ($DomTree$) order (BottomUp) at node ($X$) do
        $DF(X) \leftarrow \emptyset$
        foreach $Y \in Succ(X)$ do
            if $idom(Y) \neq X$
            then
                $DF(X) \leftarrow DF(X) \cup \{ Y \}$                    59
        foreach $Z \mid X = idom(Z)$ do
            foreach $Y \in DF(Z)$ do
                if $idom(Y) \neq X$
                then
                    $DF(X) \leftarrow DF(X) \cup \{ Y \}$                60
end
```

Figure 14.29: Computation of dominance frontiers.

In a bottom-up traversal of the dominator tree from Figure 14.27, node 6 initially appears in $DF_{local}(10)$ due to the edge from 10 to 6, and the fact that node 10 does not dominate node 6. Instead of testing for dominance, the following lemma proves that immediate dominance suffices:

Lemma 14.17 *If* $(Y, Z) \in \mathcal{E}_f$ *then* $Y \gg Z \iff Y = idom(Z)$

Proof: The $\Longleftarrow$ implication is trivial. For $\Longrightarrow$, if Y dominates Z, then the edge (Y, Z) guarantees that Y appears just before Z on all paths from *root*. Therefore, $Y = idom(Z)$. □

Thus, a better definition of DF_{local} is:

Equation 14.18 $DF_{local}(X) = \{ Y \in Succ(X) \mid idom(Y) \neq X \}$

Thus, node 6 is added to $DF(10)$ because $10 \neq idom(6)$. Node 6 then appears in $DF_{up}(10)$ by Equation 14.16.

When moving up the dominator tree to node 9, node 6 is included in $DF(9)$ because of node 10's contribution to its parent in Equation 14.16. Node 8 is included in $DF_{local}(9)$ by Equation 14.18. $DF_{up}(9)$ includes both nodes 6 and 8, which are added to $DF(3)$ when we move up the dominator tree. In computing $DF_{up}(3)$ by Equation 14.16, we find that node 1 dominates all nodes in $DF(3)$ so $DF_{up}(3) = \{ \}$. Due to our bottom-up traversal of the dominator tree, the test for general dominance in Equation 14.16 can be simplified to immediate dominance:

$$DF_{up}(Z) = \{ Y \in DF(Z) \mid idom(Y) \neq parent(Z) \}$$

The improved dominance frontier algorithm is shown in Figure 14.29. Marker 59 computes $DF_{local}(X)$ on-the-fly without needing to devote storage to it. Marker 60 operates similarly for $DF_{up}(Z)$. The dominator tree is

Node X	$Z \mid X = idom(Z)$	DF_{local}	$DF(X)$	$DF_{up}(X)$
12		{7,11}	{7,11}	{7,11}
13		{12}	{12}	{ }
11	12,13	{ }	{7,11}	{7}
2	11	{3,4}	{3,4,7}	{ }
6		{5,7}	{5,7}	{ }
4		{5,8}	{5,8}	{ }
5		{6,3}	{6,3}	{ }
10		{6}	{6}	{6}
9	10	{8}	{6,8}	{6,8}
3	9	{4}	{4,6,8}	{ }
7		{ }	{ }	{ }
8		{6}	{6}	{ }

Figure 14.30: Example of dominance frontier computation.

traversed bottom-up, visiting each node X only after visiting each of its children. The dominance frontiers for the flow graph in Figure 14.10 are shown in Figure 14.30.

14.2.9 Intervals

Optimization is primarily concerned with reducing the execution time of programs. Because most execution time is spent in the loops of programs, transformations often attempt to reduce the cost of operations that are deeply nested in the loops of a program. For example, **code motion** attempts to move code out of loops. **Reduction in strength** replaces a costly operation by a cheaper equivalent. Such optimization requires computing a program's **intervals**—a data structure that represents the looping constructs of a program. Intervals are also useful for evaluating data flow frameworks [Bur90].

We first consider the following definition of an interval based on the control flow graph:

Definition 14.19 *A* ***Cocke-Allen interval*** $I(x)$ *in* $\mathcal{G}_f$ *with* ***header*** *node* x *is a subset of* $\mathcal{N}_f$ *that contains* x *and has the following properties:*

1. *The interval can be entered only through its* ***header*** *node* x*. Thus, all edges in* $\mathcal{G}_f$ *that enter* $I(x)$ *from a node not in* $I(x)$ *must do so through* x*. More formally, if* $(y,z) \in \mathcal{E}_f$*,* $y \notin I(x)$*, and* $z \in I(x)$*, then* $z = x$*.*
2. *All nodes in* $I(x)$ *can be reached from* x *along a path contained in* $I(x)$*.*
3. *All cycles wholly contained in* $I(x)$ *must contain* x*.*

```
procedure DerivingGraphs($\mathcal{G}_f$)
    repeat
        $interval \leftarrow$ findInterval($\mathcal{G}_f$)
        $header \leftarrow interval$.getHeader( )
        $intvnodes \leftarrow interval$.getNodes( )
        foreach $(y, z) \in \mathcal{E}_f \mid y \in intvnodes$ and $z \notin intvnodes$ do      (61)
            $\mathcal{E}_f \leftarrow (\mathcal{E}_f - \{ (y, z) \}) \cup \{ (header, z) \}$
        $\mathcal{E}_f \leftarrow \mathcal{E}_f - \{ (header, header) \}$                        (62)
        $\mathcal{N}_f \leftarrow (\mathcal{N}_f - intvnodes) \cup \{ header \}$                 (63)
    until nochange
end
```

Figure 14.31: Deriving graphs of $\mathcal{G}_f = (\mathcal{N}_f, \mathcal{E}_f, root)$.

Viewed as a relation, intervals **partition** a control flow graph. A trivial partition that satisfies Definition 14.19 places each node in its own interval. This achieves the **minimum fixed point**, which is not the solution truly of interest. The algorithms discussed here compute the **maximum fixed point**, which places as many nodes as possible into the same interval.

The interval structure of a program can be computed by repeatedly finding and eliminating intervals as shown in Figure 14.31. Each time an interval is found, a new graph is essentially **derived** by replacing the nodes and edges inside the interval with a single node—the interval's **header**—which serves to summarize the eliminated nodes. Any edges from nodes within the found interval to nodes outside that interval are replaced by an edge from the interval's header at Marker (61). If the header node has a loop to itself, then this is eliminated at Marker (62). Except for the header node, all of the interval's nodes are deleted at Marker (63). This process continues until no more intervals can be found. If the final derived graph is acyclic, then the graph is **reducible** as discussed in Section 14.2.2.

Definition 14.19 does not induce unique interval partitions of the graph shown in Figure 14.32. For example, $\{2, 3, 4, 5, 8\}$ could be an interval of the graph in Figure 14.32, but so could $\{2, 3, 4, 8\}$. Definition 14.19 can be extended to induce unique partitions, and two such methods are considered below.

Cocke-Allen Method

Although there is not a unique partition of nodes that satisfies Definition 14.19, one partition of interest is due to Cocke and Allen [All70, Coc70] and to Hecht and Ullman [HU72]. Their method produces **maximum intervals** while satisfying Definition 14.19.

The algorithm shown in Figure 14.33 begins with node 0 of Figure 14.32 as the first header. Marker (65) cannot find any other nodes to place in $I(0)$

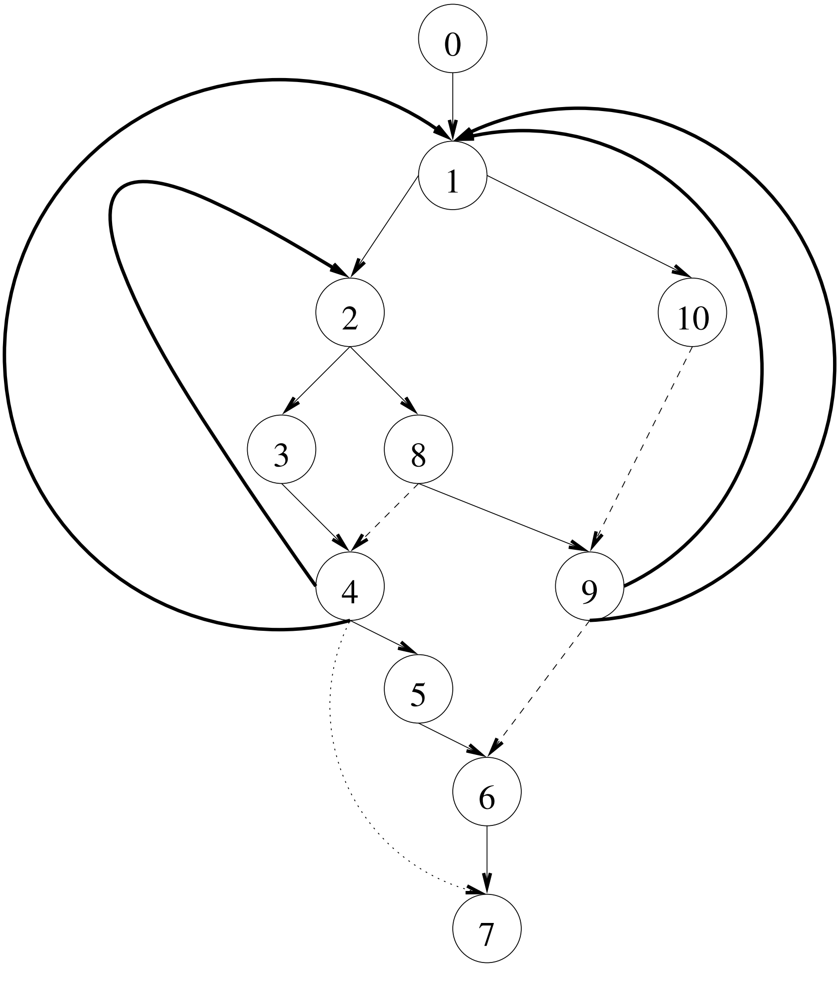


Figure 14.32: Graph for interval partitioning. The edge legend is the same as for Figure 14.13 on page 565.

```
procedure intervalsCockeAllen($\mathcal{G}_f$)
    $Nodes \leftarrow \mathcal{G}_f$
    call newHeader($Entry$)
    while $\exists\, h \in Headers \mid$ not $Processed(h)$ do
        call processHeader($h$)
        foreach $Y \mid (X, Y) \in \mathcal{E}_f$ and $X \in I(h)$ and $Y \in Nodes$ do        64
            call newHeader($Y$)

    procedure newHeader($h$)
        $Nodes \leftarrow Nodes - \{ h \}$
        $Headers \leftarrow Headers \cup \{ h \}$
        $Processed(h) \leftarrow$ false
    end
end

procedure processHeader($h$)
    $Processed(h) \leftarrow$ true
    while $\exists\, Y \in Nodes \mid Y \neq Entry$ and $\forall\, (X, Y) \in \mathcal{E}_f\ X \in I(h)$ do    65
        $I(h) \leftarrow I(h) \cup Y$
        $Nodes \leftarrow Nodes - \{ Y \}$
end
```

Figure 14.33: Cocke-Allen interval construction.

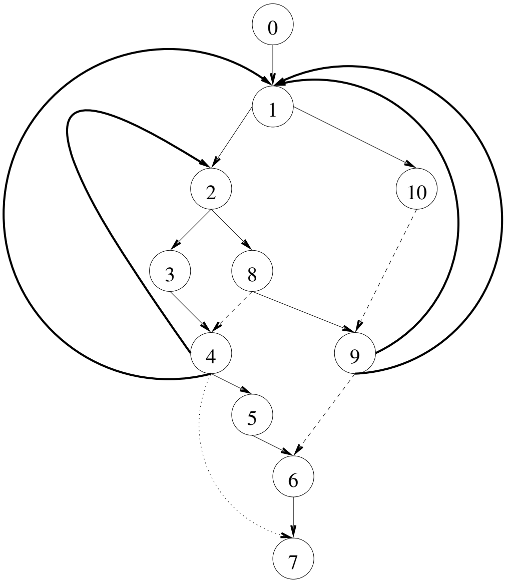
(the interval with header 0). Such nodes must have all of their predecessors in $I(0)$. For example, node 1 receives several edges from nodes outside of $I(0)$, so it cannot join $I(0)$. After Marker 64 makes a header out of node 1, then its interval can include node 10, but not node 2. The interval $I(2)$ grows to include nodes 3 and 8. After both of those nodes join $I(2)$, node 4 can join as well. Node 5's only predecessor is now in $I(2)$, so node 5 also joins $I(2)$. Node 6 cannot join $I(2)$ because of its edge from node 9, which is not in $I(2)$.

The complete interval partition is shown in Figure 14.34. This style of interval partitioning has the following disadvantages:

- The algorithm results in a sequence of derived graphs in which a given node may repeatedly appear. For example, nodes belonging to the outermost interval appear in every graph of the sequence.

- Because Cocke-Allen intervals are not strongly connected, an interval can contain nodes that are usually considered outside a loop. For example, node 5 in Figure 14.32 is an exit node from the loop comprising nodes 2, 3, 8, and 4. However, node 5 belongs to the Cocke-Allen interval $I(2)$.

- Some Cocke-Allen intervals may not correspond to loops at all. In Figure 14.32, node 7 is its own interval, but no iteration involves node 7. In



Header Node	Interval Members
0	{0}
1	{1,10}
9	{9}
2	{2,3,8,4,5}
6	{6}
7	{7}

Figure 14.34: Cocke-Allen partition of Figure 14.32.

fact, nodes 0 and 7 seem to be similar, since each is outside the scope of any iteration.

Schwartz-Sharir Method

Interval construction is typically intended to reveal the *loop structure* of a flow graph. The intervals found by the Cocke-Allen method represent loops, but a given interval can contain nodes that are typically thought to be outside of the interval's loop. In Figure 14.34, node 5 is in the Cocke-Allen interval with header 2, but that node appears to part of the outer loop (with header 1) in Figure 14.32.

The following extension to the definition of an interval can address this problem:

Definition 14.20 *A* ***Schwartz-Sharir interval*** *with* ***header*** *x is defined according to Definition 14.19 along with the following constraint:*

> *The header node x can be reached from any node in $I(x)$ along a path contained in $I(x)$.*

Definition 14.19 requires that all nodes in $I(h)$ are reachable from the header node h by paths contained in $I(h)$. The additional constraint in Definition 14.20 makes the nodes of an interval **strongly connected**. All nodes in an interval can reach each other without including any nodes outside the interval. For example, the Schwartz-Sharir interval with header 2 in Figure 14.32 excludes node 5, which is instead placed in the interval with header 1.

The algorithm in Figure 14.35 is taken from Schwartz and Sharir [SS78, SS79], which in turn is based on an algorithm by Tarjan [Tar72]. Irreducible graphs are detected in Figure 14.35 at Marker 73. Exercises 23 and 24 explore approaches for dealing with irreducible flow graphs (see Figure 14.8(b)). The algorithm's efficiency is obtained as follows:

- Nodes are considered in a "clever" order. In comparison to the algorithm in Figure 14.33, this fast algorithm considers nodes in the *reverse order* of their depth-first numbering, discovering inner intervals before outer ones.

- Throughout its analysis, the algorithm maintains **path-compressed** information to evaluate CurInt(X): the interval *currently associated* with node X. Initially, CurInt(X) returns its input node X. As the algorithm proceeds, CurInt(X) continues to return the header node of the most recently formed interval that includes, directly or indirectly, the node X.

As an example, consider how the evaluation of CurInt(3) changes as loops are discovered from innermost to outermost. Node 3 initially participates in no interval, so CurInt(3) returns 3. Node 3 subsequently joins the innermost interval with header 2; at that point, CurInt(3) = 2. This interval will eventually be incorporated into the outermost interval with header 1, at which time CurInt(3) = 1.

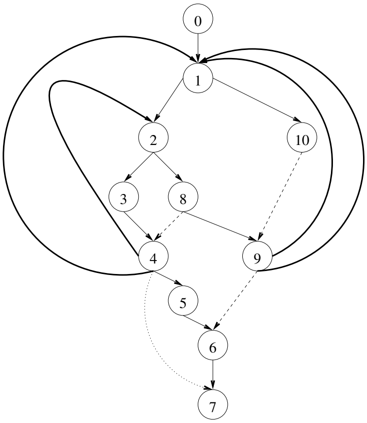
We next apply the algorithm in Figure 14.35 to the flow graph shown in Figure 14.32, with the results shown in Figure 14.36. Note that each vertex of the flow graph is already labeled with its depth-first number. Marker 66 considers the flow graph nodes in reverse order of their depth-first numbering, so that headers of inner loops are processed before headers of outer loops. Marker 68 determines if node h can be the header of an interval by looking for **back edges** to node h, applying the constant-time test described in Section 14.2.4. A back edge to h originates at some node l in the depth-first spanning subtree rooted at h. Thus, h and l are in a strongly-connected interval with header h. Node 2 is the first header so identified in Figure 14.32, as shown in Figure 14.36.

The set *ReachUnder* contains those nodes that can reach the header h by paths ending with a back edge to h. Intuitively, such nodes belong in a loop with single-entry h. The set is initialized at Marker 68 to contain the source of any back edge. Instead of adding a node v directly to *ReachUnder*, the algorithm consistently adds CurInt(v), so that a node v is represented by its

```
procedure intervalsSchwartzSharir($\mathcal{G}_f$, $DFST$)
    foreach $node \in \mathcal{N}_f$ do $head(node) \leftarrow \perp$
    for $n = |\mathcal{N}_f|$ downto 2 do                                          66
        $h \leftarrow vertex(n)$                                                   67
        $ReachUnder \leftarrow \{ \textsc{CurInt}(l) \mid (l,h) \in \mathcal{E}_f$ and $h \lhd l \}$     68
        while $\exists\, y \in (ReachUnder - \{h\}) \mid head(y) = \perp$ do        69
            $head(y) \leftarrow h$                                                 70
            foreach $X \in \{ \textsc{CurInt}(x) \mid x \in Preds(y) \}$ do        71
                if $h \ntriangleleft X$                                            72
                then
                    /★   Irreducible graph (Exercises 23 and 24)   ★/             73
                $ReachUnder \leftarrow ReachUnder \cup \{X\}$
    foreach $y \mid head(y) = \perp$ do  $head(y) \leftarrow root$                 74
    foreach $node \in \mathcal{N}_f$ do $Members(node) \leftarrow \emptyset$
    traverse tree ($DFST$) order (Pre R-L) at node ($n$) do                        75
        $Members(head(n)) \leftarrow Members(head(n)) \cup \{n\}$
end

function $\textsc{CurInt}(X)$ returns $node$
    if $head(X) = \perp$
    then  return ($X$)
    else  return ($\textsc{CurInt}(head(X))$)
end
```

Figure 14.35: Schwartz-Sharir intervals algorithm.



Header h 67	*ReachUnder* set at 69	Node y 70	New nodes 71
2	$\{4\}$	4	$\{3,8\}$
	$\{4,3,8\}$	3	$\{2\}$
	$\{4,3,8,2\}$	8	$\{2\}$
1	$\{2,6,9\}$	2	$\{2,1\}$
	$\{2,6,9,1\}$	6	$\{5,9\}$
	$\{2,6,9,1,5\}$	9	$\{2,10\}$
	$\{2,6,9,1,5,10\}$	5	$\{2\}$
	$\{2,6,9,1,5,10\}$	10	$\{1\}$
0		$\{0,1,7\}$	

Figure 14.36: The Schwartz-Sharir intervals of Figure 14.32.

most recently formed containing interval. For the header 2, *ReachUnder* is initialized at Marker 68 to $\{\,\textsc{CurInt}(4)\,\} = \{\,4\,\}$, because 4 was not previously associated with any interval.

The *ReachUnder* set is extended by the loop at Marker 69. A node y is mapped to its containing interval h at Marker 70, where $head(y)$ changes from $\perp$ to h. In our example, this establishes $head(4) = 2$. Marker 71 then considers predecessors of y, which can also reach the header by a path using a back edge. In our example, Marker 71 considers nodes 3 and 8. The mapping CurInt is applied to these nodes, and *ReachUnder* includes the unmapped nodes 3 and 8 after the loop at Marker 71. As the set *ReachUnder* grows, the loop at Marker 69 considers only unmapped nodes. Thus, the loop at Marker 69 will next choose either node 3 or node 8. If node 3 is chosen, then $head(3) = 2$ is established at Marker 70, and the loop at Marker 69 adds node 2 to *ReachUnder*.

Our example now has $ReachUnder = \{\,4, 3, 8, 2\,\}$, with nodes 2 and 8 unmapped. Our example continues as loop Marker 69 considers node 8. After mapping $Header(8) = 2$, *ReachUnder* does not change because node 8's sole predecessor is already in *ReachUnder*. Loop Marker 69 excludes the header h for the following reasons:

- Even though node 2 belongs in the interval with header 2, we leave $head(2) = \perp$, so that an interval's header node can represent its interval for subsequent containment in outer intervals.
- If the header h were processed as any other node in *ReachUnder*, then the loop at Marker 69 would include nodes that can reach h by tree, chord, or cross edges to h. Such nodes are not necessarily strongly connected with h, and *ReachUnder* already contains the appropriate predecessors of h after Marker 68.

Thus, loop Marker 69 is finished and the interval with header 2 is complete.

The algorithm continues as loop Marker 66 looks for a lower-numbered node that receives a back edge. Such a node is found when loop Marker 66 reaches node 1. The set of nodes with back edges to node 1 is $\{\,4, 8, 6, 9\,\}$. Applying CurInt to these nodes allows nodes 4 and 8 to be represented by their header node 2. At Marker 68, *ReachUnder* is initialized to $\{\,2, 6, 9\,\}$. Eventually, the loop at Marker 69 maps these nodes to the interval with header 1. The set *ReachUnder* is eventually expanded at Marker 71 to include nodes 5 and 10. When the loop at Marker 66 is finished processing the header 1, $head(y) = 1$ for $y \in \{\,2, 5, 6, 9, 10\,\}$. The header node 1 remains unmapped.

All strongly connected regions are now identified, but nodes outside any loop remain unmapped. The algorithm maps all such nodes into an interval headed by the root of the flow graph at Marker 74. Alternatively, the flow graph could be augmented with an edge from its exit to its entry, rendering

Node	Header		
	0	1	2
0	X		
1	X		
10		X	
2		X	
8			X
9		X	
3			X
4			X
5		X	
6		X	
7	X		

Figure 14.37: Nodes from Figure 14.32 listed in interval order. The interval header for each node is marked with an X.

the outermost interval strongly connected. All nodes except the flow graph *root* could then be processed by the algorithm prior to Marker 74.

Although intervals could have been constructed as they were discovered, the algorithm postpones construction until Marker 75, so that nodes can be organized into intervals using the following special order:

Definition 14.21 *Consider a flow graph* $\mathcal{G}_f = (\mathcal{N}_f, \mathcal{E}_f)$ *and its associated depth-first spanning tree* T*.*

1. *The **topological order** of* $\mathcal{G}_f$ *using* T *is defined by the partially ordered set* $TopOrder(\mathcal{G}_f, T) = (\mathcal{N}_f, \preceq)$*:*

   $$X \preceq Y \iff (X, Y) \in \mathcal{E}_f \text{ and } Y \not\preceq X$$

   Thus, nodes X *and* Y *are topologically ordered iff they are related by a tree, chord, or cross edge in their flow graph.*

2. *Given an interval* h *constructed from* $\mathcal{G}_f$ *using* T*, the **interval order** of* h *is a total ordering of the interval's nodes that respects the partial order* $TopOrder(\mathcal{G}_f, T)$*.*

As described in Section 14.5.1, algorithms that propagate information through paths of an interval are most efficient if nodes are visited in interval order. Recalling the discussion in Section 14.2.4, interval order is obtained by adding nodes to intervals in a right-to-left preorder traversal of the depth-first spanning tree. Figure 14.37 lists nodes in this order and shows how they are partitioned into intervals.

Better Implementation of CurInt

During interval analysis, nodes are incorporated into increasingly larger intervals as the loop at Marker 66 considers nodes in reverse order of their depth-first numbering. Initially, CurInt(X) = X, and throughout the algorithm, CurInt(X) returns the header of the most recently processed interval containing X. While CurInt(X) changes continually as the outer intervals containing X are discovered, X is mapped to its *immediately* containing interval exactly once at Marker 70. Therefore, throughout the algorithm, the following holds for every node X:

$$head(X) = \begin{cases} \perp & (X \text{ has not yet been associated with an interval}) \\ W & \text{with } dfn(W) < dfn(X) \end{cases}$$

Exponentiation of *head* can then be defined as the number of times the *head* map is applied:

$$\begin{array}{rcl} head^0(X) & = & X \\ head^i(X) & = & head(head^{i-1}(X)) \\ head^*(X) & = & Z \text{ such that} \\ & & \quad Z \neq \perp \text{ and} \\ & & \quad head(Z) = \perp \end{array}$$

As the algorithm discovers intervals that surround X, evaluation of CurInt(X) takes on the sequence

$$seq(X) = X, head(X), head(head(X)), \ldots, root$$

CurInt(X) always returns $head^*(X)$, which is evaluated based on the intervals discovered thus far. Initially, $head(X) = \perp$ yielding CurInt(X) = X. Eventually, $head(X)$ is mapped at Marker 70 to an outer interval that contains X. Subsequently, that interval's header is mapped to some outer interval. This continues until the outermost interval of the graph (*root*) is found. At any point in the algorithm's execution, the method CurInt(X) returns the current value of $head^*(X)$.

For example, consider node 4 from Figure 14.32. Initially, $seq(4) = 4$ = CurInt(4). The next change occurs when node 4 is mapped to header 2, at which point $seq(4) = 4, 2$ = CurInt(4). When the outer loop is processed, $seq(X) = 4, 2, 1$ = CurInt(4). Finally the outermost interval headed by *root* is processed so that $seq(X) = 4, 2, 1, 0$ = CurInt(4).

The naive implementation of CurInt(X) in Figure 14.35 visits every node in $seq(X)$ to reach its ultimate element, even though this sequence grows only at its end. The more efficient implementation shown in Figure 14.38 uses **path compression** to decrease the number of nodes that must be visited on average

```
/★ Initially, soln(X)=X                                   ★/
function CurInt( X ) returns node
    call compress( X )                                    (76)
    return ($soln(X)$)                                    (77)
end

procedure compress( X )
    if $head(soln(X)) \neq \perp$                         (78)
    then
        if $soln(X) = X$
        then
            $SameSoln \leftarrow head(X)$                 (79)
        else
            $SameSoln \leftarrow soln(X)$                 (80)
        call compress( SameSoln )                         (81)
        $soln(X) \leftarrow soln(SameSoln)$               (82)
end
```

Figure 14.38: Better implementation of CurInt.

to evaluate CurInt(X). For each node X, $soln(X)$ is the most recently computed value of CurInt(X). The call to compress at Marker (76) is responsible for updating $soln(X)$ in case $seq(X)$ has been extended since the last evaluation. The compress method not only updates $soln(X)$, it also updates $soln(Z)$ at each node Z that must be visited to compute $soln(X)$. Because such nodes are visited anyway for computing $soln(X)$, the extra updates are (asymptotically) free and they can save time later should $soln(Z)$ be required.

Based on the above definitions, $soln(X)$ should return the last element of $seq(X)$ when CurInt(X) is called. The staleness of $soln(X)$ is tested at Marker (78). If $head(soln(X)) = \perp$, then $soln(X) = seq(X)$ and the solution is current. Otherwise, a solution is recursively demanded by observing:

$$soln(X) = \begin{cases} soln(soln(X)) & \text{if } soln(X) \neq X \\ soln(head(X)) & \text{otherwise} \end{cases}$$

In other words, if $soln(X)$ was previously $Y \neq X$, then CurInt(X) = CurInt(Y) and the solution at X can be computed once the solution at Y is available. On the other hand, if $soln(X) = X$, then the solution at X should be the same as the solution at $head(X)$. The proper choice is made at Markers (79) and (80) by assigning the variable *SameSoln*. Compression is then requested for *SameSoln* at Marker (81). This solution is used at Marker (82) to update $soln(X)$. Because CurInt is recursive, the update at Marker (82) is applied to all nodes for which CurInt was called on behalf of X.

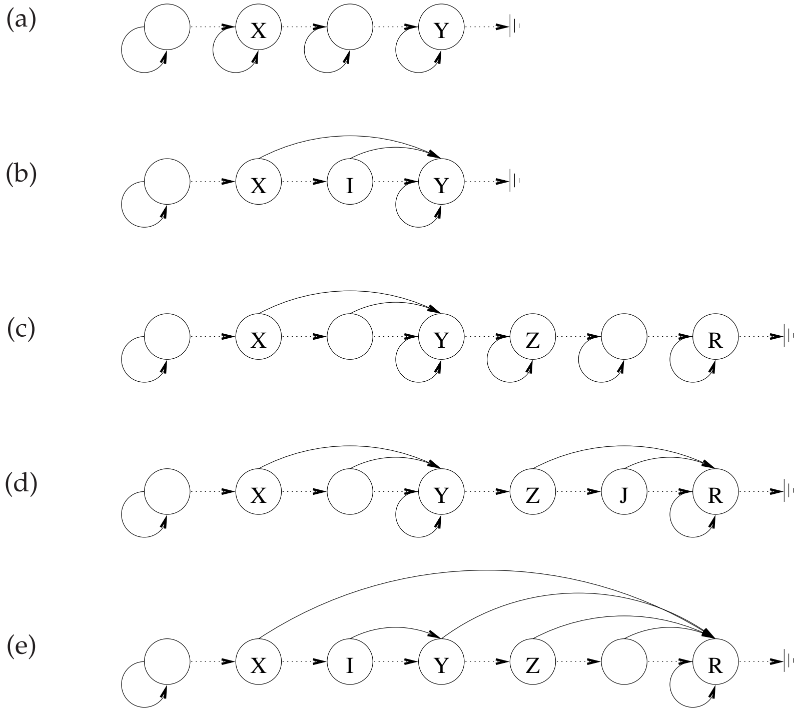


Figure 14.39: Path compression. The dashed edges represent *head* mappings and the solid edges represent *soln* values.

The path-compressing CurInt of Figure 14.38 is illustrated in Figure 14.39. The initial stage is shown in Figure 14.39(a). The *head* mappings are established as shown by the dashed edges from left to right, but CurInt has not yet been called on any node. Thus, each node's *soln* value points to itself. Figure 14.39(b) shows the results of calling CurInt(X). Because of path compression, both *soln*(X) and *soln*(I) are updated to point to Y. In Figure 14.39(c), the *head* mappings are extended from Y, as shown for nodes Z through R. Figure 14.39(d) shows the result of a subsequent call of CurInt(Z), which updates *soln*(Z) = *soln*(J) = R. Finally, Figure 14.39(e) shows the result of another call to CurInt(X). All nodes whose *head* or *soln* pointers are traversed by this call will have their *soln* values updated to R. A subsequent call of CurInt on nodes X, Y, or Z immediately returns node R. Node I's solution was not updated by previous calls to CurInt. An evaluation of CurInt(I) after Figure 14.39(e) jumps to Y and then directly to R, updating *soln*(I) = R.

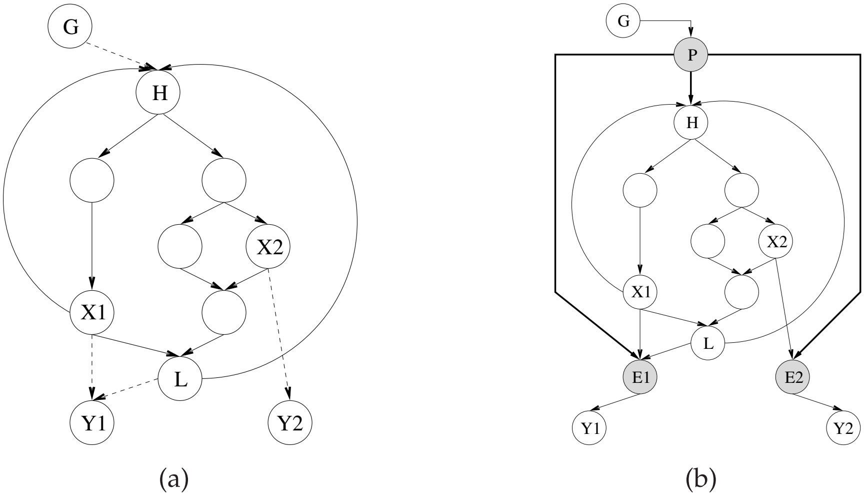


Figure 14.40: (a) Control flow graph; (b) Augmented graph with preheader P and postexits $E1$ and $E2$.

Augmented Flow Graphs

After interval analysis identifies the loop structure of a program, optimizations may try to reorganize computations inside and outside the loops. To facilitate further analysis and optimization, the flow graph can be **augmented** with explicit interval entry and exit nodes. Figure 14.40 demonstrates interval augmentation as follows:

Preheader Node P is introduced into the graph as the **preheader** for the interval with header H. Instead of entering a node at its header, each entry is redirected to P. Thus, the edge (G, H) is changed to (G, P) and a new edge (P, H) is introduced. The preheader is a convenient place to move code out of the loop. It can be suitably protected by the loop's continuation predicate if necessary.

Postexits The interval shown in Figure 14.40(a) has three exits: from $X1$ and L to $Y1$, and from $X2$ to $Y2$. For each node outside the interval that is the successor of a node inside the interval, we introduce a **postexit** node. In Figure 14.40(b), edges into $Y1$ are redirected to $E1$, and similarly with $Y2$ and $E2$. Each exit node then has a single edge to its associated node. Edges are therefore introduced from $E1$ to $Y1$ and from $E2$ to $Y2$.

The bold edges in Figure 14.40(b) connect the preheader node P with each postexit node, facilitating reduction of an interval-derived graph.

14.3 Introduction to Data Flow Analysis

As discussed in Section 14.1, an optimizing compiler is typically organized as a series of **passes**. Each pass may require approximate information about the program's runtime behavior to do its job. **Data flow frameworks** offer a unified and mathematically appealing structure for such analysis.

In Section 14.4 we present a more rigorous formulation of data flow frameworks. Here, we discuss some **data flow problems** informally, examining a few popular optimization problems and reasoning about their data flow formulation. For each problem, we are interested in the following:

- What is the effect of a code sequence on the solution to the problem?
- When branching in a program converges, how do we summarize the solution so that we need not keep track of branch-specific behavior?
- What are the best and worst possible solutions?

We use the program and **control flow graph** shown in Figure 14.41 as an example.

Local solutions to the above questions are combined by data flow analysis to arrive at a **global solution**. By local, we mean information that is present on a given edge because of an adjacent node's behavior. By global, we mean that a solution can be found for every edge of a flow graph.

14.3.1 Available Expressions

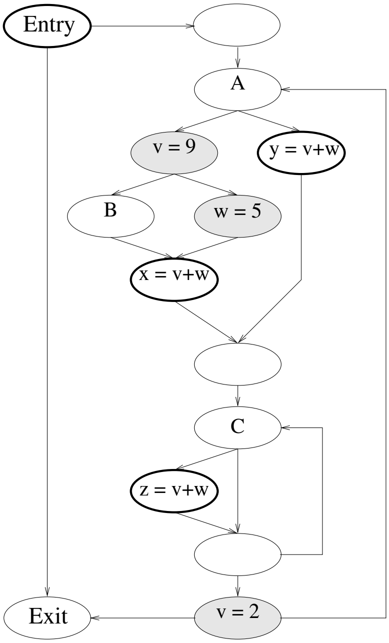
Figure 14.41 contains several computations of the expression $v + w$. If we can show that the particular value of $v + w$ computed at Marker 83 is already *available*, then there is no need to recompute the expression at Marker 83. More specifically, an expression *expr* is available at edge *e* of a flow graph if the past behavior of the program necessarily includes a computation of the value of *expr* as it would appear on edge *e*. The *available expressions* data flow problem analyzes programs to determine such information.

To solve the *available expressions* problem, a compiler must examine a program to determine that expression *expr* is available at edge *e*, regardless of how the program arrives at edge *e*. If a compiler simply executed a program to find such information, then an infinite loop could prevent the compiler from finishing. Finding such loops is in general **undecidable** [Mar03]. Compilers instead perform **static analysis**, which symbolically interprets a program and avoids infinite loops.

Returning to our example, $v + w$ is available at Marker 83 if every path arriving at Marker 83 computes $v + w$ without a subsequent change to v or w.

```
$u \leftarrow 5$
repeat
    if $r$
    then
        $v \leftarrow 9$
        if $p$
        then  $u \leftarrow 6$
        else  $w \leftarrow 5$
        $x \leftarrow v + w$
    else  $y \leftarrow v + w$
    $u \leftarrow 7$
    repeat
        if $q$
        then
            $z \leftarrow v + w$   83
    until $r$
    $v \leftarrow 2$
until $s$
```

(a)



(b)

Figure 14.41: (a) A program; (b) Its control flow graph.

In this problem, an instruction affects the solution if it computes $v + w$ or if it changes the value of v or w, as follows:

- The Entry node of the program is assumed to contain an implicit computation of $v + w$. For programs that initialize their variables, $v + w$ is certainly available after node Entry. Otherwise, $v + w$ is uninitialized, which allows the compiler to assume that the expression has *any* value it chooses.
- A node of the flow graph that computes the expression $v + w$ makes $v + w$ available on its outgoing edges.
- A node of the flow graph that assigns v or w makes $v + w$ unavailable.

  We assume an assignment to v destroys the availability of $v + w$ even if the value of v is unaffected by the assignment. For example, the assignment $v = v + 0$ does not really change v's value, but we leave the elimination of such useless code to other optimization passes.
- All other nodes have no effect on the availability of $v + w$.

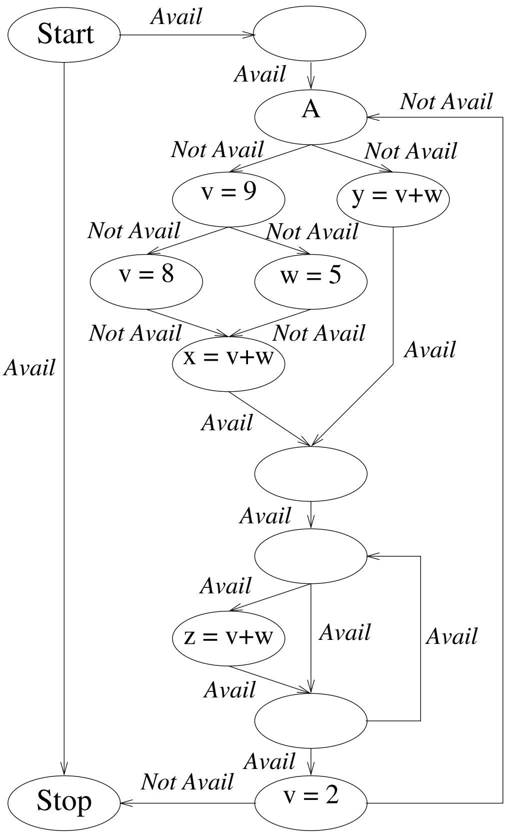


Figure 14.42: Solution throughout the flow graph of Figure 14.41(b) for the availability of expression $v + w$.

In Figure 14.41(b), the shaded nodes make $v + w$ *un*available. The nodes with dark circles make $v + w$ available. When two solutions are present at the input of a node, we summarize them by assuming the worst case. For example, the input to node A contains an implicit computation of $v + w$. However, on the loop edge into node A, $v + w$ is not available, because the node sponsoring that edge changes the value of v. We must therefore assume that the current value of $v + w$ is *not* available on entry to node A.

Based on the above reasoning, information can be pushed through the graph to reach the solution shown on each edge of Figure 14.42. The Entry and Exit nodes have been replaced by Start and Stop nodes to indicate the direction of data flow propagation. In the solution shown in Figure 14.42, $v + w$ is available on the edge entering the node that represents Marker (83) in Figure 14.41(a). Thus, the program can be optimized by eliminating that computation of $v + w$.

In this example, we explored the availability of a single expression $v + w$. Programs typically contain many expressions and compilers often generate more expressions than are written explicitly in programs (e.g., to compute the byte offset of a subscript expression). Optimizing compilers usually take one of the following approaches to identify expressions of interest:

- The compiler may identify an expression such as $v + w$ as *important*, in the sense that eliminating its computation can significantly improve the program's performance. In this situation, the optimizing compiler may selectively evaluate the availability of a single expression. SSA Form (Section 14.7) and sparse evaluation graphs [CCF91] facilitate such selective analysis.

- The compiler may compute availability of *all* expressions, without regard to the importance of the results. In this situation, it is common to formulate a *set* of expressions and compute the availability of its members. Section 14.4 and Exercise 35 consider this in greater detail.

14.3.2 Live Variables

We next examine an optimization problem related to register allocation. As discussed in Chapter 13, k registers suffice for a program whose **interference graph** is k-colorable. In this graph, each node represents one of the program's variables. An edge is placed between two nodes if their associated variables are simultaneously *live*. A variable v is *live* at control flow graph edge e if the future behavior of the program can reference the value of v that is present at edge e. In other words, the value of a live variable is potentially of future use in the program. Thus, register allocation relies on computing *live variables* to build the interference graph.

The *live variables* problem can be solved by data flow analysis techniques. It is related to *available expressions* in that they are two of the four problems commonly called the **bit-vectoring data flow problems** (Exercise 39).

In the *available expressions* solution shown in Figure 14.42, information follows the orientation of the control flow graph. Information is collected at the inedges of a node, pushed through the node, and transmitted on the node's outedges. Such data flow problems are called **forward**. On the other hand, solving the *live variables* problem requires characterizing the *future* behavior of a program. Such data flow problems are called **backward**. Information is collected at a node's outedges, pushed backward through the node, and transmitted out of the node on its inedges.

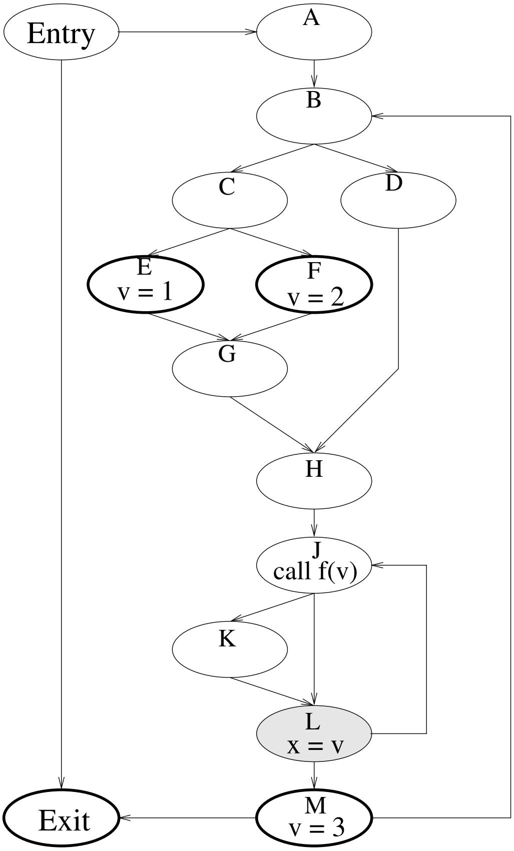


Figure 14.43: Example flow graph for *live variables*. The function `f` potentially assigns v but does not read its value.

Consider analysis of the liveness of variable v in Figure 14.43. The shaded nodes contain uses of v, which make v *live* when viewed from above such nodes. On the other hand, the dark-circled nodes destroy the current value of v. Such nodes represent future behavior that makes v not live (i.e., *dead*). At the `Exit` node, we may assume v is dead since the program is over.

Figure 14.43 contains a node with a `call` instruction. How does this node affect the liveness of v? For instructive purposes, we assume that **interprocedural analysis** reveals that the function `f` *potentially* assigns v but does not use its value. In that case, the invoked function does not make v live. However, since `f` does not always modify v, the invoked function does not make v dead. This particular node therefore has *no effect* on the liveness of v.

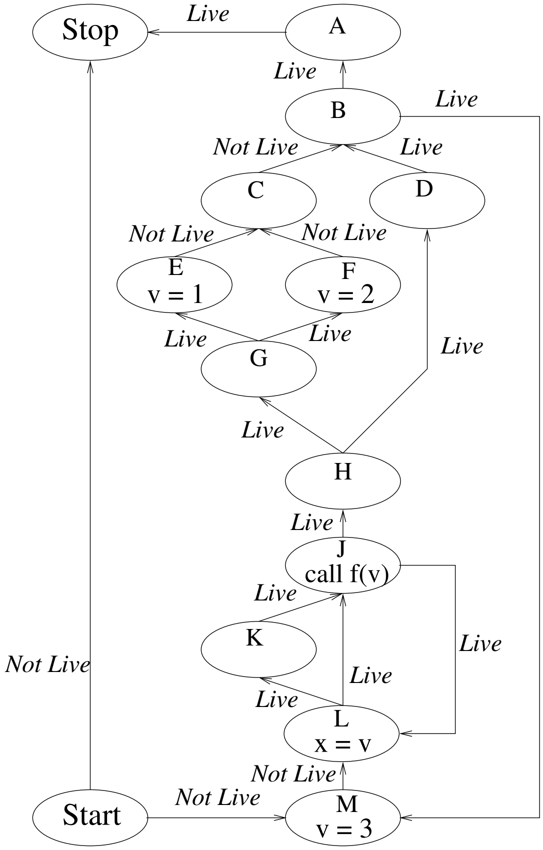


Figure 14.44: Solution for liveness of v.

The solution for liveness of v is shown in Figure 14.44. Note that the control flow edges are reversed to show how the computation is performed. Based on the definition of this problem, common points of control flow cause v to be live if *any* future behavior shows v to be live. For example, disparate solutions are combined with v live on input to (the bottom of) node B in Figure 14.44.

It would be advantageous for an optimizing compiler to show that a variable is dead. Any resources associated with a dead variable can be reclaimed by the compiler, including the variable's register or local JVM slot. Another use of *live variables* analysis is to find **potentially uninitialized variables**. Such variables are live at the Entry of a procedure.

An optimizing compiler may seek liveness information for one variable or for a set of variables. Exercise 36 considers the computation for a set.

14.4 Data Flow Frameworks

We have introduced the notion of a data flow framework informally, relying on examples drawn from optimizing compilers. In this section, we formalize the notion of a **data flow framework**. As we examine these details, it will be helpful to refer to the available expressions and *live variables* problems introduced in Section 14.3. The components of a data flow framework $\mathcal{D} = (\mathcal{G}_{eg}, L, \mathcal{F})$ are as follows:

- An **evaluation graph** $\mathcal{G}_{eg}$. This directed graph's nodes typically represent some aspect of a program's behavior. A node may represent a single instruction, a nonbranching sequence of instructions, or an entire procedure. The graph's edges represent a relation over the nodes. For example, the edges may indicate potential transfer of control by branching or by procedure call. We assume the graph's edges are oriented in the "direction" of the data flow problem.

- A **meet lattice** L. This is a mathematical structure that describes the solution space of the data flow problem and designates how to combine multiple solutions in a safe (conservative) way. It is convenient to present such lattices as **Hasse diagrams**. The **meet** of two elements can be found by tracing down the diagram from those elements until the traces first meet at a common point. For example, the lattice in Figure 14.45(a) specifies *Soln*1 and *Soln*2 meet at *Soln*3.

- A set of **transfer functions** $\mathcal{F}$. Each function models the behavior of a possible node (or path of nodes) with respect to the optimization problem under study. Figure 14.45(b) depicts a generic transfer function. A transfer function's input is the solution that holds on entry to the node. If multiple edges converge at the node, then a **meet** may be taken of those edges' solutions to form the node's input. The transfer function specifies how the node's output is computed given its input.

We next examine each of these components in some detail. Ryder and Marlowe offer a more thorough treatment of data flow frameworks and their properties [MR90].

14.4.1 Data Flow Evaluation Graph

The **data flow evaluation graph** is constructed for an optimization problem, so that evaluation of this graph produces a solution to the problem:

- Transfer functions are associated with each node.

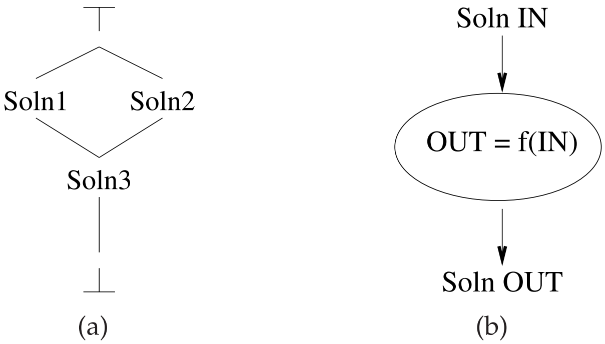


Figure 14.45: (a) A meet lattice; (b) A node's transfer function.

- Information converging at a node is combined as directed by the meet lattice.

- As described in Section 14.5, information is propagated through the data flow evaluation graph to obtain a solution.

For the problems considered here, a flow graph's nodes represent some component of a program's behavior and its edges represent potential transfer of control between nodes. In posing an optimization problem as a data flow framework, the resulting framework is said to be:

- **forward**, if the solution at a node can depend only on the program's past behavior. Evaluating such problems involves propagating information *forward* through the flow graph. Thus, the control flow graph in Figure 14.41(b) serves as the data flow evaluation graph in Figure 14.42 for analyzing the availability of the expression $v + w$ in the program of Figure 14.41(a).

- **backward**, if the solution at a node can depend only on the program's future behavior. The *live variables* problem introduced in Section 14.3.2 is such a problem. For *live variables*, a suitable data flow evaluation graph is the *reverse* control flow graph, as shown in Figure 14.44.

- **bidirectional**, if both past and future behavior is relevant.

In this chapter, we discuss only forward or backward problems, and we orient the edges of a data flow evaluation graph in the direction of the data flow problem. With this assumption, information always propagates in the direction of the graph's edges. It is convenient to augment data flow evaluation graphs with a Start and Stop node, and an edge from Start to Stop, as shown in Figures 14.42 and 14.44.

In compilers where space is at a premium, nodes of a control flow graph typically correspond to the *maximal* straight-line sequences (the **basic blocks**) of a program. While this design conserves space, program analysis and optimization must then occur at two levels: *within* and *between* the basic blocks. These two levels are called **local** and **global data flow analysis**, respectively. An extra level of analysis complicates our discussion and can increase the expense of writing, maintaining, and documenting an optimizing compiler. We therefore formulate data flow evaluation graphs whose nodes model the effects of perhaps a single JVM or MIPS® instruction.

14.4.2 Meet Lattice

As with all lattices, the **meet lattice** represents a partial order imposed on a set. Formally, the meet lattice is defined by the tuple

$$L = \left(A, \top, \bot, \preceq, \wedge\right)$$

which has the following components:

- A **solution space** A. In a data flow framework, the relevant set is the space of all possible solutions to the data flow problem. Exercise 36 considers the *live variables* problem, posed over a set of n variables. Since each variable is either live or not live, the set of possible solutions contains 2^n elements. Fortunately, we need not enumerate or represent all elements in this large set. In fact, some data flow problems (e.g., *constant propagation* discussed in Section 14.6) have an infinite solution space.

- The **meet operator** $\wedge$. The partial order present in the lattice directs how to combine (summarize) multiple solutions to a data flow problem. In Figure 14.42, the first node of the outer loop receives two edges—$v + w$ is available on one edge and not available on the other. The meet operation ($\wedge$) serves to summarize the two solutions. Mathematically, $\wedge$ is associative and commutative, so multiple solutions are easily summarized by applying $\wedge$ pairwise in any order.

- Distinguished elements $\top$ and $\bot$. Lattices associated with data flow frameworks always include the following distinguished elements of A:

  - $\top$ intuitively is the solution that allows the most optimization.
  - $\bot$ intuitively is the solution that prevents or inhibits optimization.

- The comparison operator $\preceq$. The meet lattice includes a reflexive partial order, denoted by $\preceq$. Given two solutions a and b from set A, it must be true that $a \preceq b$ or $a \npreceq b$. If $a \preceq b$, then solution a is no better than solution b. Further, if $a \prec b$, then solution a is strictly worse than b—optimization based on solution a will not be as good as with solution b. If $a \npreceq b$, then solutions a and b are incomparable.

  For example, consider the problem of *live variables*, computed for the set of variables $\{ v, w \}$. As discussed in Section 14.3.2, the storage associated with variables found *not* to be live can be reused. Thus, optimization is improved when fewer variables are found to be live. So, the set $\{ v, w \}$ is worse than the set $\{ v \}$ or the set $\{ w \}$. In other words, $\{ v, w \} \preceq \{ v \}$ and $\{ v, w \} \preceq \{ w \}$. However, the solution $\{ v \}$ cannot be compared with the set $\{ w \}$ ($\{ v \} \npreceq \{ w \}$). In both cases, one variable is live, and data flow analysis cannot prefer one to the other for *live variables*.

At this point, it is important to develop an intuitive understanding of the lattice, especially its distinguished elements $\top$ and $\bot$. For each analysis problem there is some solution that admits the greatest amount of optimization. This solution is always $\top$ in the lattice. Recalling *available expressions*, the best solution would make *every* expression available—all recomputations could then be eliminated. Correspondingly, $\bot$ represents the solution that admits the least amount of optimization. A simple device for remembering this arrangement is that $\top$ is always drawn at the top of a lattice diagram, as in Heaven (i.e., good optimization); $\bot$ is always drawn at the bottom, as in Hell (i.e., poor optimization).

For *available expressions*, $\bot$ implies that *no* expressions can be eliminated. For *live variables*, $\top$ represents that no variables are live, while $\bot$ represents that all variables are live. Recall that liveness means only that a variable's current value *might* be used in the future. Assurance that a variable's current value is useful is a different optimization problem, as considered by Exercise 38.

While the informal notions of $\top$ and $\bot$ are helpful for understanding data flow frameworks, a more rigorous specification of the lattice's properties is given in Figure 14.46.

Some texts introduce the **join lattice**, in which the best solution is $\bot$ and the worst solution is $\top$. Fortunately, it is possible to study all of data flow analysis using only the **meet lattices** we have introduced here. Intuitively, a join lattice can be turned into a meet lattice by flipping it upside down. The resulting data flow framework then solves the **complement** of the join lattice's problem. For example, a *dead variables* analysis could be performed on a join lattice, but we can solve *live variables* instead using a meet lattice.

Finally, we can consider data flow evaluation in the context of a meet lattice. While this is discussed more fully in Section 14.5, we can identify some points (elements) of interest in the lattice in addition to $\top$ and $\perp$:

- Some point in the lattice represents the best solution for a given data flow problem. That solution holds no matter what path is taken at runtime in the control flow graph. If each possible path is considered separately, then the best solution would be the **meet** of each of those path's solutions. Most programs contain loops and have apparently an infinite number of such paths.

  Nonetheless, for some problems (Section 14.5.4), it is possible to compute the lattice element known as the **meet over all paths** (MOP) solution even for programs with an infinite number of possible paths.

- Consider any lattice element b such that $b \preceq MOP$. Such an element is **safe** in the sense that optimization based on b cannot contradict information on any possible program path. The $\perp$ element is always safe but results in the least amount of optimization.

  Correspondingly, consider any element a for which $MOP \prec a$. Such an element is **unsafe** in the sense that optimization based on a may not properly preserve a program's meaning. The $\top$ element may or may not be unsafe, depending on MOP, because $MOP \preceq \top$.

- Data flow evaluation (Section 14.5) terminates by finding a safe solution at or below MOP. This solution is called the **maximum fixed point** (MFP) as it represents the fixed point of the computation and it is as good a solution as can be computed by an iterative approach that summarizes a node's inputs using **meet**.

These issues are considered again in Section 14.5.4.

14.4.3 Transfer Functions

Our data flow framework needs a mechanism to describe the effects of a fragment of program code—specifically, the code represented by a path through the flow graph. Consider a single node of the data flow evaluation graph. Referring to Figure 14.45(b), a solution is present on entry to the node and the transfer function is responsible for converting this input solution to a value that holds after the node executes. Mathematically, the node's behavior can be modeled as a *function* whose domain and range are the meet lattice A.

We denote the set of all such **transfer functions** in a data flow framework as $\mathcal{F}$. Each function must be **total**—defined on every possible input. Moreover, we shall require each function to behave **monotonically**—the function cannot produce a better result when presented with a worse input:

Property	Explanation
$a \wedge a = a$	The combination of two identical solutions is trivial.
$a \preceq b \iff a \wedge b = a$	If a is worse than b, then combining a and b must yield a; if $a = b$, then the combination is simply a, as above.
$a \wedge b \preceq a$ $a \wedge b \preceq b$	The combination of a and b can be no better than a or b.
$a \wedge \top = a$	Since $\top$ is the best solution, combining with $\top$ changes nothing.
$a \wedge \bot = \bot$	Since $\bot$ is the worst solution, any combination that includes $\bot$ will be $\bot$.

Figure 14.46: Meet lattice properties.

Definition 14.22 *A data flow framework is **monotone** iff*

$$(\forall\, a, b \in A)\ (\forall\, f \in \mathcal{F})\ \ a \preceq b \iff f(a) \preceq f(b)$$

Consider the *available expressions* problem, posed for the expressions $v + w$, $w + y$, and $a + b$. Figure 14.47 shows some fragments of a program and explains the transfer function that models each fragment's effects. The last example in Figure 14.47 shows the most general form of a transfer function for this problem. This is often written in the form $f(in) = (in - KILL) \cup GEN$, where *KILL* and *GEN* are node-specific constants that represent the expressions that become unavailable and available, respectively, due to the node's behavior. The complete set of transfer functions $\mathcal{F}$ for *available expressions* includes all such functions for every possible value of *KILL* and *GEN*. If there are n expressions in an *available expressions* problem, then there are 2^n possible values for *KILL*. The same argument holds for *GEN*, so that the total size of $\mathcal{F}$ is $O(2^n)$.

Because transfer functions are *mathematical*, they can model not only the effects of a single node but also the effects along any *path* through a program. If a node with transfer function f is followed by a node with transfer function g, then the cumulative effect of both nodes on input a is modeled by $g(f(a))$. In other words, any potential program behavior—brief or lengthy—is modeled

Fragment	Transfer Function	Explanation
v+w	$f(in) = in \cup \{v + w\}$	Regardless of which expressions are available on entry to this node, expression $v + w$ becomes available after the node. The other expressions are not affected by this node.
v = 9	$f(in) = in - \{v + w\}$	The assignment to v potentially changes the value of $v + w$, and the node includes no recomputation of this expression. Regardless of which expressions are available on entry to this node, expression $v+w$ is *not* available after the node. The same would be true of any expression that mentions v or w, but the availability of expression $a + b$ is not affected by this node.
print("hello")	$f(in) = in$	This node affects no expression; thus, the solution on exit from the node is identical to the solution on entry.
y = v+w	$f(in) = (in - \{w + y\})$ $\cup \{v + w\}$	This node makes $w + y$ unavailable because it changes y, but it makes $v + w$ available. This is the most general form of a transfer function for *available expressions*.

Figure 14.47: Data flow transfer functions for *available expressions*.

by some transfer function in $\mathcal{F}$. A transfer function can be applied to any lattice value a to obtain the compile-time estimation of the program fragment's behavior given the conditions represented by a.

14.5 Evaluation

Having specified a data flow problem as a data flow framework, we now turn to evaluating the framework to obtain an answer to the associated problem. Each flow graph node provides an equation in terms of the solution presented into that node. The challenge here is to determine an assignment of solutions from the lattice A that satisfies all of the equations while providing the best possible optimization. Section 14.5.1 describes an iterative approach to evaluate data flow frameworks. Each node initially asserts a solution of $\top$, which we take on faith as correct for now. We revisit the issue of **initialization** in Section 14.5.2. Evaluation continues until convergence is achieved with no changes in solution throughout the graph. The speed of reaching convergence and the quality of the solution are discussed in Sections 14.5.3 and 14.5.4.

14.5.1 Iteration

The most straightforward approach to evaluating a data flow framework is simply to iterate over the nodes and edges of the evaluation graph until convergence is reached. The simple iterative evaluation algorithm is shown in Figure 14.48. On visiting a given node Y, Marker (87) computes the input for the transfer function at Y as the **meet** of the current solutions at Y's predecessors in the evaluation graph. Recall that the edges of the evaluation graph are already oriented in the direction of the data flow problem. Marker (88) then establishes the new solution at node Y. Marker (89) detects if the new solution differs from the previous solution at Y. If the solution has changed, then Marker (90) forces another round of node evaluations.

When the algorithm has finished, we have computed the data flow problem's MFP solution. We say the solution is the **maximum** fixed point because it is as high (toward $\top$) in the lattice as possible while still being safe. This is related to the framework's initialization, as discussed in Section 14.5.2.

We next consider applying this algorithm to the example shown in Figure 14.49. Marker (86) does not specify an order in which nodes should be considered. If we consider nodes in the order $[\,2,5,4,3,1\,]$ then the evaluations occur as shown in Figure 14.50. This evaluation required four passes over $\mathcal{N}_{eg}$ as follows:

1. The input IN_Y to each node Y is the initialized value $\top$. Because the output of node 4 changes from its initialized value of $\top$ to its computed value $\bot$, another pass is required.

```
foreach $Y \in \mathcal{N}_{eg}$ do  $Soln(Y) \leftarrow \top$        84
repeat                                                              85
    $change \leftarrow$ false
    foreach $Y \in \mathcal{N}_{eg}$ do                             86
        $OldSoln \leftarrow Soln(Y)$
        $IN_Y \leftarrow \bigwedge_{X \in Preds(Y)} (Soln(X))$      87
        $Soln(Y) \leftarrow f_Y(IN_Y)$                              88
        if $Soln(Y) \neq OldSoln$                                   89
        then
            $change \leftarrow$ true                                90
until $change$ = false
```

Figure 14.48: Simple iterative evaluation. Inputs are a data flow framework $\mathcal{D}$ and an evaluation flow graph $\mathcal{G}_{eg}$. The algorithm computes $Soln(Y)$ for the solution at the output of each node Y.

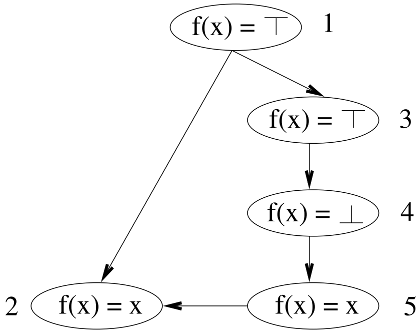


Figure 14.49: Example for iterative evaluation. Each node's transfer function and depth-first numbering are shown inside and adjacent to the node, respectively.

Node Y	*Preds*(Y)	IN_Y Marker 87	*Soln*(Y) Marker 88	Change? Marker 89
2	{1,5}	$\top$	$\top$	
5	{4}	$\top$	$\top$	
4	{3}	$\top$	$\bot$	**true**
3	{1}	$\top$	$\top$	
1	{ }	$\top$	$\top$	
2	{1,5}	$\top$	$\top$	
5	{4}	$\bot$	$\bot$	**true**
4	{3}	$\top$	$\bot$	
3	{1}	$\top$	$\top$	
1	{ }	$\top$	$\top$	
2	{1,5}	$\bot$	$\bot$	**true**
5	{4}	$\bot$	$\bot$	
4	{3}	$\top$	$\bot$	
3	{1}	$\top$	$\top$	
1	{ }	$\top$	$\top$	
2	{1,5}	$\bot$	$\bot$	
5	{4}	$\bot$	$\bot$	
4	{3}	$\top$	$\bot$	
3	{1}	$\top$	$\top$	
1	{ }	$\top$	$\top$	

Figure 14.50: Iterative evaluation using the node order $[2,5,4,3,1]$.

2. In the prior pass, node 4's value changed. Node 5's transfer function copies its input to its output, so node 5's output changes from $\top$ to $\bot$ during this pass. Thus, another pass is required.

3. When node 2 is considered in this pass, its input (formed by the *meet* of the output of nodes 1 and 5) is $\bot$. Node 2's transfer function copies its input to its output, so the output of node 5 changes from $\top$ to $\bot$.

4. Nothing changes in this pass, so iterative evaluation is finished.

The following observations can help to improve the performance of iterative evaluation:

- The change in solution at a given node Y need not require another round of evaluation of *all* nodes in $\mathcal{N}_{eg}$. The output of a transfer function can change only if its input changes. Thus, the loop at Marker 86 need only consider those nodes whose predecessors have a different solution. The set of nodes requiring evaluation can be maintained by a **worklist**, initialized to $\mathcal{N}_{eg}$, and updated to include all successors of a node Y whose solution has changed. Elements can be taken from the *worklist* for processing until the *worklist* is empty.

```
foreach $Y \in \mathcal{N}_{eg}$ do
    $Soln(Y) \leftarrow \top$                                          (91)
    $NeedEvaluate(Y) \leftarrow$ true
$NodeOrder \leftarrow$ [ Right-to-left preorder ]
repeat
    $again \leftarrow$ false
    foreach $\left(Y \in \mathcal{N}_{eg}\right)$ order ($NodeOrder$) do          (92)
        if $NeedsEvaluate(Y)$                                          (93)
        then
            $NeedsEvaluate(Y) \leftarrow$ false
            $OldSoln \leftarrow Soln(Y)$
            $IN_Y \leftarrow \bigwedge_{X \in Preds(Y)} (Soln(X))$                     (94)
            $Soln(Y) \leftarrow f_Y(IN_Y)$                                  (95)
            if $Soln(Y) \neq OldSoln$
            then
                foreach $Z \in Succs(Y)$ do
                    $NeedEvaluate(Z) \leftarrow$ true                        (96)
                    $again \leftarrow again$ or $Z \lhd Y$                        (97)
until $again$ = false
```

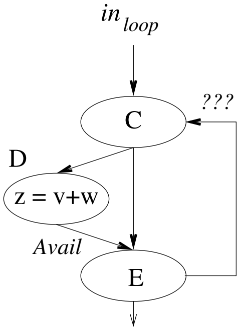
Figure 14.51: Better iterative evaluation.

- Nodes could be considered in a better order at Marker (86). If nodes are chosen in order [1, 3, 4, 5, 2], then 2 passes suffice instead of the 4 passes shown in Figure 14.50. Actually, the solution is achieved in a single pass, but the extra pass is required by Figure 14.48 to detect termination.

  To obtain the fastest results, a node *Y* should be visited only after all of its predecessors have updated their solutions. When an evaluation graph has a cycle, such ordering is possible only to a certain extent. The algorithm shown in Figure 14.11 is based on Definition 14.21. Nodes are visited in a right-to-left preorder traversal of their depth-first numbering (**interval order**). Except for back edges, this traversal visits all predecessors of *Y* before visiting *Y*. The right-to-left preorder traversal of the nodes in Figure 14.49 is [1, 3, 4, 5, 2].

- The outer loop at Marker (85) need execute only if information changes along a **back edge**. The test for a back edge ($\lhd$) at Marker (97) can be performed in **constant time**, as discussed in Section 14.2.4.

These improvements are incorporated in the algorithm shown in Figure 14.51. Marker (92) considers the nodes in interval order, using right-to-left preorder. Only those nodes marked for evaluation are considered at Marker (93).



(a)

Node	Transfer Function		
C	$f_C(in_C)$	$=$	in_C
D	$f_D(in_D)$	$=$	$\top$
E	$f_E(in_E)$	$=$	in_E

(b)

Figure 14.52: Is $v + w$ available throughout this loop? (a) Subgraph of Figure 14.41; (b) Transfer functions.

Marker (96) marks only those nodes for evaluation that are successors of a node whose output solution has changed. Marker (97) requests another pass over the nodes only if information changes along a back edge.

14.5.2 Initialization

We now return to the discussion of how to **initialize** a framework's solutions for iterative evaluation. In the algorithms of Figures 14.48 and 14.51, Markers (84) and (91) set all solutions initially to $\top$. For *available expressions*, this means that every node is initially assumed to make every expression available. This approach may seem unsound because we are initially assuming something that will likely turn out to be false. However, when the algorithm of Figure 14.51 is applied to the *available expressions* problem given in Figure 14.41, the answer shown in Figure 14.42 is correctly obtained (Exercise 29).

In fact, if we initially assume that all solutions are $\perp$ instead of $\top$, then we do not always compute the best possible answer (Exercise 30). If we view a data flow problem as seeking the solution to a set of equations, then an interesting issue can arise when a flow graph has loops. Figure 14.52 is a subgraph of our example from Figure 14.41(b). The edges in the subgraph show the interdependence of one node's solution on other nodes.

We know that the best, correct solution for this subgraph should show $v+w$ available everywhere (Figure 14.42). But how is such a solution determined? Figure 14.52(b) shows the transfer functions for the subgraph's nodes. Let in_{loop} denote the input to the loop—the input to node C from outside the loop, shown as *Avail* in Figure 14.52(a). The inputs to C and E are mathematically modeled as follows.

$$\begin{aligned}
in_E &= f_D(in_D) \wedge f_C(in_C) \\
&= \top \wedge f_C(in_C) \\
&= f_C(in_C) \\
in_C &= f_E(in_E) \wedge in_{loop} \\
&= in_E \wedge in_{loop} \\
&= f_C(in_C) \wedge in_{loop}
\end{aligned}$$

In other words, the input to node *C* depends on the output of node *C*! This circularity seems unresolvable, unless we reason that the first time we evaluate *C*'s transfer function, we can assume some prior result.

Computationally, the same problem arises. The iterative approach described in Section 14.5.1 will encounter *C* as the first node in Figure 14.52. The input to *C* must be evaluated at that point, but that input depends on the result coming from node *E*, which has not yet been examined. Because the graph contains a loop, looking at *E* before *C* does not offer any relief.

When we first evaluate the solution for node *C*, we have the following choices concerning the solution from *E*:

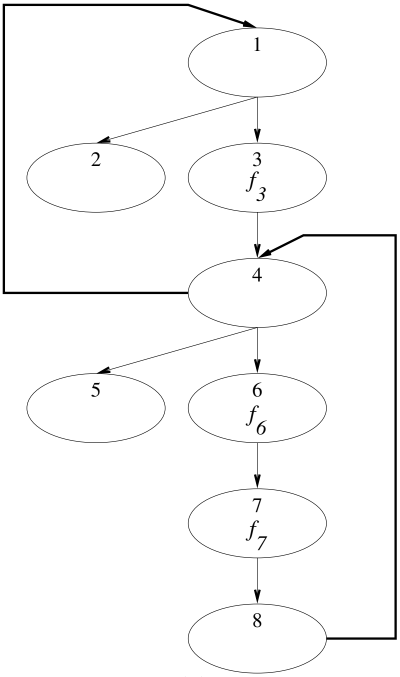
$$\begin{aligned}
f_E(in_E) &= \perp \text{ } (\textbf{optimistic}) \\
f_E(in_E) &= \top \text{ } (\textbf{pessimistic})
\end{aligned}$$

It is safe to assume that $v + w$ is *not available* previously from *E*, in which case we initially assume pessimistically that $f_E(in_E) = \perp$. However, when $\perp$ meets in_{loop}, we obtain $\perp$ as the input to node *C*. Propagation forward would then show $v+w$ to be unavailable everywhere in Figure 14.52(a), except on the edge marked *Avail*. This solution is safe, but not as good as we can obtain.

It is more daring to make the optimistic assumption that $v + w$ *is* available: $f_E(in_E) = \top$. Based on this assumption, we obtain the result shown in Figure 14.42, with $v + w$ available everywhere in the inner loop. We can safely initialize all solutions to $\top$, counting on the meet operator to combine all facts safely throughout the computation. If a given point in the flow graph is unsafe at $\top$, then it will be lowered in the lattice as evaluation proceeds to a safe value.

14.5.3 Termination and Rapid Frameworks

Because data flow problems are solved at compile-time, they are expected to terminate. Monotonicity (Definition 14.22) helps in this regard, because solutions at any point in a flow graph can only move toward $\perp$ as evaluation proceeds. However, to ensure convergence, the distance from any data flow solution to $\perp$ must be bounded:



(a)

Data Flow Framework Specification

A	$2^{\{1,2,3,4,5,6,7,8\}}$
$\top$	$\emptyset$
$\bot$	$\{1,2,3,4,5,6,7,8\}$
$a \preceq b$	$a \supseteq b$
$a \wedge b$	$a \cup b$
Node	**Transfer Function**
3	$f_3(in) = in \cup \{3\}$
6	$f_6(in) = in \cup \{6\}$
7	$f_7(in) = in \cup \{7\}$
all others	$f(in) = in$

(b)

Figure 14.53: (a) Flow graph; (b) Data flow framework.

Definition 14.23 *A lattice has* ***finite descending chains*** *if* $\forall\ a \in A$*, the path from* a *to* $\bot$ *in the lattice is finite.*

The above requirement does not insist that A is finite, but it does require a limit on the number of times a solution can move toward $\bot$ without actually reaching $\bot$. Iterative evaluation can continue only if the solution at some node continues to change. For monotone frameworks (Definition 14.22), a solution at any point in the flow graph progresses toward $\bot$ if it changes at all. Thus, termination is guaranteed for any data flow problem whose lattice has finite descending chains.

Termination is clearly essential for the optimization phase of a compiler. Because an optimization phase typically calls for the solution of multiple data flow problems, some understanding of the computational cost of each problem is necessary for crafting the optimization phase. In this section we study a class of data flow problems that converge as quickly as can be expected.

A flow graph and data flow framework are given in Figure 14.53. The meet lattice's domain A is the set of all subsets (**power set**) of $\{1,2,3,4,5,6,7,8\}$. If it-

Node	*Preds*(Y)	IN_Y	*Soln*(Y)	*NeedEvaluate*(Z)	Again?
Y		Marker 94	Marker 95	Marker 96	Marker 97
1	$\{4\}$	$\{\}$	$\{\}$	$\{\}$	
3	$\{1\}$	$\{\}$	$\{3\}$	$\{4\}$	
4	$\{3,8\}$	$\{3\}$	$\{3\}$	$\{5,6,1\}$	**true**
6	$\{4\}$	$\{3\}$	$\{3,6\}$	$\{7\}$	
7	$\{6\}$	$\{3,6\}$	$\{3,6,7\}$	$\{8\}$	
8	$\{7\}$	$\{3,6,7\}$	$\{3,6,7\}$	$\{4\}$	**true**
5	$\{4\}$	$\{3\}$	$\{3\}$	$\{\}$	
2	$\{1\}$	$\{\}$	$\{\}$	$\{\}$	
1	$\{4\}$	$\{3\}$	$\{3\}$	$\{2,3\}$	
3	$\{1\}$	$\{3\}$	$\{3\}$	$\{4\}$	
4	$\{3,8\}$	$\{3,6,7\}$	$\{3,6,7\}$	$\{5,6,1\}$	**true**
6	$\{4\}$	$\{3,6,7\}$	$\{3,6,7\}$	$\{7\}$	
7	$\{6\}$	$\{3,6,7\}$	$\{3,6,7\}$	$\{\}$	
8	Not evaluated				
5	$\{4\}$	$\{3,6,7\}$	$\{3,6,7\}$	$\{\}$	
2	$\{1\}$	$\{3\}$	$\{3\}$	$\{\}$	
1	$\{4\}$	$\{3,6,7\}$	$\{3,6,7\}$	$\{2,3\}$	
3	$\{1\}$	$\{3,6,7\}$	$\{3,6,7\}$	$\{4\}$	
4	$\{3,8\}$	$\{3,6,7\}$	$\{3,6,7\}$	$\{\}$	
6	Not evaluated				
7	Not evaluated				
8	Not evaluated				
5	Not evaluated				
2	$\{1\}$	$\{3,6,7\}$	$\{3,6,7\}$	$\{\}$	

Figure 14.54: Evaluation of the framework from Figure 14.53.

erative evaluation proceeds using the graph's interval order $[1,3,4,6,7,8,5,2]$ then the computation proceeds as shown in Figure 14.54.

The first pass propagates the solution $\{3,6,7\}$ to node 8. After that first pass, information from node 8 is available for node 4. After the second pass, the information from node 8, present now at node 4, is available for node 1. This example demonstrates that for some frameworks the number of passes required for convergence is related to the number and structure of the graph's back edges. Each pass can propagate all available information forward in topological order, but extra passes are required to propagate information along the back edges.

If p is the longest path in the flow graph comprised only of back edges, then the length of p determines how many passes are required to obtain convergence

```
$b \leftarrow 15$
$w \leftarrow 3$
$d \leftarrow 127$
/⋆ At this point, b takes 4 bits; w takes 2 bits; d takes 7 bits ⋆/
$q \leftarrow w$                                                  98
$a \leftarrow b$
$w \leftarrow a$                                                  99
$b \leftarrow a \odot d$                                          100
```

Figure 14.55: Program for the *number of bits* problem. The ⊙ operator performs some bit-wise operation, creating a result whose size is the same as the larger of the two operands.

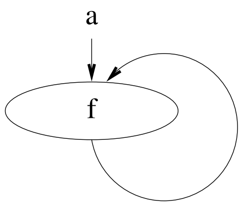


Figure 14.56: Flow graph to illustrate a rapid framework.

if the data flow problem is **rapid** [KU76, Ros78]:

Definition 14.24 *A data flow framework is* ***rapid*** *iff*

$$\forall\, a \in A, \forall\, f \in \mathcal{F},\ a \wedge f(\top) \preceq f(a)$$

To appreciate this definition, consider the flow graph shown in Figure 14.56. The path through the node will apply the transfer function f. Iterative evaluation would compute the loop edge as having the value $f(a)$. The next time through, evaluation will compute the meet of the two values, $a \wedge f(a)$, as the new input to f, computing $f(a \wedge f(a))$. This process continues until the output of the node stops "lowering" in the lattice.

If the framework in Figure 14.56 is rapid, then for every $a \in A$, $a \wedge f(\top) \preceq f(a)$. In other words, f acting on the best possible information ($\top$) can meet a, and the result is no better than if f were to act on a itself. In terms of the data flow lattice, $a \wedge f(\top)$ arrives at an element that is the same as, or lower in the lattice than, $f(a)$, which drives us toward convergence at least as well as if f had a chance to act on a in the evaluation.

Exercises 38, 32, and 42 investigate the rapidness of the frameworks we have studied thus far. It is also instructive to study the following data flow framework that is *not* **rapid**. Although most computers provision an entire word of storage to hold an integer value, program optimization can try to determine the *number of bits* that are actually required to represent all values held by a given variable name. This analysis involves examining the values that a given name can hold. Of course, if such information is unavailable or too difficult to discern, then $\perp$ can indicate that a name should occupy a full word.

Figure 14.55 shows a simple program (no branches or loops) for analyzing the right *number of bits*. After Marker 98, q requires 2 bits, because the value held in w requires that many bits. After $a \leftarrow b$, a requires the same 4 bits needed for b. After Marker 99, w also needs 4 bits. The assignment at Marker 100 takes 7 bits—the maximum number of bits taken by a (4) and d (7).

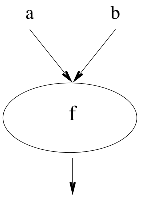
To study the rapidness of the *number of bits* problem, Figure 14.57 is the flow graph from Figure 14.53(a), with the statements from Figure 14.55 placed in some of the graph's nodes. Iterative evaluation of this problem proceeds as shown in Figure 14.58. Any "news" about the problem that hits node 8 takes two more passes to propagate to node 1. If the *number of bits* problem were **rapid**, then any instance of the problem for the flow graph shown in Figure 14.53(a) should converge in 3 passes, because of the number and structure of that graph's back edges. However, Figure 14.58 shows 7 iterations are required for convergence. Exercise 50 explores this further.

14.5.4 Distributive Frameworks

We next turn to an examination of the **quality** of the solution computed by data flow analysis. We begin with the following lemma:

Lemma 14.25 *Given a monotone data flow framework* $\mathcal{D} = (\mathcal{G}_{eg}, L, \mathcal{F})$,

$$(\forall\ f \in \mathcal{F})\ (\forall\ a, b \in A)\ \ f(a \wedge b) \preceq f(a) \wedge f(b)$$



Proof: Left as Exercise 40. □

Lemma 14.25 is the essence of a data flow framework's approximation of a program's actual behavior. On arrival to the node shown in Lemma 14.25, one of a or b is presented. The effect of executing f on either input is either $f(a)$ or $f(b)$. Thus, f's behavior given either input is best summarized as $f(a) \wedge f(b)$. However, iterative analysis does not allow f to act separately on a or b. Instead, $a \wedge b$ is computed, and f is applied to that lattice element.

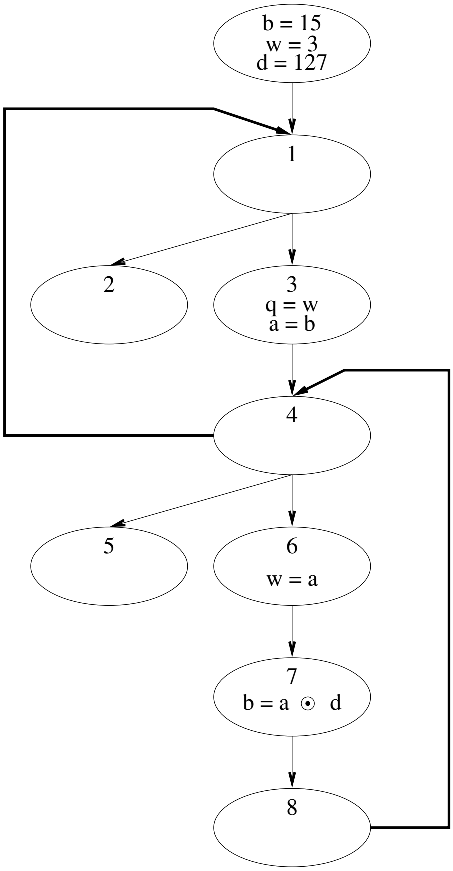


Figure 14.57: Another instance of the *number of bits* problem.

Lemma 14.25 effectively states that the result computed by iterative analysis ($f(a \wedge b)$) can be no better than what would actually occur when the program takes either path ($f(a) \wedge f(b)$).

For some data flow problems, we are fortunate in that we can strengthen Lemma 14.25 to obtain the following:

Definition 14.26 *A data flow framework $\mathcal{D} = (\mathcal{G}_{eg}, L, \mathcal{F})$ is **distributive** iff*

$$(\forall\ f \in \mathcal{F})\ (\forall\ a, b \in A)\ \ f(a \wedge b) = f(a) \wedge f(b)$$

For such frameworks, the meet operator ($\wedge$) loses no information, and we obtain the best solution possible, assuming that any path through the program

Node	*Preds*(*Y*)	IN_Y	*Soln*(*Y*)		Again?
Y		94	95	96	97
1	{4}	[7,4,0,2,0]	[7,4,0,2,0]	{2,3}	
3	{1}	[7,4,0,2,0]	[7,4,4,2,2]	{4}	
4	{3,8}	[7,4,4,2,2]	[7,4,4,2,2]	{5,6,1}	**true**
6	{4}	[7,4,4,2,2]	[7,4,4,4,2]	{7}	
7	{6}	[7,4,4,4,2]	[7,7,4,4,2]	{8}	
8	{7}	[7,7,4,4,2]	[7,7,4,4,2]	{4}	**true**
5,2	Don't matter				
1,3	Don't matter				
4	{3,8}	[7,7,4,4,2]	[7,7,4,4,2]	{5,6,1}	**true**
6,7,8,5,2	Don't matter				
1	{4}	[7,7,4,4,2]	[7,7,4,4,2]	{2,3}	
3	{1}	[7,7,4,4,2]	[7,7,7,4,4]	{4}	
4	{3,8}	[7,7,7,4,4]	[7,7,7,4,4]	{5,6,1}	**true**
6	{4}	[7,7,7,4,4]	[7,7,7,7,4]	{7}	
7	{6}	[7,7,7,7,4]	[7,7,7,7,4]	{8}	
8	{7}	[7,7,7,7,4]	[7,7,7,7,4]	{4}	**true**
5,2	Don't matter				
1,3	Don't matter				
4	{3,8}	[7,7,7,7,4]	[7,7,7,7,4]	{5,6,1}	**true**
6,7,8,5,2	Don't matter				
1	{4}	[7,7,7,7,4]	[7,7,7,7,4]	{2,3}	
3	{1}	[7,7,7,7,4]	[7,7,7,7,7]	{4}	
4	{3,8}	[7,7,7,7,7]	[7,7,7,7,7]	{5,6,1}	**true**
6	{4}	[7,7,7,7,7]	[7,7,7,7,7]	{7}	
7	{6}	[7,7,7,7,7]	[7,7,7,7,7]	{8}	
8	{7}	[7,7,7,7,7]	[7,7,7,7,7]	{4}	**true**
5,2	Don't matter				
1,3	Don't matter				
4	{3,8}	[7,7,7,7,7]	[7,7,7,7,7]	{5,6,1}	**true**
6,7,8,5,2	Don't matter				
1	{4}	[7,7,7,7,7]	[7,7,7,7,7]	{2,3}	
3	{1}	[7,7,7,7,7]	[7,7,7,7,7]	{4}	
4	{3,8}	[7,7,7,7,7]	[7,7,7,7,7]	{5,6,1}	
6	{4}	[7,7,7,7,7]	[7,7,7,7,7]	{7}	
7	{6}	[7,7,7,7,7]	[7,7,7,7,7]	{8}	
8	{7}	[7,7,7,7,7]	[7,7,7,7,7]	{4}	
5	{4}	[7,7,7,7,7]	[7,7,7,7,7]	{ }	
2	{1}	[7,7,7,7,7]	[7,7,7,7,7]	{ }	

Figure 14.58: Evaluation of Figure 14.57. Each 5-tuple represents the *number of bits* solution for the 5 variables $[d,b,a,w,q]$.

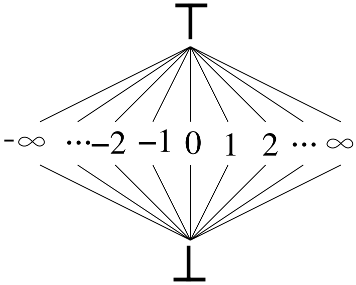


Figure 14.59: Lattice for constant propagation of a single value.

can be taken. This solution is called the MOP solution. Thus, for distributive frameworks, MOP=MFP. Examples of distributive frameworks include the *available expressions* and *live variables* problems. Exercises 44, 46, and 49 explore the distributive nature of data flow frameworks in greater detail.

14.6 Constant Propagation

The last data flow problem we study in detail is *constant propagation*, which determines values that are constant over all executions of a program. Most programmers do not intentionally introduce constant expressions. However, such expressions often arise as artifacts of program translation. Interprocedurally, constants can develop when a general method is specialized with arguments that are constant.

We begin by considering the lattice that models a single constant value, as shown in Figure 14.59. The lattice is conceptually infinite in width, unless some bound is placed on constant values. No one constant is related ($\leq$) to any other, so the constants are all at the same level in the lattice. The distinguished element $\bot$ represents that a value is *not* constant. The lattice reflects that if two different constants meet ($\wedge$), then the result is $\bot$.

The distinguished element $\top$ requires some explanation. Lattice axioms require that $\forall\, a \in A,\ a \wedge \top = a$. Thus, $\top$ cannot be any particular constant. Instead, $\top$ represents a value that could be chosen by the compiler to suit its purposes. Recalling that $\top$ is the initialized value for data flow evaluation, an uninitialized variable can take on any value of interest without contradiction.

More generally, consider the program whose control flow graph is shown in Figure 14.60. Some nodes assign constant values to variables. In other nodes, constant values may combine to create other constant values. We can describe constant propagation as a data flow problem as follows:

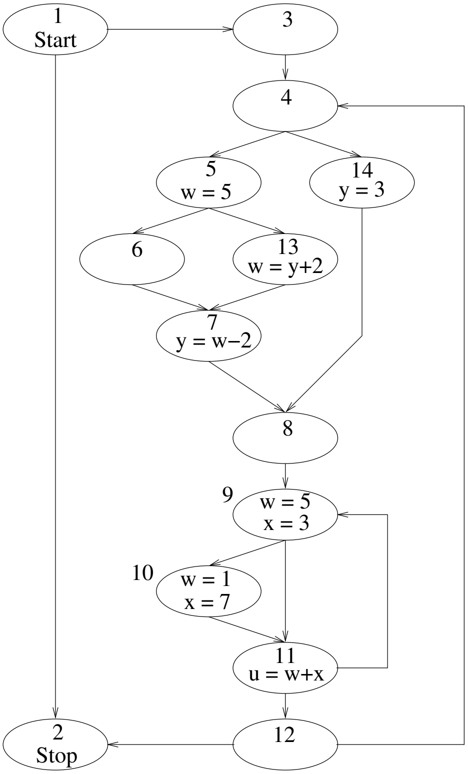


Figure 14.60: A program for constant propagation. The depth-first number of each node is shown near the top of each node.

- We pose the constant propagation over a program's *variables*. If the program contains an expression or subexpression of interest, then this can be assigned to a temporary variable. Constant propagation can then try to discover a constant value for the temporary variable.

- For each variable, we formulate the three-tiered lattice shown in Figure 14.59.

  - $\top$ means that the variable is considered a constant of an (as yet) undetermined value.
  - $\perp$ means that the expression is not constant.
  - Otherwise, the expression has a constant value found in the middle layer.

- The meet operator is applied using the lattice shown in Figure 14.59. The lattice is applied separately for each variable of interest.

- The transfer function at node Y interprets the node by substituting the node's incoming solution for each variable used in the node's expression. Suppose node v is computed using the variables in set U. The solution for variable v after node Y is computed as follows:

  - If any variable in U has value $\bot$, then v has value $\bot$. For example, if x has value $\bot$, then the expression $w + x$ has value $\bot$ even if w is constant or $\top$.
  - Otherwise, if any variable in U has value $\top$, then v has value $\top$. For example, if y has value $\top$, then the expression $y + 2$ is also $\top$.
  - Finally, if all variables in U have constant value, then the expression is evaluated and the constant value is assigned to v.

Figure 14.61 shows the evaluation of Figure 14.60 by the algorithm from Figure 14.51. Initially, the value of every solution is $\top$. Each node is then considered in interval order: $[1, 3, 4, 14, 5, 13, 6, 7, 8, 9, 10, 11, 12, 2]$. Nodes ⑤ and ⑭ are straightforward, in that they assert values for w and y, respectively, regardless of their input solutions. When ⑬ is evaluated, the current value for y is $\top$, which could be any constant. Thus, with $w = y + 2$, w can also be any constant and its solution is emitted as $\top$. When ⑦ is evaluated, the input value for y is computed as the meet of ⑥'s value of 5 with ⑬'s value of $\top$. Thus, at least for this iteration, w has the value 5 and ⑦ computes y using $y = w - 2 = 3$. It is important to notice that y is ultimately found to be the constant 3 only because we initially gave it a value of $\top$.

The rest of the nodes are processed in the first iteration as shown in Figure 14.61, but no constants are found except those emitted by a single node based on an assignment from a constant. In the second iteration, the solutions for all the variables are propagated along the back edges, but all solutions converge in this iteration.

The example in Figure 14.60 serves to illustrate the following features of *constant propagation*:

- The graph has a path to ⑬ that avoids initialization of y. Data flow evaluation in Figure 14.61 nonetheless finds that y is constant almost everywhere in the flow graph. Although an uninitialized variable may indicate a programming error, the optimizer can assume an uninitialized variable has any value of choice without fear of contradiction.

  If a programming language's semantics insist on implicit initialization of all variables (e.g. to 0), then such initialization must be represented by an assignment to these variables at the Start node.

Node *Y*	IN_Y Marker 94				*Soln*(*Y*) Marker 95			
	u	*w*	*x*	*y*	*u*	*w*	*x*	*y*
①③④	No change				⊤	⊤	⊤	⊤
⑭	④ ⊤	④ ⊤	④ ⊤	④ ⊤	⊤	⊤	⊤	3
⑤	④ ⊤	④ ⊤	④ ⊤	④ ⊤	⊤	5	⊤	⊤
⑬	⑤ ⊤	⑤ 5	⑤ ⊤	⑤ ⊤	⊤	⊤	⊤	⊤
⑥	⑤ ⊤	⑤ 5	⑤ ⊤	⑤ ⊤	⊤	5	⊤	⊤
⑦	⑥ ⑬ ⊤ ∧ ⊤ = ⊤	⑥ ⑬ 5 ∧ ⊤ = 5	⑥ ⑬ ⊤ ∧ ⊤ = ⊤	⑥ ⑬ ⊤ ∧ ⊤ = ⊤	⊤	5	⊤	3
⑧	⑦ ⑭ ⊤ ∧ ⊤ = ⊤	⑦ ⑭ 5 ∧ ⊤ = 5	⑦ ⑭ ⊤ ∧ ⊤ = ⊤	⑦ ⑭ 3 ∧ 3 = 3	⊤	5	⊤	3
⑨	⑧ ⑪ ⊤ ∧ ⊤ = ⊤	⑧ ⑪ 5 ∧ ⊤ = 5	⑧ ⑪ ⊤ ∧ ⊤ = ⊤	⑧ ⑪ 3 ∧ ⊤ = 3	⊤	5	3	3
⑩	⑨ ⊤	⑨ 5	⑨ 3	⑨ 3	⊤	1	7	3
⑪	⑩ ⑨ ⊤ ∧ ⊤ = ⊤	⑩ ⑨ 1 ∧ 5 = ⊥	⑩ ⑨ 7 ∧ 3 = ⊥	⑩ ⑨ 3 ∧ 3 = 3	⊥	⊥	⊥	3
⑫	⑪ ⊥	⑪ ⊥	⑪ ⊥	⑪ 3	⊥	⊥	⊥	3
②	① ⑫ ⊤ ∧ ⊥ = ⊥	① ⑫ ⊤ ∧ ⊥ = ⊥	① ⑫ ⊤ ∧ ⊥ = ⊥	① ⑫ ⊤ ∧ 3 = 3	⊥	⊥	⊥	3
①③	No change				⊤	⊤	⊤	⊤
④	③ ⑫ ⊤ ∧ ⊥ = ⊥	③ ⑫ ⊤ ∧ ⊥ = ⊥	③ ⑫ ⊤ ∧ ⊥ = ⊥	③ ⑫ ⊤ ∧ 3 = 3	⊥	⊥	⊥	3
⑭	④ ⊥	④ ⊥	④ ⊥	④ 3	⊥	⊥	⊥	3
⑤	④ ⊥	④ ⊥	④ ⊥	④ 3	⊥	5	⊥	3
⑬	⑤ ⊥	⑤ 5	⑤ ⊥	⑤ 3	⊥	5	⊥	3
⑥	⑤ ⊥	⑤ 5	⑤ ⊥	⑤ 3	⊥	5	⊥	3
⑦	⑥ ⑬ ⊥ ∧ ⊥ = ⊥	⑥ ⑬ 5 ∧ 5 = 5	⑥ ⑬ ⊥ ∧ ⊥ = ⊥	⑥ ⑬ 3 ∧ 3 = 3	⊥	5	⊥	3
⑧	⑦ ⑭ ⊥ ∧ ⊥ = ⊥	⑦ ⑭ 5 ∧ ⊥ = ⊥	⑦ ⑭ ⊥ ∧ ⊥ = ⊥	⑦ ⑭ 3 ∧ 3 = 3	⊥	⊥	⊥	3
⑨	⑧ ⑪ ⊥ ∧ ⊥ = ⊥	⑧ ⑪ ⊥ ∧ ⊥ = ⊥	⑧ ⑪ ⊥ ∧ ⊥ = ⊥	⑧ ⑪ 3 ∧ ⊤ = 3	⊥	5	3	3
⑩	⑨ ⊥	⑨ 5	⑨ 3	⑨ 3	⊥	1	7	3
⑪⑫②	No change				⊥	⊥	⊥	3

Figure 14.61: Evaluation of *constant propagation* for Figure 14.60.

- Both w and x have constant (albeit different) values at nodes ⑩ and ⑨. When a meet is taken coming into ⑪, w's value is computed as the meet of 1 and 5, which yields $\perp$. Similarly, the value for x coming into ⑪ is $\perp$. Thus, the expression $w + x$ inside ⑪ is computed as $\perp$.

  While the variables w and x have different values coming into ⑪, their sum does not. If the transfer function at ⑪ is applied to the values emitted from ⑨, then $w + x = 5 + 3 = 8$. Applying the transfer function to the output of ⑩ yields $w + x = 1 + 7 = 8$. The meet of those two values finds $w = 8$, but this solution is not computed by the iterative algorithm of Figure 14.51.

  This observation is actually a proof that *constant propagation* is *not* a **distributive framework**.

14.7 SSA Form

SSA form is introduced in Chapter 10 as an intermediate language in which each variable name is assigned exactly once. Figure 10.5 on page 412 shows a program before and after its translation into SSA form. We can now study how this translation is accomplished, as it is based on some of the advanced compiler structures presented in this chapter. Figure 14.62 shows a program and its control flow graph. The SSA construction algorithm requires the graph's dominator tree and dominance frontiers, which are shown in Figure 14.63.

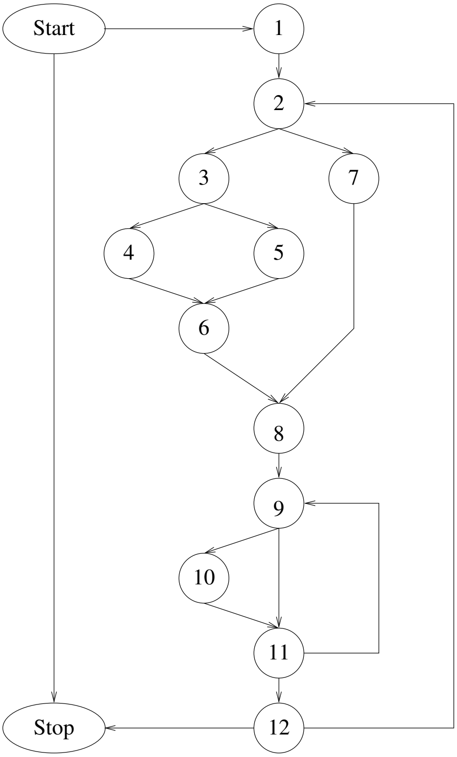
Based on these structures, there are two phases to the SSA construction algorithm. The algorithm shown Figure 14.64 computes SSA form variable by variable, with the phases shown in Figures 14.65 and 14.67.

In the first phase, the location of each ϕ-function is determined. Each ϕ-function represents the convergence (i.e., meet) of two or more names of a given variable. The **arity** (number of parameters) for a given ϕ-function is determined by the number of edges into the node containing the function. All nodes with ϕ-functions must have at least two incoming edges. In Figure 14.62(b), examples of such nodes include 8, 9, and 11, but node 10 would never host a ϕ-function.

While having at least two inedges is necessary for a node to host a ϕ-function, it is not sufficient. At least two distinct names for a given variable must reach a node to require a ϕ-function for the name. If Figure 14.62(b) contained an assignment to a variable x in node 1, then the only node to receive a ϕ-function for x would be the Stop node. While nodes such as 8 have two inedges, the same name for variable x flows on both edges, so no ϕ-function is necessary.

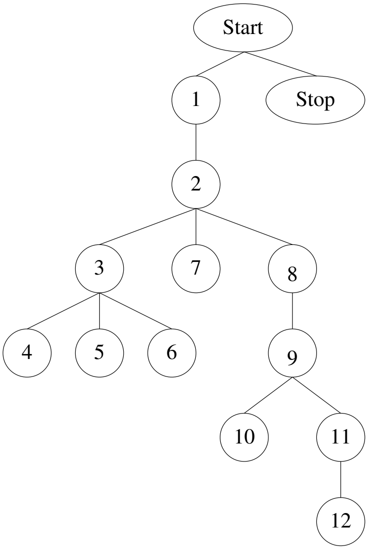
```
$i \leftarrow 1$                          ①
$j \leftarrow 1$
$k \leftarrow 1$
$l \leftarrow 1$
repeat                                    ②
    if $p$
    then
        $j \leftarrow i$                  ③
        if $q$
        then
            $l \leftarrow 2$              ④
        else
            $l \leftarrow 3$              ⑤
        $k \leftarrow k + 1$              ⑥
    else
        $k \leftarrow k + 2$              ⑦
    call PRINT$(i, j, k, l)$              ⑧
    repeat                                ⑨
        if $r$
        then
            $l \leftarrow l + 4$          ⑩
                                          ⑪
    until $s$
    $i \leftarrow i + 6$                  ⑫
until $t$
```

(a)



(b)

Figure 14.62: (a) Program and (b) its control flow graph. The node's numbers correspond to the program's markers, not to a depth-first numbering. This example is from [CFR+91].



(a)

Node Y	$DF(Y)$
Start	{ }
1	{Stop}
2	{2, Stop}
3	{8}
4	{6}
5	{6}
6	{8}
7	{8}
8	{2, Stop}
9	{2, 9, Stop}
10	{11}
11	{2, 9, Stop}
12	{2, Stop}
Stop	{ }

(b)

Figure 14.63: (a) Dominator tree and (b) Dominance frontiers for the control flow graph in Figure 14.62.

```
foreach V ∈ Variables do
    call PLACEPHIS(V)        (101)
    call RENAME(V)           (102)
```

Figure 14.64: Algorithm for computing SSA form.

14.7.1 Placing ϕ-Functions

The structure we use for determining where to place ϕ-functions is the **dominance frontier graph**, which is summarized for our example in Figure 14.63. If a definition of a variable V occurs at node X, then $DF(X)$ is the set of nodes where that definition will meet other definitions. Thus a ϕ-function for V is required at every node in $DF(Y)$. Suppose node 4 in Figure 14.62 contained an assignment to variable V. Consulting $DF(4)$ in Figure 14.63, node 6 requires a ϕ-function. We know the form of the code introduced at node 6 must be:

$$V = \phi(V, V)$$

At node 6, two different names for V will reach the node, but we do not yet know which ones. The result of the ϕ-function is a new assignment to V. That

```
/*   Called from Marker (101)                                            */
procedure PLACEPHIS( $V$ )
    foreach $node \in \mathcal{N}_{cg}$ do $node.hasPhi \leftarrow node.processed \leftarrow$ false   (103)
    foreach $def \in defs(V)$ do call ADDNODE( $def$.GETNODE( ) )                                  (104)
    $worklist \leftarrow \emptyset$
    while $worklist \neq \emptyset$ do                                                               (105)
        $X \leftarrow worklist$.PICKANDREMOVE( )
        foreach $Y \in DF(X)$ do
            if not $Y.hasPhi$                                                                        (106)
            then
                $Y.hasPhi \leftarrow$ true
                At node $Y$, place $V \leftarrow \phi(V, \ldots, V)$
                call ADDNODE( $Y$ )                                                                  (107)
end

procedure ADDNODE( $node$ )
    if not $node.processed$
    then
        $worklist \leftarrow worklist \cup \{ node \}$
        $node.processed \leftarrow$ true
end
```

Figure 14.65: Placement of ϕ-functions.

assignment may in turn meet other names for V, so the ϕ-placement algorithm iterates over dominance frontiers until all nodes needing ϕ-functions have them.

The algorithm for placing ϕ-functions is given in Figure 14.65. The method PLACEPHIS is called separately for each variable V in a program. Prior to processing the variable V, Marker (104) sets the per-node flags *hasPhi* and *processed* to **false**.

- The *hasPhi* flag keeps track of whether its associated node already has a ϕ-function for the current variable V. Only one such function is needed at any node.

- The *processed* flag keeps track of whether a definition of V has been put on the *worklist* for the current variable V.

Marker (105) considers definition sites for V in any order. For each node X that defines V, Marker (106) ensures that every node Y in $DF(X)$ has a ϕ-function for V. Recall that the arity of the ϕ-function is determined by the number of inedges to node Y. Each ϕ-function placed in the program is itself a definition site for V. Marker 107 makes sure that the definition site is considered by the algorithm for further ϕ placement. Figure 14.66 shows the results of placing ϕ-functions for our example.

14.7.2 Renaming

The final step in constructing SSA form is to rename all of the definition sites so they are unique. While we simply add subscripts to create unique names, the variables i_2 and i_3 are as distinct from each other as are x and y. The subscripts enable us to visually track the origin of each name and to see the progress of the renaming algorithm.

The primary structure used in the renaming algorithm is the **dominator tree**, although the control flow graph is also consulted. Each definition site is given a unique name by adding a subscript, as described above. The challenge for this part of SSA construction is determining the unique definition site that reaches a given use of a variable name. There are two cases:

- Each original (non-ϕ) use of a variable name v is reached by the definition of v that appears in the node that most closely dominates that use.
- A given use of v in a ϕ-function is reached by the definition of v that flows on its associated inedge to the node containing the ϕ-function. From the definition of **dominance frontiers**, if a ϕ-function appears at some node Z, then Z is in the dominance frontier of X, and X must dominate some predecessor Y of Z. The definition of v that reaches Y is the definition that reaches the use of v using the edge (Y, Z) into the ϕ-function at Z.

  When the algorithm is at a node Y, it can check whether a ϕ-function exists at a successor Z of Y and forward the appropriate name for v to the ϕ-function.

The algorithm for renaming variables is in Figure 14.67, and the results of applying the algorithm to our example are shown in Figure 14.68.

Markers 108 and 109 initialize the *stack* and *version* variables, which are used in the renameHelper method. The *stack* keeps track of the version of the current variable V that reaches any ordinary uses. The *version* variable keeps track of the name that will be created for the next definition of V ($V_{version}$). The renaming algorithm makes the following assumptions:

- The *Start* node is assumed to contain a definition of every variable, which the algorithm will number as version 0. Thus, the *stack*, while initially empty, receives the definition of V from *Start* in the initial call to renameDomtree.

```
$i \leftarrow 1$                        ①
$j \leftarrow 1$
$k \leftarrow 1$
$l \leftarrow 1$
repeat                                  ②



    if $p$
    then
        $j \leftarrow i$                ③
        if $q$
        then
            $l \leftarrow 2$            ④
        else
            $l \leftarrow 3$            ⑤
                                        ⑥
        $k \leftarrow k + 1$
    else
        $k \leftarrow k + 2$            ⑦
                                        ⑧


    call PRINT$(i, j, k, l)$
    repeat                              ⑨

        if $r$
        then
            $l \leftarrow l + 4$        ⑩
                                        ⑪

    until $s$
    $i \leftarrow i + 6$                ⑫
until $t$
```

(a)

```
$i \leftarrow 1$                        ①
$j \leftarrow 1$
$k \leftarrow 1$
$l \leftarrow 1$
repeat                                  ②
    $i \leftarrow \phi(i, i)$
    $j \leftarrow \phi(j, j)$
    $k \leftarrow \phi(k, k)$
    $l \leftarrow \phi(l, l)$
    if $p$
    then
        $j \leftarrow i$                ③
        if $q$
        then
            $l \leftarrow 2$            ④
        else
            $l \leftarrow 3$            ⑤
        $l \leftarrow \phi(l, l)$       ⑥
        $k \leftarrow k + 1$
    else
        $k \leftarrow k + 2$            ⑦
    $j \leftarrow \phi(j, j)$           ⑧
    $k \leftarrow \phi(k, k)$
    $l \leftarrow \phi(l, l)$
    call PRINT$(i, j, k, l)$
    repeat                              ⑨
        $l \leftarrow \phi(l, l)$
        if $r$
        then
            $l \leftarrow l + 4$        ⑩
        $l \leftarrow \phi(l, l)$       ⑪

    until $s$
    $i \leftarrow i + 6$                ⑫
until $t$
```

(b)

Figure 14.66: (a) Program and (b) ϕ placement.

- At Marker 110, all uses of V in a node X are assumed to be **upwards exposed**, meaning that they are not reached by any definition of V in node X. If a node X has internal definitions and uses of V, then X could always be split into multiple nodes such that all uses are upwards exposed in each node.

- All definitions are **killing**, which means that the associated name is completely and certainly defined at the definition site. Exercise 56 considers the issue of arrays, which are typically not completely defined by an assignment. For example, an assignment to $A[i]$ changes only part of the name A. Exercise 57 considers the issue of method calls, which may not certainly define a name. For example, the name v may be assigned conditionally within a called method.

```
/⋆ Called from Marker (102)                                          ⋆/
procedure rename( V )
    stack ← new stack( )                                              (108)
    version ← 0                                                       (109)
    call renameHelper( Start, V )
end

procedure renameHelper( X, V )
    foreach use ∈ X.getOrdinaryUses( V ) do                           (110)
        call use.replaceName( stack.getTOS( ) )
    if X.containsDef( V )                                             (111)
    then
        def ← X.getDef( V )
        call def.replaceName( version )
        version ← version + 1
        call stack.push( def )
    foreach $(X, Y) \in \mathcal{E}_{cf}$ do                          (112)
        if Y.containsPhi( V )
        then
            phiUse ← Y.getPhiUse( V, X )
            call phiUse.replaceName( stack.getTOS( ) )

    foreach C ∈ X.getDomChildren( ) do                                (113)
        call renameHelper( C, V )

    if X.containsDef( V )                                             (114)
    then call stack.pop( )
end
```

Figure 14.67: Algorithm to rename variables.

```
$i \leftarrow 1$                                ①
$j \leftarrow 1$
$k \leftarrow 1$
$l \leftarrow 1$
repeat                                          ②
    $i \leftarrow \phi(i,i)$
    $j \leftarrow \phi(j,j)$
    $k \leftarrow \phi(k,k)$
    $l \leftarrow \phi(l,l)$
    if $p$
    then
        $j \leftarrow i$                        ③
        if $q$
        then
            $l \leftarrow 2$                    ④
        else
            $l \leftarrow 3$                    ⑤
        $l \leftarrow \phi(l,l)$                ⑥
        $k \leftarrow k+1$
    else
        $k \leftarrow k+2$                      ⑦
    $j \leftarrow \phi(j,j)$                    ⑧
    $k \leftarrow \phi(k,k)$
    $l \leftarrow \phi(l,l)$
    call PRINT($i,j,k,l$)
    repeat                                      ⑨
        $l \leftarrow \phi(l,l)$
        if $r$
        then
            $l \leftarrow l+4$                  ⑩
        $l \leftarrow \phi(l,l)$                ⑪

    until $s$
    $i \leftarrow i+6$                          ⑫
until $t$
```

(a)

```
$i_1 \leftarrow 1$                              ①
$j_1 \leftarrow 1$
$k_1 \leftarrow 1$
$l_1 \leftarrow 1$
repeat                                          ②
    $i_2 \leftarrow \phi(i_3,i_1)$
    $j_2 \leftarrow \phi(j_4,j_1)$
    $k_2 \leftarrow \phi(k_5,k_1)$
    $l_2 \leftarrow \phi(l_9,l_1)$
    if $p$
    then
        $j_3 \leftarrow i_2$                    ③
        if $q$
        then
            $l_3 \leftarrow 2$                  ④
        else
            $l_4 \leftarrow 3$                  ⑤
        $l_5 \leftarrow \phi(l_3,l_4)$          ⑥
        $k_3 \leftarrow k_2+1$
    else
        $k_4 \leftarrow k_2+2$                  ⑦
    $j_4 \leftarrow \phi(j_3,j_2)$              ⑧
    $k_5 \leftarrow \phi(k_3,k_4)$
    $l_6 \leftarrow \phi(l_2,l_5)$
    call PRINT($i_2,j_4,k_5,l_6$)
    repeat                                      ⑨
        $l_7 \leftarrow \phi(l_9,l_6)$
        if $r$
        then
            $l_8 \leftarrow l_7+4$              ⑩
        $l_9 \leftarrow \phi(l_8,l_7)$          ⑪

    until $s$
    $i_3 \leftarrow i_2+6$                      ⑫
until $t$
```

(b)

Figure 14.68: (a) Program with ϕ-functions and (b) variables renamed.

Exercises

1. Using a common programming language, construct a program whose control flow graph is the one shown in Figure 14.41.

2. Cycles in a procedure call graph do not necessarily indicate recursion. Write code for methods P, Q, and R so that the methods are not recursive, yet they have Figure 14.8(b) as their procedure call graph.

3. Arguments concerning the likely structure of control flow graphs do not easily extend to procedure call graphs. In general, we should not expect structured or reducible procedure call graphs. Consider the recursive-descent parser shown in Figure 5.7 on page 151. Build its procedure call graph and analyze its structure.

4. For a procedure call graph, invocation of procedure P implies that all DFST ancestors of P have been invoked. Thus, at runtime, the maximum depth of a method-call stack is at least the height of a graph's DFST. Given a cycle-free call graph for a program P, devise an algorithm that computes the maximum depth of a method-call stack for P.

5. The algorithm in Figure 14.9 creates a DFST by picking a node Y at Marker ㉖ such that (X, Y) is an edge in the DFST if Y has not previously been discovered.

   (a) For an irreducible flow graph (e.g., Figure 14.8(b)), show that the edges of the flow graph that are identified as **back edges** depend on the order in which edges are considered by Marker ㉖.

   (b) Prove that for a reducible flow graph, the same back edges are found in any depth-first traversal that starts at the flow graph's *root* node.

   (c) How many distinct DFSTs can be found for a flow graph $\mathcal{G}_f = (\mathcal{N}_f, \mathcal{E}_f)$?

6. Prove the following theorem:

   Theorem 14.27 *A flow graph $\mathcal{G}_f$ is* ***reducible*** *iff for all back edges (X, Y), Y dominates X*

7. Analyze the worst-case time complexity for the dominators algorithm given in Figure 14.17.

8. Dominance is defined by Definition 14.3 on page 566. A related concept is **postdominance**, which can be defined as follows:

   Definition 14.28 *A graph has a distinguished* ***exit node*** *if it contains one node z such that z has no successors and z can be reached from all nodes in the graph.*

   Definition 14.29 *If a graph $\mathcal{G}_f$ has a distinguished exit node z, then the* ***reverse*** *of $\mathcal{G}_f$ is the flow graph defined by $(\mathcal{N}_f, \{\, (x, y) \mid (y, x) \in \mathcal{E}_f \,\}, z)$ and $\mathcal{G}_f$ is said to be* ***reversible****.*

   Definition 14.30

   - *Node Z* ***postdominates*** *node Y if every path from Y includes node Z. A node always postdominates itself.*
   - *Node Z* ***strictly postdominates*** *Y if Z ≠ Y and Z postdominates Y.*
   - *The* ***immediate postdominator*** *of node Y is the* closest *strict postdominator of Y.*
   - *The* ***postdominator forest*** *for $\mathcal{G}_f$ has nodes $\mathcal{N}_f$; Z is a parent of Y in this forest if and only iff Z immediately postdominates Y.*
     If $\mathcal{G}_f$ has an exit node, then the postdominator forest is a tree.

   Draw the postdominator tree for each of the flow graphs shown in Figure 14.15.

9. Given the definition of **postdominance** from Exercise 8, prove the following theorem:

   Theorem 14.31 *Node X dominates node Y in a reversible flow graph $\mathcal{G}_f$ iff node Y postdominates node X in the reverse of $\mathcal{G}_f$.*

10. Use Theorem 14.31 to create an algorithm that computes the postdominators of a flow graph.

11. Consider the following definition of **control dependence**:

    Definition 14.32

    - *A node Z is* ***control dependent*** *on a node X (using edge $e = (X, Y)$) if Z postdominates a successor Y of X but Z does not strictly postdominate X.*
    - *Let $CD(X)$ denote the set of nodes that are control dependent on X:*

      $$Z \in CD(X) \iff \exists Y \mid Z \text{ is control dependent on } X \text{ using edge } (X, Y)$$

    - *A* ***control dependence graph*** *for $\mathcal{G}_{cf}$=($\mathcal{N}_{cf}$,$\mathcal{E}_{cf}$) is defined as*

      $$\mathcal{G}_{cd} = (\mathcal{N}_{cf}, \{ (X, Z) \mid Z \in CD(X) \})$$

    Build a control dependence graph for each of the graphs shown in Figure 14.15.

12. Based on the Definitions 14.15, 14.30, and 14.32, investigate the relationship between dominance frontiers and the control dependence graph. Show how to construct control dependence graphs using some simple graph transformations and the algorithm given in Figure 14.29.

13. Prove Theorem 14.2.

14. The table shown in Figure 14.14 uses $dfn(X) \geq dfn(Y)$ as part of the test for determining a cross edge. What happens if that test is changed to $dfn(X) > dfn(Y)$?

15. Devise an algorithm that computes semidominators for the control flow graphs of **structured programs**.

16. Devise an algorithm that computes the dominator tree for the control flow graphs of **structured programs**.

17. Nodes are not considered in any particular order by Marker (33) in the dominators algorithm given in Figure 14.17.

    (a) Devise a node ordering that generally provides for the best efficiency.

    (b) Compare the efficiency of your better node ordering with the evaluation shown in Figure 14.18.

    (c) For reducible flow graphs, how many passes are needed for your node ordering to converge for the dominance computation?

18. Consider a DFST T and the right-to-left preorder traversal given in Figure 14.11. Prove that nodes are visited in the same order by a **reverse postorder traversal**. Such a traversal is accomplished by first listing the nodes in postorder and then visiting the nodes in reverse of their appearance on that listing.

19. Marker 31 initializes $dom(X)$ (the dominators of node X) to be $\mathcal{N}_f$ (all nodes in the flow graph). Based on the relationship between a node's dominators and a graph's DFST, what set of nodes is more suitable for initializing $dom(X)$?

20. Prove Lemma 14.12. For intuition, consult the example in Figure 14.23.

21. Prove Lemma 14.13.

22. Prove the correctness of Marker 55 in the algorithm of Figure 14.24.

23. Show how to adapt the algorithm in Figure 14.35 so that irreducible intervals are resolved in favor of strong connectivity, at the expense of having multiple entries to irreducible loops.

24. Show how to adapt the algorithm in Figure 14.35 so that irreducible intervals are resolved in favor of being single-entry, at the expense of losing strong connectivity of irreducible loops.

25. Recall the transformation experienced by the inner loops as the program in Figure 14.1 was optimized into the program shown in Figure 14.5. Apply these same transformations to the outer loops of Figure 14.5.

26. **Dead code elimination** removes computations from a program that do not affect the program's output. Consider a simple programming language with the usual assignment and arithmetic operations. There are no looping or conditional statements. The language includes the statement `print` *var*, which prints the contents of the specified variable.

    Every statement of the form `print` *var* is live, and so are all statements that contribute to the computation of variables whose values are printed.

    Design a data flow framework that determines those computations that can be removed as dead code.

27. Some computations in a program may be **unreachable**, in the sense that no control flow path can cause such statements to execute. Using control flow analysis techniques, determine which statements of a program are unreachable.

28. The store to memory through $\star a$ in Figure 14.5 appears in the most deeply nested loop. Describe the analysis and transformations that are necessary to move the store out of the loop and show the result of the optimization.

29. Apply the algorithm of Figure 14.51 to the *available expressions* problem given in Figure 14.41 using a table such as the one shown in Figure 14.50 to display the computation. You should obtain the solution shown in Figure 14.42.

30. Repeat Exercise 29, but modify the algorithm in Figure 14.51 at Marker 84 so that all nodes' solutions are initialized to $\bot$ instead of $\top$. How does your result differ from the solution shown in Figure 14.42?

31. Apply the algorithm of Figure 14.51 to the *live variables* problem given in Figure 14.43 using a table such as the one shown in Figure 14.50 to display the computation. You should obtain the solution shown in Figure 14.44.

32. Consider the data flow problem given in Figure 14.53. Assume that every transfer function for such a framework is of the form $f(in) = (in - KILL) \cup GEN$, where $KILL$ and GEN are function-specific constants. Prove that such a data flow framework is **rapid**. Your proof must be based on the framework, not on the specific flow graph shown in Figure 14.53(a).

33. Verify that the data flow framework given in Figure 14.53 obeys the meet-lattice properties that appear in Figure 14.46.

34. Consider an instance of the *available expressions* data flow problem that is defined for n expressions.

    (a) Draw the lattice for $n = 1$, clearly labeling $\top$ and $\bot$.

    (b) Draw the meet lattice for $n = 3$, clearly labeling $\top$ and $\bot$.

    (c) Describe the meet lattice for an arbitrary value of n. What do the "levels" of the lattice represent?

35. The **bit-vectoring data flow problems** earn their name from a common representation for finite sets—the **bit vector**. In this representation, a slot is reserved for each element in the set. If $e \in S$, then e's slot is **true** in the bit vector that represents set S.

    Describe how bit vectors can be applied to the *available expressions* problem for a set of n expressions. In particular, describe how a bit vector is affected by

    (a) the transfer function at a node.

    (b) the application of the meet operator.

36. The *live variables* problem for a set of n variables can be solved as a data flow problem.

    (a) Define the formal framework, using the components given in Section 14.4. The transfer function at node Y is defined by the following formula (from Section 14.4.3):

    $$f_Y(in) = (in - Kill_Y) \cup Gen_Y$$

    Figure 14.47 shows how transfer functions model a node's behavior for *available expressions*. Offer a similar set of diagrams and transfer functions for *live variables*. In particular, explain how $Kill_Y$ and Gen_Y are determined for *live variables* for a node Y.

    (b) Now consider the use of bit vectors to solve *live variables*. The transfer function can be implemented as described in Exercise 35. How is the meet operation performed? What are $\top$ and $\bot$?

37. Some optimization problems, such as *constant propagation*, are concerned with the flow of values in a program. One way to track such values is to distinguish each definition of a given variable (e.g., `x`) by giving each definition a unique name (as with SSA form).

    Each assignment to `x` is called a **definition site** (**def** for short) of `x`. If there are multiple defs of `x`, then these are suitably renamed so that they are distinct. For example, the def of `x` at node 3 might be renamed as an assignment to $\texttt{x}_3$. Unlike SSA form, the renaming of `x` to $\texttt{x}_3$ is still a def of `x` and not a def of a completely new variable name.

    The *reaching definitions* problem can be stated as follows:

    - The only nodes of interest are those that assign to a variable. The transfer function for all other nodes is simply $f(in) = in$.

- Some assignments to a variable completely and certainly redefine the variable. In such cases, the node generates a new def of the variable and **kills** all other defs of the same variable name. Nodes E and F in Figure 14.43 completely and certainly define the variable v. All other defs of v that reach node E cannot reach any further, and the node generates the def v_E that propagates out of node E.
- Some assignments assign to a name without certainty. For example, the call to `f` at node J in Figure 14.43 might modify v (assuming v is passed by reference), but it is difficult to say with certainty that v will be modified. Such an assignment is called a **wounding definition**. Any defs of v that reach node J continue to propagate forward, along with v_J defined at node J.
- Some assignments assign to a name, but not completely. For example, an assignment to the array element $A[i]$ assigns to the name A but does not completely modify A. Such an assignment is also treated as a **wounding definition**.
- When multiple paths converge at a node, the set of defs that propagates forward is the *union* of the defs that propagate on the edges into the node.

(a) Is this a forward or backward problem?

(b) What is the value of $\top$ (the best solution)?

(c) Describe how to model a node's behavior with a transfer function, using Figure 14.47 as a guide.

(d) How are solutions summarized at common control flow points?

(e) Formally define the components (Section 14.4) of the *reaching definitions* data flow framework.

(f) Prove or disprove that your framework is **rapid**.

(g) Prove or disprove that your framework is **distributive**.

38. Liveness shows that a variable is potentially of future use in a program. The *very busy expressions* problem determines if an expression's current value is *certainly* of future use. An expression e is very busy at a point P in a control flow graph if every path after point P contains a use of the current value of e at P.

(a) Is this a forward or backward problem?

(b) What is the value of $\top$ (the best solution)?

(c) Describe the effects of a node on an expression, using Figure 14.47 as a guide.

(d) How are solutions summarized at common control flow points?

(e) How would you determine *very busy expressions* for a *set* of expressions?

(f) Formally define the framework for *very busy expressions*.

(g) Prove or disprove that your framework is **rapid**.

(h) Prove or disprove that your framework is **distributive**.

39. The following data flow problems are known as the **bit-vectoring data flow problems**:

- *available expressions* (Section 14.3.1 and Exercise 35)
- *live variables* (Section 14.3.2 and Exercise 36)
- *very busy expressions* (Exercise 38)
- *reaching definitions* (Exercise 37)

Summarize these problems by entering each into its proper position in the following table:

	Forward	Backward
Any path		
All paths		

The columns refer to whether information is pushed forward or backward to achieve a solution to the problem. The rows refer to how information is summarized by the meet ($\wedge$) operator: information is conserved if it occurs on any paths or all paths.

40. From Definition 14.22 and the lattice properties given in Figure 14.46, prove Lemma 14.25.

41. Prove the following lemma:

Lemma 14.33 *For a lattice where the meet operator ($\wedge$) is set intersection ($\cap$) or set union ($\cup$):*

- $(\forall\, x \in A)(\forall\, y \in A)(\forall\, z \in A)\ (x \wedge y) \cup z = (x \cup z) \wedge (y \cup z)$
- $(\forall\, x \in A)(\forall\, y \in A)(\forall\, z \in A)\ (x \wedge y) \cap z = (x \cap z) \wedge (y \cap z)$

42. Prove that all four bit-vectoring data flow problems in Exercise 39 are **rapid**. Instead of writing four separate proofs, base your proof on the generic form of a transfer function

$$\begin{aligned} f_Y(in) &= (in - Kill_Y) \cup Gen_Y \\ &= (in \cap NKill_Y) \cup Gen_Y \end{aligned}$$

and Lemma 14.33. The above reformulation of $NKill_Y$ as the complement of $KILL_Y$ allows the transfer function to be stated in terms of set union and intersection.

43. Prove or disprove that constant propagation is a **rapid** data flow problem.

44. Prove or disprove that *available expressions* is a **distributive** data flow problem (Definition 14.26).

45. Generalize the proof from Exercise 44 to prove or disprove that all four bit-vectoring data flow problems in Exercise 39 are **distributive**. Use Lemma 14.33 and the generic form of the transfer function given in Exercise 42.

46. Prove or disprove that constant propagation is a **distributive** data flow problem.

47. Consider generalizing the problem of constant propagation to that of *range analysis*. For each variable, we wish to associate a minimum and maximum value, such that the actual value of the variable (at that site in the program) at runtime is guaranteed to fall between the two values. For example, consider the following program.

```
$x \leftarrow 5$
$y \leftarrow 3$
if $p$
then
    $z \leftarrow x + y$                                        115
else
    $z \leftarrow x - y$                                        116
$w \leftarrow z$
```

After their assignment, variable x has range $5 \ldots 5$ and variable y has range $3 \ldots 3$. The effect of Marker 115 gives z the range $8 \ldots 8$. The effect of Marker 116 gives z the range $2 \ldots 2$. The assignment for w therefore gets the range $2 \ldots 8$.

(a) Sketch the data flow lattice for a single variable. Be specific about the values for $\top$ and $\bot$.

(b) Is this a forward or backward propagation problem?

(c) If the variable v could have range r_1 or r_2, describe how to compute the meet of these two ranges.

48. Prove or disprove that *range analysis* is a **rapid** data flow problem.

49. Prove or disprove that *range analysis* is a **distributive** data flow problem.

50. Figure 14.58 shows the evaluation of the *number of bits* problem for the flow graph shown in Figure 14.57. The number of iterations taken to reach convergence proves that the *number of bits* problem is not **rapid**.

    Construct a different proof, based on Definition 14.24. In other words, find an f and a for an instance of the *number of bits* problem that violates Definition 14.24. *Hint:* The instance given in Figure 14.57 provides inspiration for finding a suitable f and a.

51. Given a monotone data flow framework, is it decidable whether a node's transfer function always returns the same value? In other words, can it be decided for an arbitrary $f \in \mathcal{F}$ that $(\exists\, k \in A)\ (\forall\, a \in A)\ f(a) = k$? If this is undecidable, provide a proof. It is decidable, provide an algorithm.

52. Given a monotone data flow framework, is it decidable whether a node's transfer function always returns its input? In other words, can it be decided for an arbitrary $f \in \mathcal{F}$ that $(\forall\, a \in A)\ f(a) = a$? If this is undecidable, provide a proof. It is decidable, provide an algorithm.

53. Consider the following definition:

    Definition 14.34 *A data flow framework is **idempotent** iff*

    $$(\forall\, a \in A)\ (\forall\, f \in \mathcal{F})\ f(f(x)) = f(x)$$

    Prove or disprove that the **bit-vectoring data flow problems** are idempotent.

54. Verify that the dominator and dominance frontiers shown in Figure 14.63 are correct for the flow graph shown in Figure 14.62(b).

55. Figure 14.65 computes the location of ϕ-functions one variable at a time. Before the algorithm moves from one variable to the next, the flags *hasPhi* and *processed* are reset to **false**.

    Devise a more efficient algorithm that does not require resetting the flags between variables. *Hint:* you can change the type of *hasPhi* and *processed* from *Boolean* to *integer*.

56. SSA form requires that a definition of a name completely define that name. Array assignments typically modify part, but not all, of the named array. For example, assignment to $A[i]$ leaves all elements of A other than $A[i]$ unchanged.

    Develop an approach for translating array assignments and references into SSA form, so that each assignment to an array *does* completely define the named array.

57. SSA form requires that each definition site for a name be explicitly represented in a program. Consider a call to FOO(v) where FOO may, or may not, assign v. For example, the only assignment to v in FOO may be programmed to occur only if $v = 0$.

    Develop an approach for translating method calls into SSA form, so that each call that may modify a given variable v surely does so every time the method is called. *Hint:* if a method does not assign its own value to v, then what value should v have on return from that method?

58. A reference r is said to **may alias** a name n if a load or store of r can be a load or store of n. This information is needed by Exercise 59 to determine the program names that may be affected by a given reference.

    The set of all names a given reference may alias is called that reference's **may alias** set. Investigate how **may alias** sets can be computed using data flow analysis techniques. Compare the techniques you discover by their cost and accuracy. What are the consequences of over- or under-determining the correct may-alias sets for a reference?

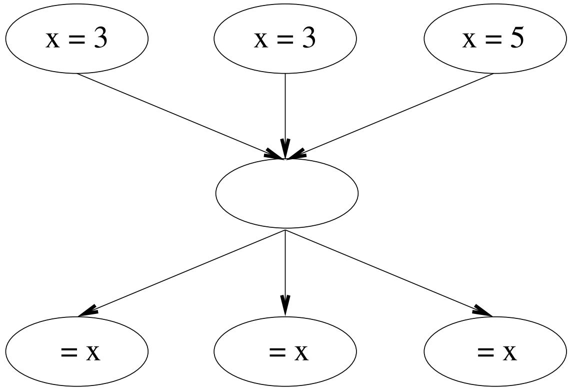


Figure 14.69: Constant propagation example.

59. The SSA form discussion in Section 14.7 is limited to programs whose references to names are explicit. If a programming language includes pointers, then the names possibly affected by a given pointer indirection are not explicitly stated in the program.

    Develop an approach for translating programs with pointers into SSA form. Distinguish between pointer references as follows:

    - Some pointer references **may alias** a given set of names. Stores or loads through such pointer references may, or may not, affect those names.
    - Some pointer references **must alias** a given set of names. Stores or loads through such pointer references are guaranteed to affect those names.

60. Compute SSA form for the program shown in Figure 14.60.

61. The data flow framework for *constant propagation* can be applied to programs in SSA form by applying the **meet** operator at ϕ-functions [WZ91]. Compare the efficiency and results of evaluating *constant propagation* on Figure 14.60 in its original form and in its SSA form.

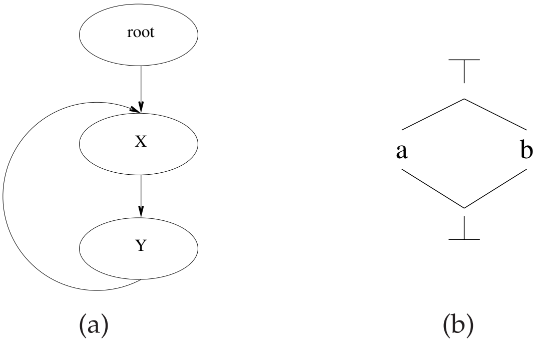


Figure 14.70: (a) Control flow graph; (b) Data flow lattice.

62. Another approach to *constant propagation* is to compute *reaching definitions* (Exercise 37) for each variable. The potentially constant value of a given use of a variable v is determined by computing the **meet** of the reaching definitions' solutions for *constant propagation*.

    (a) Consider the control flow graph shown in Figure 14.69. Compute *reaching definitions* for the flow graph and then analyze where meets are required for *constant propagation*.

    (b) Now compute SSA form for Figure 14.69 and recompute a solution for *constant propagation* using *reaching definitions*.

    (c) How many meets are computed in each approach? What property of SSA form makes problems like *constant propagation* easier to solve?

63. Many optimizations and transformations have been proposed in literature that rely on SSA form. Some examples include the following:

    - The *constant propagation* problem with consideration for branches that cannot be taken [WZ91].
    - A *value numbering* algorithm [AWZ88].

    Using a digital library [ACM], investigate these and other algorithms that compute or use SSA form.

64. SSA form has been implemented in the **GNU Compiler Collection** (GCC) suite. Investigate the implementation and compare the algorithms that construct and use SSA form to the classic algorithms given in this book.

65. Consider a data flow framework with the lattice shown in Figure 14.70(b). The transfer functions associated with Figure 14.70(a) are as follows:

$$\begin{array}{rcl} \forall\ in\ f_{root}(in) & = & \top \\ f_X(\top) & = & \bot \\ f_X(a) & = & a \\ f_X(b) & = & b \\ f_X(\bot) & = & \top \\ \forall\ in\ f_Y(in) & = & in \end{array}$$

What happens when the algorithm of Figure 14.51 is applied to this framework and flow graph? What causes that behavior?

66. Reconsider Exercise 65, but with the following transfer functions:

$$\begin{array}{rcl} \forall\ in\ f_{root}(in) & = & a \\ f_X(\top) & = & \top \\ f_X(a) & = & b \\ f_X(b) & = & a \\ f_X(\bot) & = & \bot \\ \forall\ in\ f_Y(in) & = & in \end{array}$$

67. Section 11.2.2 on page 424 described the code that must be generated for a JVM, which includes a bound on how much stack is needed to perform the operations within a method. Each JVM instruction affects the stack in a predictable way, based on the number of operands it must pop and the results it must push when the instruction has completed. For example, the `iadd` instruction pops two operands and pushes one result on the stack.

    (a) Investigate Java's rules concerning how the stack can be manipulated by a method and develop a data flow framework that determines the maximum number of stack slots needed for performing the instructions within a given method.

    (b) Describe $\top$, $\bot$, and the rest of the lattice.

    (c) How does **meet** ($\wedge$) work for your framework?

    (d) In what sense are two elements of the lattice related by $\preceq$?

    (e) Prove or disprove that your framework is **rapid**.

    (f) Prove or disprove that your framework is **distributive**.

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Abbreviations

Pseudocode Guide

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