Logic design

Binary operation

- Many object and phenomena can represent binary data

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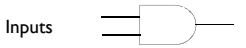
- Many mathematical concepts for binary data have similar structure
  - Set operation:
    - Basic element: Empty set/Universal set
    - Operation: UNION, INTERSECTION, COMPLEMENT
  - Logic operation:
    - Basic element: True/False
    - Operation: AND, OR, NOT,…

More of those mathematical properties will be taught in Discrete Math

Boolean operation

- Any operation that manipulates one or more true/false values
  - Can be used to operate on bits
- Specific operations
  - AND
  - OR
  - XOR
  - NOT
- Gates are devices that produce the output of a Boolean operation when given the operation's input values

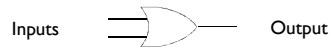
AND



- The AND Operation

0	1	0	1
AND 0	AND 0	AND 1	AND 1
0	0	0	1

OR



- The OR Operation

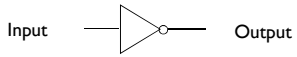
```
  0     1     0     1
OR 0  OR 0  OR 1  OR 1
  0     1     1     1
```

XOR (eXclusive OR)

- The OR Operation

```
  0     1     0     1
XOR 0 XOR 0 XOR 1 XOR 1
  0     1     1     0
```

NOT



- The NOT Operation

```
NOT 0   NOT 1
  1       0
```

Truth table

- Enumerates all the input combinations and the corresponding output of Boolean operations
- Four commonly used Boolean operators

Input		Output
A	B	A.and.B
0	0	0
0	1	0
1	0	0
1	1	1

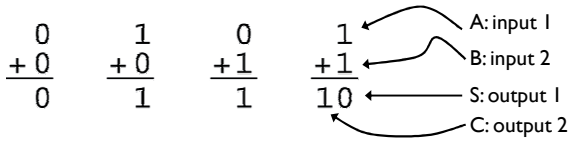
Input		Output
A	B	A.or.B
0	0	0
0	1	1
1	0	1
1	1	1

Input		Output
A	B	A.xor.B
0	0	0
0	1	1
1	0	1
1	1	0

Input	Output
A	Not A
0	1
1	0

Boolean functions

- Almost all binary operations of computers can be carried out by logic gates
- Example: One bit adder
  - Two inputs and two outputs (S: sum, C: carry out)

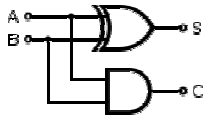


Implementation of an one bit adder

- The truth table of an one-bit adder

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

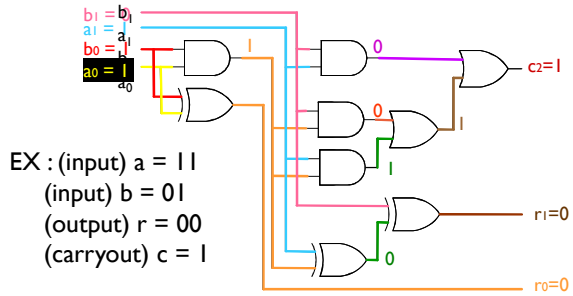
- Compare it to the truth table of Boolean function AND, OR, XOR, NOT
  - S = A .XOR. B
  - C = A .AND. B



More of those will be taught in '邏輯設計'.

One bit full adder

- Equations : (sum) $s_i = a_i \oplus b_i \oplus c_i$
  (carry out); $c_{i+1} = a_ib_i + a_ic_i + b_ic_i$

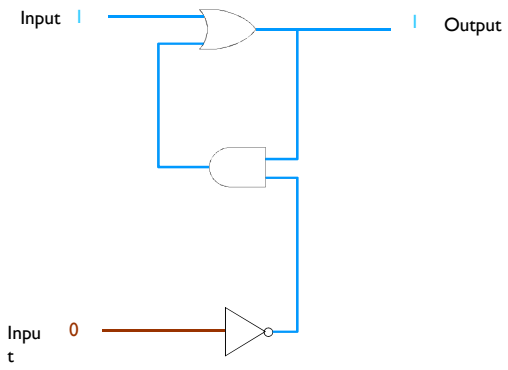


EX : (input) a = 11
(input) b = 01
(output) r = 00
(carryout) c = 1

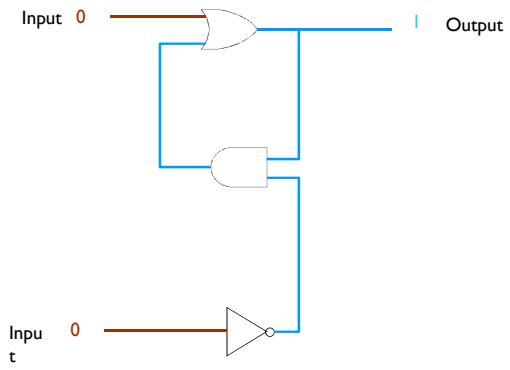
Flip-flops

- **Flip-flop** = a circuit built from gates that can store one bit of data.
  - Has an input line which sets its stored value to 1
  - Has an input line which sets its stored value to 0
  - While both input lines are 0, the most recently stored value is preserved
- A flip-flop is ideal for the storage of a bit within a computer
  - A flip-flop loses data when its power is turned off

A Simple Flip-Flop Circuit

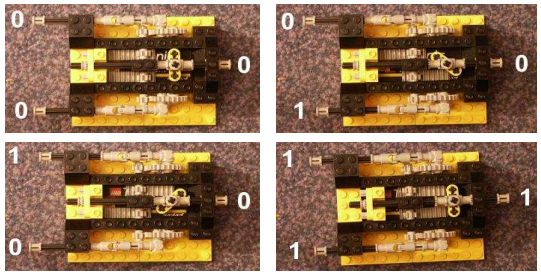


A Simple Flip-Flop Circuit

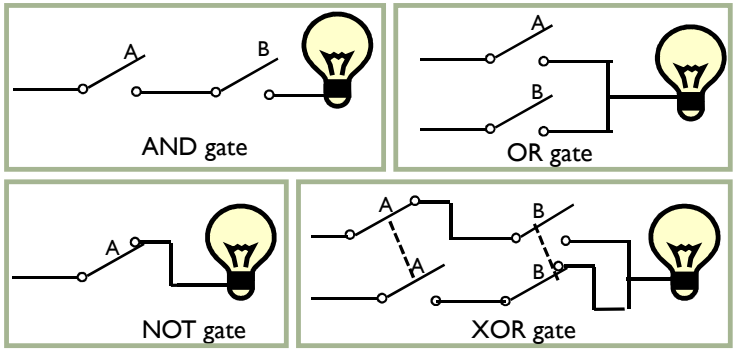


What can be a gate?

- LEGO's "mechanical gates"
  - The AND gate



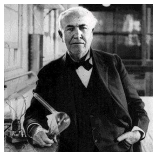
Circuit gates

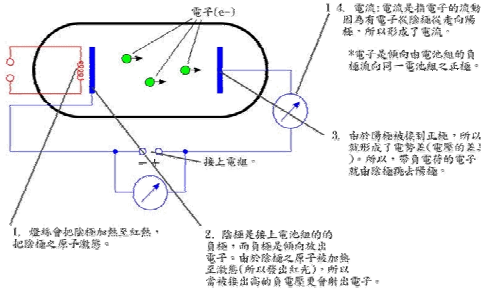


- Can we flip the switches without hands?

Electronic switch

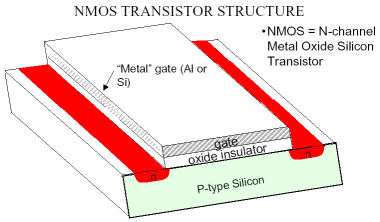
- The earliest one is the vacuum tube
  - 1884, Thomas Edison



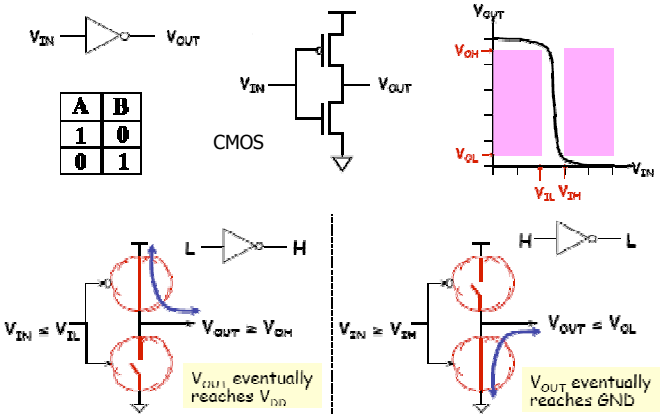


Transistor

- The problems of vacuum tubes are slow, large, expensive, and easy to break.
- Transistor can make it faster, smaller, and more robust.



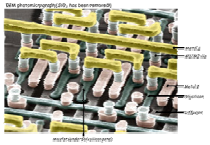
Transistors for logic gates



- More of those will be taught in '電子電路'

Integrated circuit (IC)

- An electronic circuit consisted of transistors and other components in the thin substrate of semiconductor material.
- Also known as **IC**, **microchip**, or **chip**.
- Invented by Jack Kilby and Robert Noyce
  - 2000 Nobel Prize in Physics
- **VLSI**:Very-Large-Scale IC
  - More than million transistors



- More in 'VLSI設計'