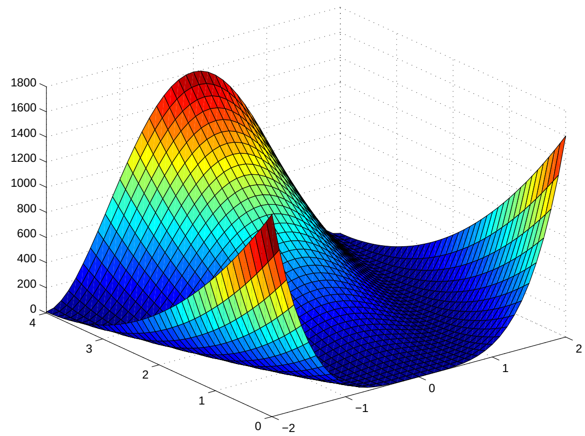
CS5321 Numerical Optimization Homework 3

Due April 11

1. (100%) The Rosenbrock function $f(x, y) = (1-x)^2 + 100(y - x^2)^2$ is shown below, whose minimizer is at $(1, 1)$.[1]



   (a) Derive the gradient and the Hessian of $f(x, y)$.

$$\nabla f = \begin{pmatrix} 400x^3 - 400xy + 2x - 2 \\ 200y - 200x^2 \end{pmatrix}, \nabla^2 f = \begin{pmatrix} 1200x^2 - 400y + 2 & -400x \\ -400x & 200 \end{pmatrix}$$

   (b) Implement the backtracking line-search method, and make it a function. Your input should at least include

   - The current solution $\vec{x}_k$.
   - The search direction $\vec{p}_k$.
   - The function that can evaluate $f(\vec{z})$ for a given $\vec{z}$.
   - The function that can evaluate $\nabla f(\vec{z})$ for a given $\vec{z}$.

   and your output is the step length α that satisfies the Wolfe conditions or the Goldstein condition. (Just choose one to implement.)

   (c) Implement (1) the steepest descent method (2) Newton's method (3) SR1 or BFGS (4) CG or truncated Newton (CG with limited iterations). Use your line search algorithm to find the best step length. Use $(x_0, y_0) = (-1.2, 1.0)$ as the initial guess and compare their results. (Do not use any

[1] You can find reference of this function in MO and Wikipedia.

symbolic computation of Matlab, like sub or diff, or eval. Write your own function, gradient, and Hessian subroutines, and use them in your code. Also, do NOT implement your own linear solvers or rank 1 updates. Use $H\backslash\vec{b}$ or $B\backslash\vec{p}$ in Newton's method and SR1.)