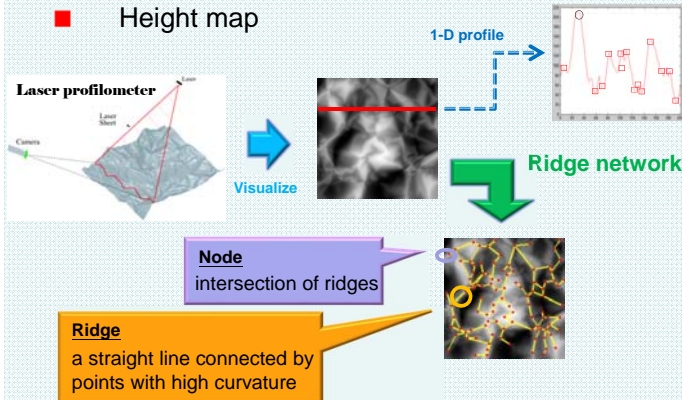# Automatic Ridge Network Detection in Crumpled Paper Based on Graph Density
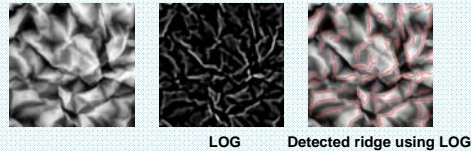
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Introduction

Data representation of crumpled paper

- Height map

  Laser profilometer

  Visualize

  1-D profile

  Ridge network

  Node intersection of ridges

  Ridge a straight line connected by points with high curvature

Challenge on automatic detection

  LOG   Detected ridge using LOG

- Complex structure
  - Fragmental and discontinuous results using conventional image processing methods

Goal

- To automate the ridge network detection
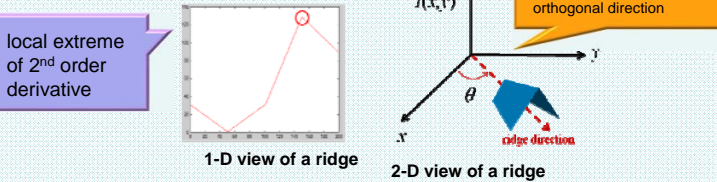- Also maintaining good continuity

Automatic ridge network detection

Modeling ridge network as a weight graph G = (**V**,**E**)

- Node detection
  - Set of nodes **V**
- Ridge response calculation
  - Edge weight for all edges in **E**
- Network detection
  - Determining a sub-graph G'=(**V**,**E'**), **E'** ⊆ **E** with maximal graph density

Ridge response

- Observation

  local extreme of 2nd order derivative

  different behaviors on 2nd order derivative
  - almost zero along ridge direction
  - local extreme along the orthogonal direction

  $I(x,y)$

  1-D view of a ridge   **2-D view of a ridge**

  θ ridge direction

- Formulation

$$F(x,y,\theta,\sigma) = \left| g(x,y,\sigma) * \frac{\partial^2 I(x,y)}{\partial y_\theta^2} \right| - \left| g(x,y,\sigma) * \frac{\partial^2 I(x,y)}{\partial x_\theta^2} \right|$$
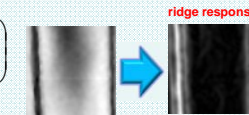
$g(x,y,\sigma)$: Gaussian smoothing filter

θ : ridge direction

$x_\theta = x\cos\theta + y\sin\theta$

$y_\theta = y\cos\theta - x\sin\theta$

$$\hat{\theta}(x,y) = \max_{\hat{\theta}}\left(\sum_{(x',y')\in N_{x_\theta}} \left| G * \left(\frac{\partial^2 I(x',y')}{\partial y_\theta^2}\right)\right| - \sum_{(x',y')\in N_{y_\theta}} \left| G * \left(\frac{\partial^2(x',y')}{\partial x_\theta^2}\right)\right| \right)$$

$N_{x_\theta}, N_{y_\theta}$: neighboring pixels of (x,y) along x_θ and y_θ, respectively
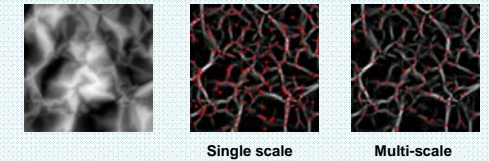
ridge response

Non-maximum suppression (NMS)

Node response

- Local extreme on the 2nd order derivative along two intersected ridges

$$R(x,y,\sigma) = \left| \left(G(x,y,\sigma) * \frac{\partial^2 I(x,y)}{\partial x_{\hat{\theta}}^2} \right) \cdot \left(G(x,y,\sigma) * \frac{\partial^2 I(x,y)}{\partial y_{\hat{\theta}}^2} \right) \right|$$

- Multi-scale response

$$R'(x,y) = \prod_{\sigma_i} R(x,y,\sigma_i)$$

(Combining responses from small to large scales)

  Single scale   Multi-scale

Ridge network detection

- Criterion
  - Planar graph
  - Connectivity
  - Maximal graph density

$$\rho(G') = \frac{\sum_{e_k \in E'} w(e_k)}{|E'|} \quad \text{where} \quad w(e_k) = \frac{\sum_{(x,y)\in e_k} F(x,y,\hat{\theta},\sigma)}{\#\ \text{pixels on } e_k}$$

- Edge linking
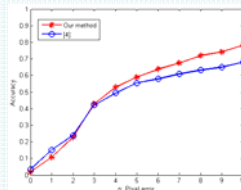 - Greedy-based approach
    - Including ridges in decreasing order of edge weight
    - Checking planarity constraint
   - Checking connectivity constraint

Experimental results

- Performance evaluation of node detection
  - Average accuracy from 304 image patches
  - Cumulative score

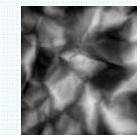$$accuracy = \frac{\#\ corrected\ nodes}{\#\ nodes\ detected}$$

[4] R. Laganiere and R. Elias, "The Detection of Junction Feature in Images," *ICASSP*, 2004.

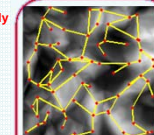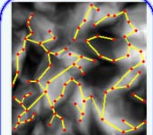- Comparison of edge linking result

  Nodes detected by [4]   Nodes detected by our approach
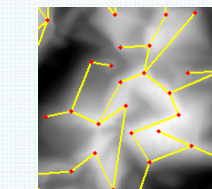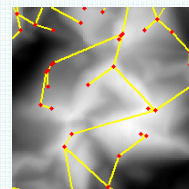
- Detected network by different node sets

  Crumpled paper 1

  Crumpled paper 2

  automatically detected

  labeled manually