Multi-Source Broadcasting and Multicasting in Wireless Ad Hoc Networks*

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1 Introduction

Recently, wireless ad hoc networks have received significant attention due to potential applications in various situations such as battlefields, emergency disaster relief and so on [3, 8]. Unlike traditional wired networks, there is no wired backbone infrastructure installed for wireless ad hoc networks. Each node in the network has an omnidirectional antenna to transmit and receive signals. In a general power attenuation model [6, 8], the signal strength decreases to $1/d^{\alpha}$, where d is the distance between sender and receiver and α is a constant (typically between 2 and 6) depending on the communication environments.

Thus, a node u successfully receives a signal transmitted from a node s if $C(s) \geq ||su||^{\alpha}$, where $C(s)$ is the initial signal power cost from s and $||su||$ is the Euclidean distance between s and u. We call $||su||^{\alpha}$ the *power distance* from s to u. All nodes within the signal power range from u can receive the signal as well due to the omnidirectional antenna. See Figure 1 for an example.

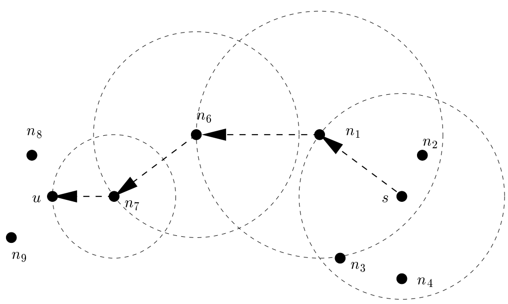

Figure 1: s sends a signal to n_1. Note that nodes n_2, n_3, n_4 can also receive the same signal as they are within distance $\|sn_1\|$ from s. n_1 then forwards the signal to n_6. So on so forth, finally the signal reaches u.

A crucial issue in wireless ad hoc networks is to support traditional communications such as broadcasting, multicasting or gossiping. Extensive research has been done on single source broadcasting and single source multicasting in ad hoc networks [2, 3, 7, 8]. In practice, there may be several source nodes processing the same information and each of which needs to deliver the information to a subset of the destinations. This motivates our investigation on the multi-source broadcasting and multicasting problems in wireless ad hoc networks.

Given k source nodes, a broadcasting network from k sources is a forest $\mathcal{F}$ of k trees that spans all nodes in V, where each tree contains one source. To compute the broadcasting cost for each tree $T \in \mathcal{F}$, we orient T to a directed tree rooted at the source in T. Let $f(T,u)$ denote the required transmission power of node u in T.

$$f(T,u) = \begin{cases} 0 & \text{if } u \text{ is a leaf} \\ \max_{uv \in T} ||uv||^{\alpha} & \text{otherwise} \end{cases}$$

In other words, $f(T,u)$ is the largest power distance among all pairs between u and its children in T. The *broadcasting cost* $C(\mathcal{F})$ of a given forest $\mathcal{F}$, is the sum of transmission powers of all nodes in $\mathcal{F}$. Let $T(u)$ denote the tree containing u in $\mathcal{F}$.

$$C(\mathcal{F}) = \sum_{T \in \mathcal{F}} \sum_{u \in T} f(T,u) \qquad (1)$$

The *minimum energy multi-source broadcasting problem* is to compute $\mathcal{F}$ at the minimum cost for a given set of nodes V including source set S. If sources only cast signals to destination nodes in V, then the similarly formed problem is called *minimum energy multi-source multicasting problem.*

Clementi et al. [4] proved that the problem is NP-hard for $\alpha > 1$ in Euclidean space with dimension $d \geq 2$, which consequently implies that the problem with k-source is NP-hard for $\alpha > 1$ and $d \geq 2$. Wan et al. [7] showed that the *Euclidean Minimum Spanning Tree* approximates the minimum energy single-source broadcasting problem with approximation ratio c, where $6 \leq c \leq 12$. Althaus et al. [1] pointed out that an approximate minimum Steiner tree gives a ρc approximation algorithm for the single source multicasting problem, where ρ is the approximation ratio for Steiner tree. These problems in

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a general graph were also investigated. Details of the related work are omitted in this extended abstract.

In this paper, we propose approximation algorithms for the minimum energy multi-source broadcasting and multicasting problems (Sections 3 and 4). To do these, we use the algorithm for the *minimum spanning forest problem* introduced in Section 2 as a subroutine.

2 Minimum Spanning Forest

Let $G = (V, E, W)$ be a connected weighted graph with vertex set V, edge set E and positive weight function W for edges in E. Suppose V contains a set $S = \{s_1, s_2, \ldots, s_k\}$ of k sources. A *spanning forest* is a set of trees that spans all the nodes in V, in which each tree contains at least one source. The *minimum spanning forest (MSF)* is a spanning forest that minimizes the sum of weights on the edges. Since all edges have positive weights, each tree in the MSF contains exactly one single source. For simplicity, we assume that each edge has a distinct weight.

Suppose T is the minimum spanning tree of G. Let $E_{\max}$ denote the set of maximum weight edges of the paths for all pairs of sources in T. We prove that $|E_{\max}| = k - 1$. Our algorithm to compute the MSF of G prunes all edges of $E_{\max}$ successively out of T from leaves towards center.

3 Multi-Source Broadcasting

Wan et al. [7] showed that the minimum spanning tree (MST) T is a constant approximation of the single-source broadcasting problem. We show that minimum spanning forest MSF $\mathcal{F}$ is a c-approximation of the multi-source broadcasting problem by a similar logic, where $6 \leq c \leq 12$. The approximation algorithm runs in $O(n \log n)$ time.

4 Multi-Source Multicasting

Unlike broadcasting, multicasting only requires to deliver messages to a subset D of nodes (destinations). We adapt the 2-approximation algorithm [5] for the Steiner tree problem to approximate the multi-source multicasting problem. We sketch the algorithm here. Consider the weighted complete graph $G = (V, E, W)$ with edges E connecting all pairs of nodes in V and weights W to be the power distances between the endpoints of the edges. Let $K = S \cup D$, where S is the set of sources. We first compute the complete network N for K using weights of the shortest paths (in G) as edge weights for all edges connecting node pairs in K. Then the minimum spanning tree T of N is computed. Some edges in T are pruned away to obtain MSF $\mathcal{F}$ as Section 2. Finally, we obtain a Steiner forest $\mathcal{F}_K$ by placing back the original paths in G corresponding to edges in $\mathcal{F}$ with some minor modifications. We prove that $\mathcal{F}_K$ is a $2c$-approximation to the minimum energy multi-source multicasting problem, where $6 \leq c \leq 12$. The whole approximation algorithm runs in $O(n^2)$ time.

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