

# Monte Carlo Path Tracing and Caching Illumination

An Introduction

## Beyond Ray Tracing and Radiosity

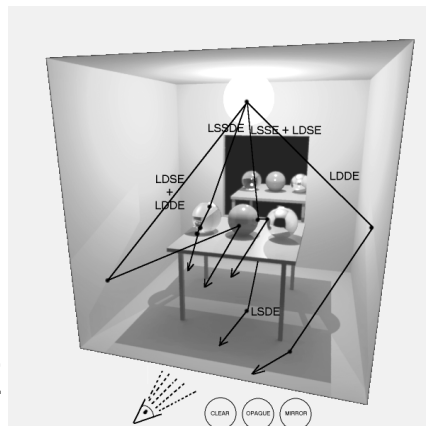
- What effects are missing from them?
  - Ray tracing: missing indirection illumination from diffuse surfaces.
  - Radiosity: no specular surfaces
- Let's classify the missing effects more formally using the notation in Watt's 10.1.3 (next slide)

# Path Notation

- Each path is terminated by the eye and a light:
  - E: the eye
  - L: the light
- Types of Reflection (and transmission):
  - D: Diffuse
  - S: Specular
  - Note that the “specular” here means mirror-like reflection (single outgoing direction). Hanrahan’s SG01 course note has an additional “glossy” type.

# Path Notation

- A path is written as a regular expression.
- Examples:
  - Ray tracing:  $LD[S^*]E$
  - Radiosity:  $LD^*E$
- Complete global illumination:  $L(D|S)^*E$



## Bi-direction Ray Tracing

- Also called two-pass ray tracing.
- Note that the Monte Carlo technique is not involved.
- The concept of “caching illumination” (as a mean of communication between two passes.) -- After the first pass, illumination maps are stored (cached) on diffuse surfaces.

## Multi-pass Methods

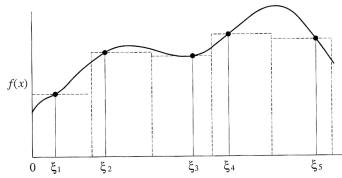
*Note: don't confuse “multi-pass” with “bi-directional” or the multiple random samples in Monte Carlo methods.*

- $LS^*DS^*E$  is included in bi-directional ray tracing.
- How about the interaction between two diffuse surfaces? (radiosity déjà vu?)

# Monte Carlo Integration

- Estimate the integral of  $f(x)$  by taking random samples  $\xi$  and evaluate  $f(\xi)$ .
- Variance of the estimate decreases with the number of samples taken (N):

$$\sigma^2 = \frac{1}{N} \left( \int f^2(x) dx - f^2(\xi) \right)$$



# Biased Distribution

- What if the probability distribution ( $p(x)$ ) of the samples is not uniform?
- Example:
  - What is the expected value of a flawless dice?
  - What if the dice is flawed and the number 6 appears twice as often as the other numbers?
  - How to fix it to get the same expected value?

$$\text{Assume } X_i \in [0,1], E[f(X_i)] = \frac{1}{N} \sum_{i=1}^N f(X_i) \text{ if not biased}$$

$$E[f(X_i)] = \sum_{i=1}^N \frac{f(X_i)}{p(X_i)} \text{ if biased}$$

## Noise in Rendered Images

- The variance (in estimation of the integral) shows up as noise in the rendered images.



## Importance Sampling

- One way to reduce the variance (with a fixed number of samples) is to use more samples in more “important” parts.
- Brighter illumination tends to be more important.
- More detail in Veach’s thesis and his “Metropolis Light Transport” paper.

## Monte Carlo Path Tracing

- Apply the Monte Carlo techniques to solve the integral in the rendering equation.
- Questions are:
  - What is the cost?
  - How to reduce the variance (noise)?

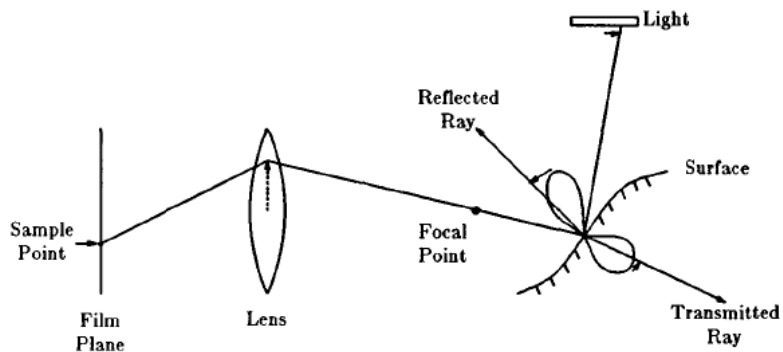
## Integrals

- In rendering equation:
  - Reflection and transmission.
  - Visibility
  - Light source
- In image formation (camera)
  - Pixel
  - Aperture
  - Time
  - Wavelength

# Effects

- By distributing samples in each integral, we get different effects:
  - Reflection and transmission → blurred
  - Visibility → fog or smoke
  - Light source → penumbras and soft shadow
- In image formation (camera)
  - Pixel → anialiasing
  - Aperture → depth of field
  - Time → motion blur
  - Wavelength → dispersion

## Typical Distributed Ray Path



## Summary

- Monte Carlo path (ray) tracing is an elegant solution for including diffuse and glossy surfaces.
- To improve efficiency, we have (at least) two weapons:
  - Importance sampling
  - Caching illumination

## Exercises (Food for Thought)

- Can the multi-pass method (i.e., light-ray tracing, radiosity, then eye-ray tracing) replace the Monte Carlo path tracing approach? (*Hint: glossy?*)
- What are the differences between Cook's distributed ray tracing and a complete Monte Carlo path tracing?



# References

- Pharr's chapters 14-16.
- Watt's Ch.10 (especially 10.1.3, and 10.4 to 10.9)
- Or, see SIGGRAPH 2001 Course 29 by Pat Hanrahan for a different view.
- After that, you shall be ready for more advanced topics, such as:
  - Global Illumination Using the Photon Maps  
by H. W. Jensen