Advanced Numerical Method Homework 1

Matrix Multiplication

Due: Oct 12, 2011

1. (20%) Let $A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1P} \\ A_{21} & A_{21} & \cdots & A_{2P} \\ \vdots & \vdots & & \vdots \\ A_{M1} & A_{M2} & \cdots & A_{MP} \end{pmatrix}$ and $B = \begin{pmatrix} B_{11} & B_{12} & \cdots & B_{1N} \\ B_{21} & B_{21} & \cdots & B_{2N} \\ \vdots & \vdots & & \vdots \\ B_{P1} & B_{P2} & \cdots & B_{MN} \end{pmatrix}$. Assume $\dim(A)_2 = \dim(B)_1$ and $\dim(A_{IK})_2 = \dim(B_{KJ})_1$ for $I = 1, \ldots, M$, $J = 1, \ldots, N$, and $K = 1, \ldots, P$. Show that for

$$C = AB = \begin{pmatrix} C_{11} & C_{12} & \cdots & C_{1N} \\ C_{21} & C_{21} & \cdots & C_{2N} \\ \vdots & \vdots & & \vdots \\ C_{M1} & C_{M2} & \cdots & C_{MN} \end{pmatrix}, C_{IJ} = \sum_{K=1}^{P} A_{IK} B_{KJ}.$$

2. (40%) Implement matrix-matrix multiplication using the following methods
   (a) Basic formulation
   (b) Block formulation
   (c) Numerical library from processor venders.

   Compare their performance for different matrix sizes. For block formulation, test different block sizes.

3. (40%) Implement polynomial multiplication by using the following methods
   (a) Direct method.
   (b) Divide and conquer method.
   (c) Fast Fourier method.

   Compare their performance for different sizes of polynomial.