CS5321 Numerical Optimization Homework 1

One dimensional problems

Due March 5, 2012

A function $f(x)$ is called unimodal if for some value m, it is monotonically decreasing for $x \leq m$ and monotonically increasing for $x \geq m$. In that case, the minimum value of $f(x)$ is $f(m)$ and there are no other local minima. For example, $f(x) = x(1 - x)$ is a unimodal function, whose minimum is at $x = 0.5$. Note: a unimodal function need not be continuous.

Suppose the only operation you can do is to input a value x and get back the function value $f(x)$. Furthermore, suppose $f(x)$ is unimodal, and it is defined in the interval $[a, b]$.

1. (50%) Design and implement an algorithm to find the minima of $f(x)$ to the precision $\epsilon = 10^{-6}$. Your Matlab code should look like follows.

```
function [x, fx] = find_min(f, a, b)
% This program should return the minimizer and
% the minimum of a given unimodal function f(x),
% which is defined in [a,b].
% ===========================================
% to get function evaluation f(t) at t, using
    s = feval(f, t)
```

   (a) How to decide the search direction?

   (b) How to decide the step length?

   (c) How do you ensure the answer is good enough? ($|x - x^*| < 10^6$)

   (d) How many function evaluations is needed to find the desired solution? (Your answer should include a, b, and the desired accuracy ϵ.)

   (e) (bonus 10%) Implement and analyze "golden section search". (http://en.wikipedia.org/wiki/Golden_section_search)

2. (50%) If $f(x)$ is continuous and differentiable, how to use this property to design your algorithm? Your code should be able to detect the case that the minima is on the boundary point, a or b. But you may assume that there is no saddle point. The Matlab interface should look like

```
function [x, fx] = find_min2(f, g, a, b)
```

where $g(x) = f'(x)$.

(a) How to decide the search direction?

(b) How to decide the step length?

(c) How do you ensure the answer is good enough? $(|x - x^*| < 10^6)$

Submission instructions

- Make your writing homework a pdf file.

- Programming homework should be submitted in .m files.

- Zip (or other compression methods) all your files into a single file, called "hw[xx]_[your student id].zip".
  For instance, "hw01_100012345.zip".

- Send the zipped file to TA *combinedofme222@yahoo.com.tw* using the title "CS5321 HW[xx] [studentid]".