$$\min_{x,y} f(x,y) = x^4 + 2x^3 + 24x^2 + y^4 + 12y^2$$

$$\nabla f(x,y) = \begin{pmatrix} 4x^3 + 6x^2 + 48x \\ 4y^3 + 24y \end{pmatrix}$$

$$\nabla^2 f(x,y) = \begin{pmatrix} 12x^2 + 12x + 48 & 0 \\ 0 & 12y^2 + 24 \end{pmatrix}$$

Minimizer is at $(x,y) = ?$

Quadratic model

At $(x, y) = (2, 1)$, $f(2, 1) = 141$

$$\nabla f(2,1) = \begin{pmatrix} 152 \\ 28 \end{pmatrix} \qquad \nabla^2 f(2,1) = \begin{pmatrix} 120 & 0 \\ 0 & 36 \end{pmatrix}$$

$$\begin{aligned} m(p) &= f(2,1) + \nabla f(2,1)^T p + \frac{1}{2} p^T \nabla^2 f(2,1) p \\ m(x,y) &= 60(2-x)^2 + 18(1-y)^2 + \\ &\quad 152(2-x) + 28(1-y) + 141 \end{aligned}$$

Newtons direction $p^{\mathrm{N}} = -(\nabla^2 f)^{-1} \nabla f = \begin{pmatrix} -1.266 \\ -0.777 \end{pmatrix}$

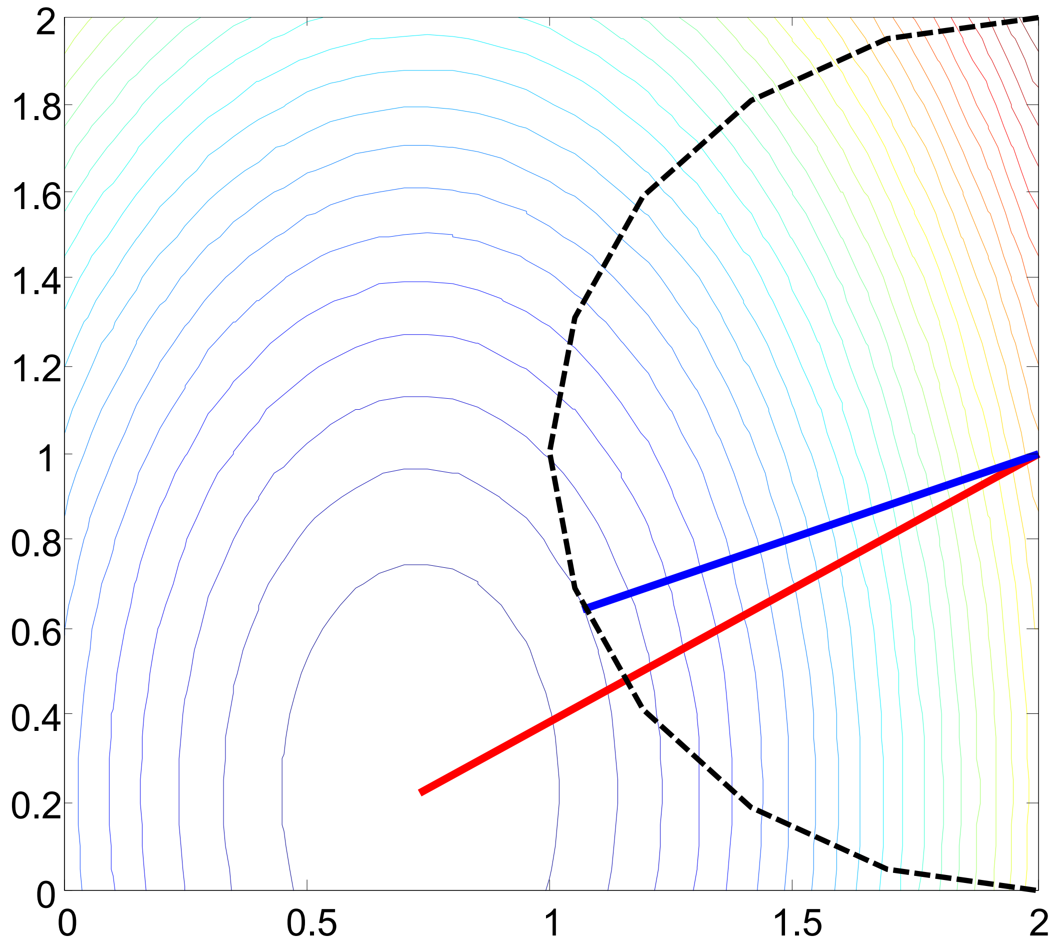
Trust region $\Delta=1$

- $\|p^N\|=1.48 > \Delta$
- Find λ s.t. $(B+\lambda I)\, p^*=-g$ and $\lambda(\Delta - \|p^*\|)=0$

$$B+\lambda I = \begin{pmatrix} 120+\lambda & 0 \\ 0 & 36+\lambda \end{pmatrix} \quad p = \begin{pmatrix} \frac{-152}{120+\lambda} \\ \frac{-28}{36+\lambda} \end{pmatrix}$$

$$\|p\| = 1 \Rightarrow \left(\frac{152}{120+\lambda}\right)^2 + \left(\frac{28}{36+\lambda}\right)^2 = 1$$

$$\text{The solution } \lambda \approx 42.655 \quad p^* = \begin{pmatrix} -0.9345 \\ -0.3560 \end{pmatrix}$$



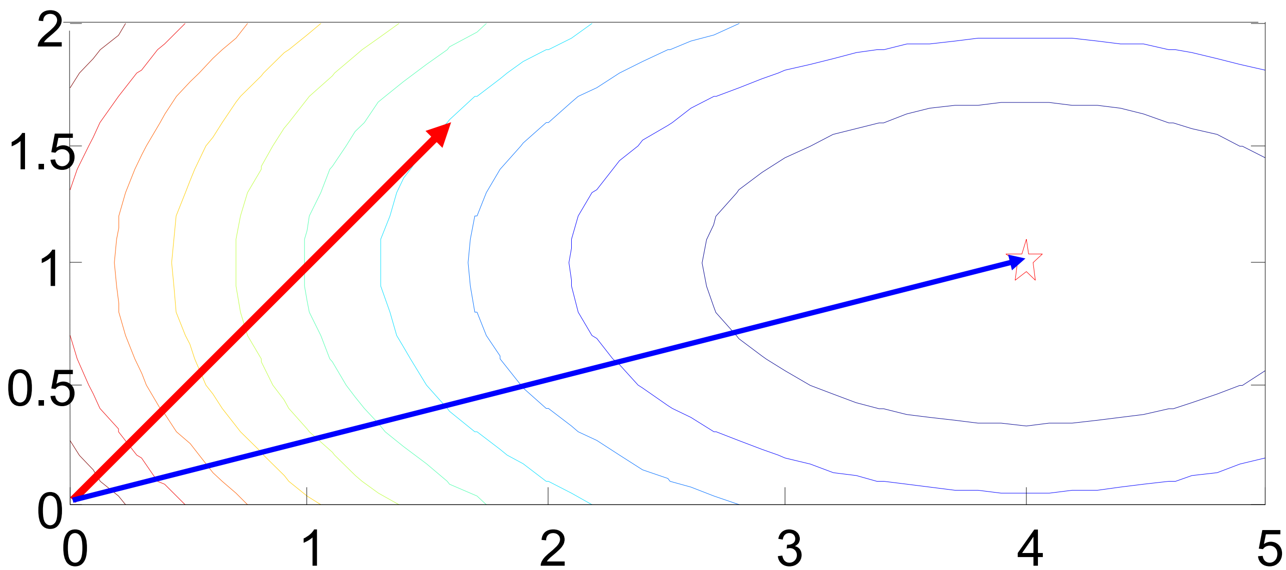
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2

Dogleg method

Consider $f(x) = \frac{1}{2}x^T Q x - c^T x$

$$Q = \begin{pmatrix} 1/4 & 0 \\ 0 & 1 \end{pmatrix}, c = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, x_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$p^{\mathrm{U}} = \begin{pmatrix} 1.6 \\ 1.6 \end{pmatrix}, p^{\mathrm{B}} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}, p^{\mathrm{B}} - p^{\mathrm{U}} = \begin{pmatrix} 2.4 \\ -.6 \end{pmatrix}$$



$$p(\tau) = \begin{cases} \tau p^U & 0 \le \tau \le 1 \\ p^U + (\tau - 1)(p^B - p^U) & 1 < \tau \le 2 \end{cases}$$

Δ	λ	min f(p)	τ	f(p(τ))
1	.9212	-1.1478	0.625	-1.1017
2	.2912	-1.8935	1.064	-1.7116
3	.0998	-2.3331	1.551	-2.3193