CS5500 Computer Graphics, Handout (May 1, 2006)

Consider two end points $P_1=(x_1, y_1, z_1)$ and $P_2=(x_2, y_2, z_2)$, and a in-between point $P_3=(1-t)P_1+(t)P_2$

After projection, P_1, P_2, and P_3 are projected to (x'_1, y'_1), (x'_2, y'_2), (x'_3, y'_3) in screen coordinates. Assume $(x'_3, y'_3)=(1-s)(x'_1, y'_1) + s(x'_2, y'_2)$.

(x'_1, y'_1), (x'_2, y'_2), (x'_3, y'_3) are obtained from P_1, P_2, P_3 by:

$$\begin{bmatrix} x'_1\, w_1 \\ y'_1\, w_1 \\ z'_1\, w_1 \\ w_1 \end{bmatrix} = M \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix}, \qquad \begin{bmatrix} x'_2\, w_2 \\ y'_2\, w_2 \\ z'_2\, w_2 \\ w_2 \end{bmatrix} = M \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x'_3\, w_3 \\ y'_3\, w_3 \\ z'_3\, w_3 \\ w_3 \end{bmatrix} = M \begin{bmatrix} x_3 \\ y_3 \\ z_3 \\ 1 \end{bmatrix} = M((1-t)\begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} + t \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix})$$

Interpolate first! then project

$$= (1-t)M \begin{bmatrix} x_1 \\ y_1 \\ z_1 \\ 1 \end{bmatrix} + tM \begin{bmatrix} x_2 \\ y_2 \\ z_2 \\ 1 \end{bmatrix}$$

$$= (1-t) \begin{bmatrix} x'_1\, w_1 \\ y'_1\, w_1 \\ z'_1\, w_1 \\ w_1 \end{bmatrix} + t \begin{bmatrix} x'_2\, w_2 \\ y'_2\, w_2 \\ z'_2\, w_2 \\ w_2 \end{bmatrix}$$

When P_3 is projected to the screen, we get (x'_3, y'_3) by dividing by w, so:

$$(x'_3, y'_3) = \left(\frac{(1-t)x'_1\, w_1 + t \cdot x'_2\, w_2}{(1-t)w_1 + t \cdot w_2}, \frac{(1-t)y'_1\, w_1 + t \cdot y'_2\, w_2}{(1-t)w_1 + t \cdot w_2}\right)$$

But remember that $(x'_3, y'_3)=(1-s)(x'_1, y'_1) + s(x'_2, y'_2)$
We have $(1-s)x'_1+s\, x'_2 = ((1-t)x'_1 w_1+t\, x'_2 w_2)/((1-t)w_1+t\, w_2)$

We may rewrite s in terms of t, w_1, w_2, x'_1, and x'_2.
In fact, $s = \dfrac{t \cdot w_2}{(1-t)w_1 + t \cdot w_2}$, or conversely $t = \dfrac{s \cdot w_1}{s \cdot w_1 + (1-s)w_2}$
Surprisingly, x'_1 and x'_2 disappear.