Theory of computation

Can computer do anything?

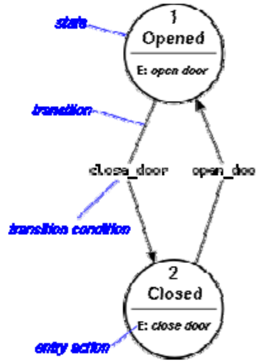
- Which computer are we talking about?
  - Supercomputers, Cloud, PC, iPhone, quantum computer, …
  - We will use an abstract model (Turing machine).
- There are two important problems
  - Can computer solve all kinds of problems?
    - There are some problems unsolvable by today's machines or any future algorithmic machine.
      - Ex: The halting problem
  - Which problems can be solved efficiently by computers?
    - There are problems too complex to be solvable in practice.
      - The P-NP classification

Classification of machines and problems

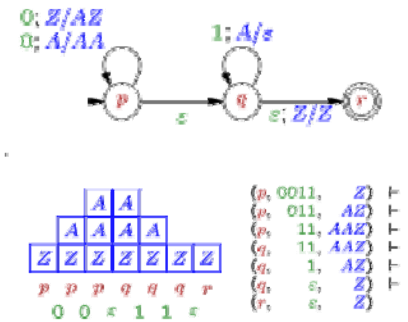
- Classification of machine power (Chomsky hierarchy)
  - Finite state machine (No memory)
  - Pushdown automata (Using a stack as memory only)
  - Tuning machine (Most of current computers)
- Classification of problem difficulty
  - Polynomial time solvable problems
    - The lower bound of problem complexity. For example, sorting
  - Non-deterministic polynomial time solvable problems
  - NP-complete/NP-hard problems
    - All NP problems can be solved via transforming to this class of problems. Ex: 3-SAT problems
  - Non-solvable problems (by the tuning machine model)

FSM and Pushdown Automata

- FSM example



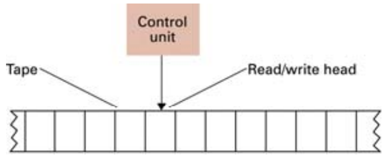
- PDA example
  - Recognize $\{0^n 1^n | n>=0\}$



From wikipedia

Turing machines

- Introduced by Alan M. Turing in 1936
- Conceptual device that consists
  - A control unit that can read and write symbols on a tape
  - The tape extends indefinitely at both ends
  - Each cell on the tape can store a finite set of symbols

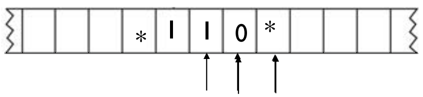


- At any time, it must be in one of a finite number of states
  - Computation starts in the start state, and stops in the halt state

Turing machine operation

- Inputs at each step
  - State
  - Value at current tape position
- Actions at each step
  - Write a value at current tape position
  - Move read/write head
  - Change state

An example of a Turing machine



Current state	Current cell content	Value to write	Direction to move	New state to enter
START	*	*	Left	ADD
ADD	0	1	Right	RETURN
ADD	1	0	Left	CARRY
ADD	*	*	Right	HALT
CARRY	0	1	Right	RETURN
CARRY	1	0	Left	CARRY
CARRY	*	1	Left	OVERFLOW
OVERFLOW	*	*	Right	RETURN
RETURN	0	0	Right	RETURN
RETURN	1	1	Right	RETURN
RETURN	*	*	No move	HALT

Church-Turing thesis

- Church-Turing thesis: a Turing machine can compute any computable function.
  - Not proven, but generally accepted
- Function
  - A mapping of a set of input values and a set of output values.
  - Each input is assigned a single output
- Computing a function
  - Determining the output value associated with a given input
- Noncomputable function
  - A function that cannot be computed by any algorithm

Which problems cannot be solved by TM?

- Any problem that can be solved on a computer has a solution expressed in some language
  - Any programming language comprising the features of this language can surely express a solution to the problem
- The halting problem: for a given program (encoded as a bit stream), return 1 if the program will eventually halt, or 0 if the program will run forever
- A wrong algorithm: run the program to see if it can halt.
  - If the program halts, then return 1.
  - If the program doesn't halts for 10 years, ….
- A problem is solvable means it needs be answered in a finite number of operations.

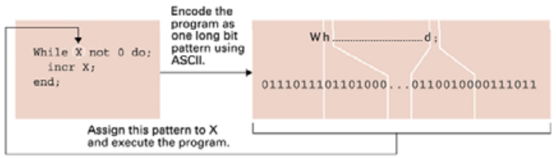
An example

```
While x is not 0
   x = x + 1
end
```

- Consider this program
  - x is a positive integer
    - Not considering the finite representation
  - If the input x is 0, the program halts.
  - Otherwise, it does not halt.

Self-reference and self-terminating

- Self-reference: use the encoded program as the input

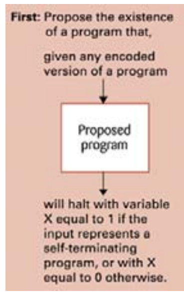


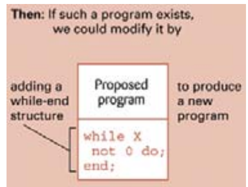
- Self-terminating: if a program with self-reference can halt, then it is called self-terminating.
  - The example program is not self-terminating.

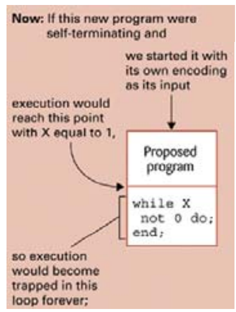
The undecibility of the halting program

Theorem: the halting problem is noncomputable.

Proof:





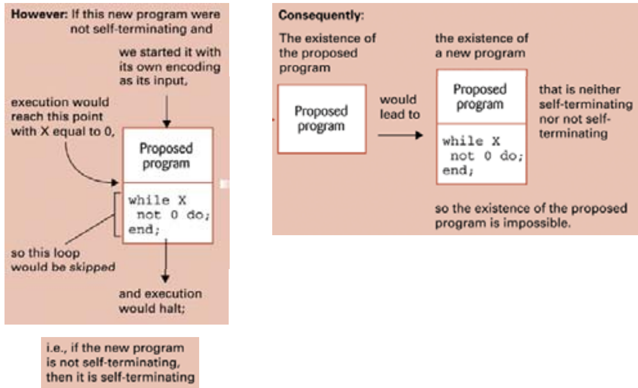


Is this new program self-terminating?

Maybe,…maybe not
But must be one of them.

i.e., if the new program is self-terminating, then it is not self-terminating.

Proof (continue)



Problem classification

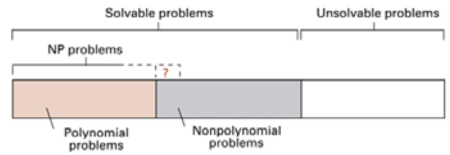
- Among solvable problems, some problems appear easier than the others.
- How to classify problems based on their difficulties?
  - Classification may be based on time, space, or other computing resources.
    - Unless otherwise noted, "complexity" means "time complexity."
  - Answer: The complexity of a problem is measured by the time complexity of the "best" algorithm to solve it.
- Unfortunately, finding a best solution or knowing it is the best is difficult for most problems.
  - Ex: The complexity of "searching a list" is O(N).

Class P

- Class P is the set of decision problems that can be solved by a Turing machine in a polynomial time.
  - Decision problem is a problem whose answer is either yes or no.
    - The halting problem is a decision problem.
  - If the problem size is N, polynomial time means the running time is dominated by a polynomial function of N
  - Exponential function $f(N)=2^N$ is always larger than the polynomial $p(N)=N^k$ for any constant k if N is large enough.
- Most computer scientists consider the problems in class P can be solved practically

Class NP

- Class NP is the set of problems that the "yes"-answers can be verified by a Turing machine in polynomial time.
  - The halting problem is in not NP.
- A million dollar question: P=NP?



  - The Clay Math Institute's first millennium prize problem
  - A new proof by Vinay Deolalikar (Aug 2010)
    - http://www.win.tue.nl/~gwoegi/P-versus-NP/Deolalikar.pdf