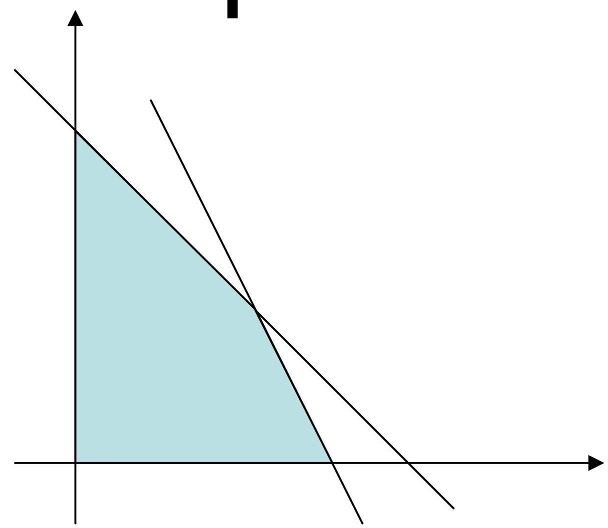
Example from Chap 13

$$\begin{aligned} \min_{x} \quad & -x_1 + x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 40 \\ & 2x_1 + x_2 \leq 60 \\ & x_1, x_2 \geq 0 \end{aligned}$$



$$\begin{aligned} \min_{x} \quad & c^T x \\ \text{s.t.} \quad & A^T x = b \\ & x \geq 0 \end{aligned}$$

$$c = \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{pmatrix} \quad b = \begin{pmatrix} 40 \\ 60 \end{pmatrix}$$

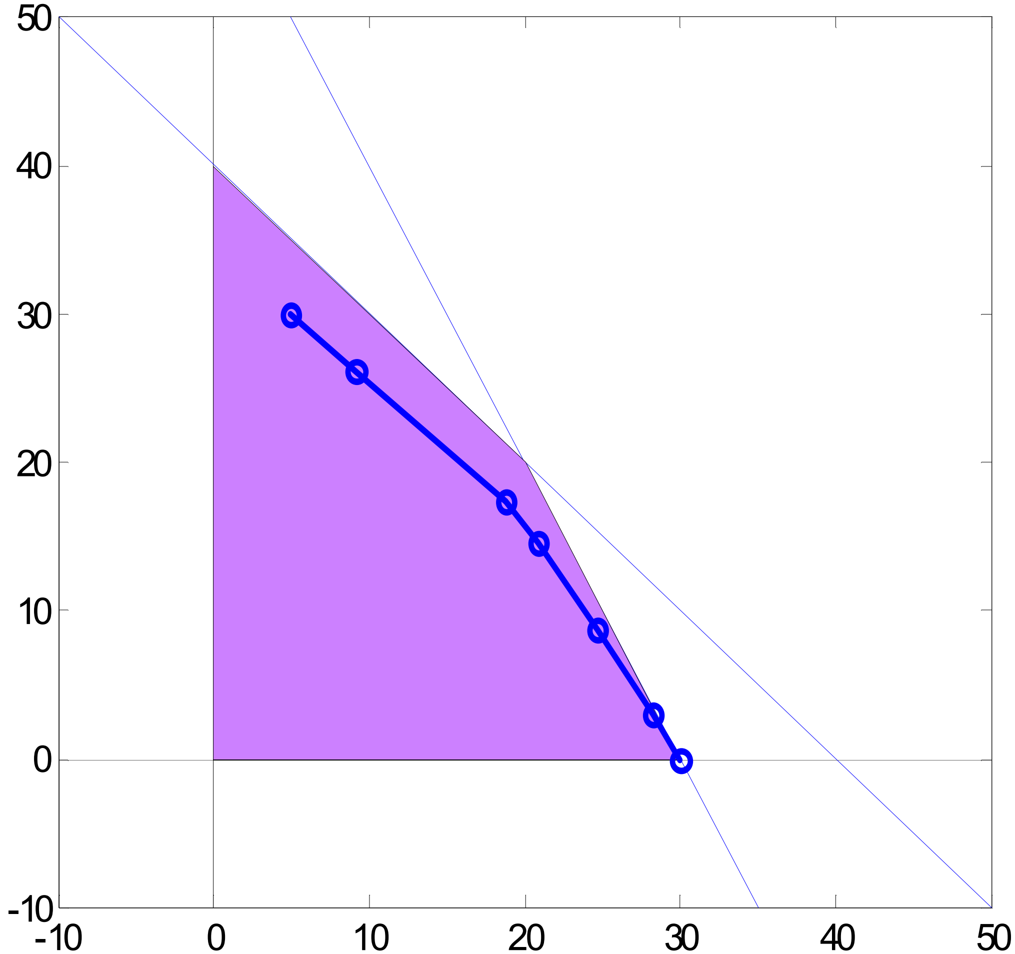
Newton's method

$$F(x, \lambda, s) = \begin{pmatrix} A^T\lambda + s - c \\ Ax - b \\ XSe \end{pmatrix} = 0, \text{ for } x, s \geq 0$$

$$J = \begin{pmatrix} \nabla_x r_c & \nabla_\lambda r_c & \nabla_s r_c \\ \nabla_x r_b & \nabla_\lambda r_b & \nabla_s r_b \\ \nabla_x r_{XS} & \nabla_\lambda r_{XS} & \nabla_s r_{XS} \end{pmatrix} = \begin{pmatrix} 0 & A^T & I \\ A & 0 & 0 \\ S & 0 & X \end{pmatrix}$$

$$J(x, \lambda, s) \begin{pmatrix} \Delta x \\ \Delta \lambda \\ \Delta s \end{pmatrix} = -F(x, \lambda, s)$$

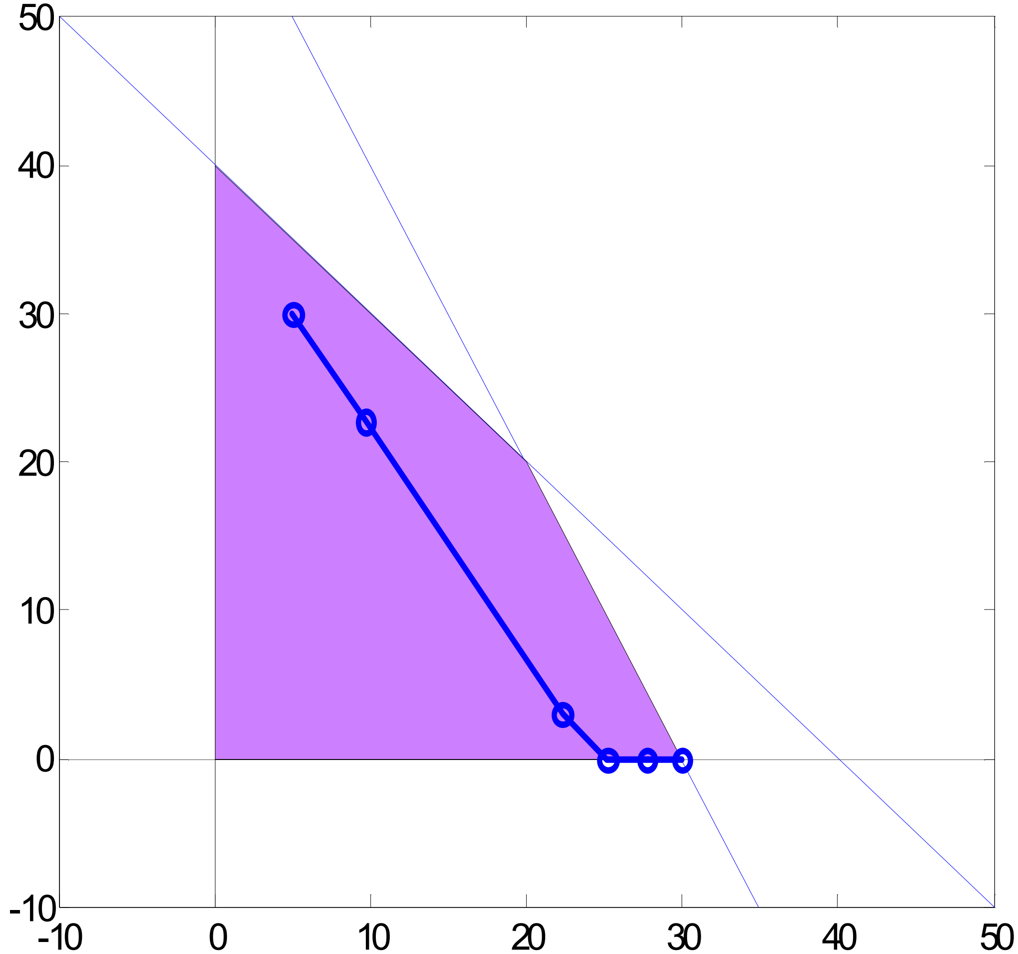
$$(x_{k+1}, \lambda_{k+1}, s_{k+1}) = (x_k, \lambda_k, s_k) - \alpha_k J(x_k, \lambda_k, s_k)^{-1} F(x_k, \lambda_k, s_k)$$



$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 5 \\ 30 \\ 5 \\ 10 \end{pmatrix}$$

$$s = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} = \begin{pmatrix} .1 \\ 2 \\ .1 \\ .1 \end{pmatrix}$$

$$\lambda = A^{-T}(c - s) = \begin{pmatrix} -0.96 \\ -0.06 \end{pmatrix}$$



$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 5 \\ 30 \\ 5 \\ 10 \end{pmatrix}$$

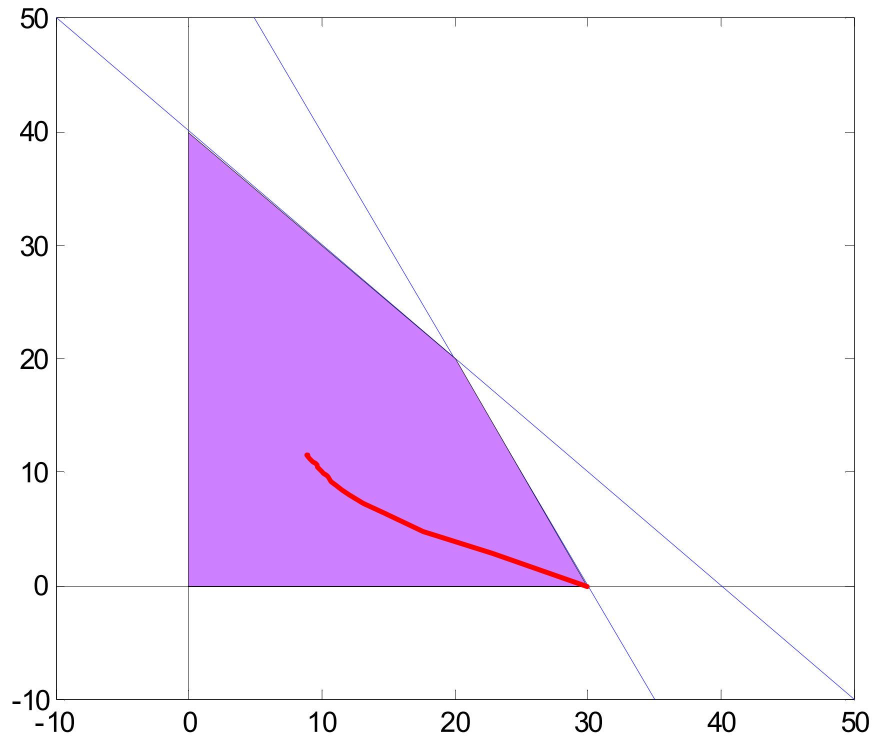
$$s = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} = \begin{pmatrix} .1 \\ .1 \\ .1 \\ .1 \end{pmatrix}$$

$$\lambda = A^{-T}(c - s) = \begin{pmatrix} 0.266 \\ -0.366 \end{pmatrix}$$

The central path

$$F(x,\lambda,s) = \begin{pmatrix} A^T\lambda + s - c \\ Ax - b \\ XSe - \tau e \end{pmatrix} = 0, \text{ for } x, s \geq 0$$

- The trajectory that solves F=0 for τ = [0,300]



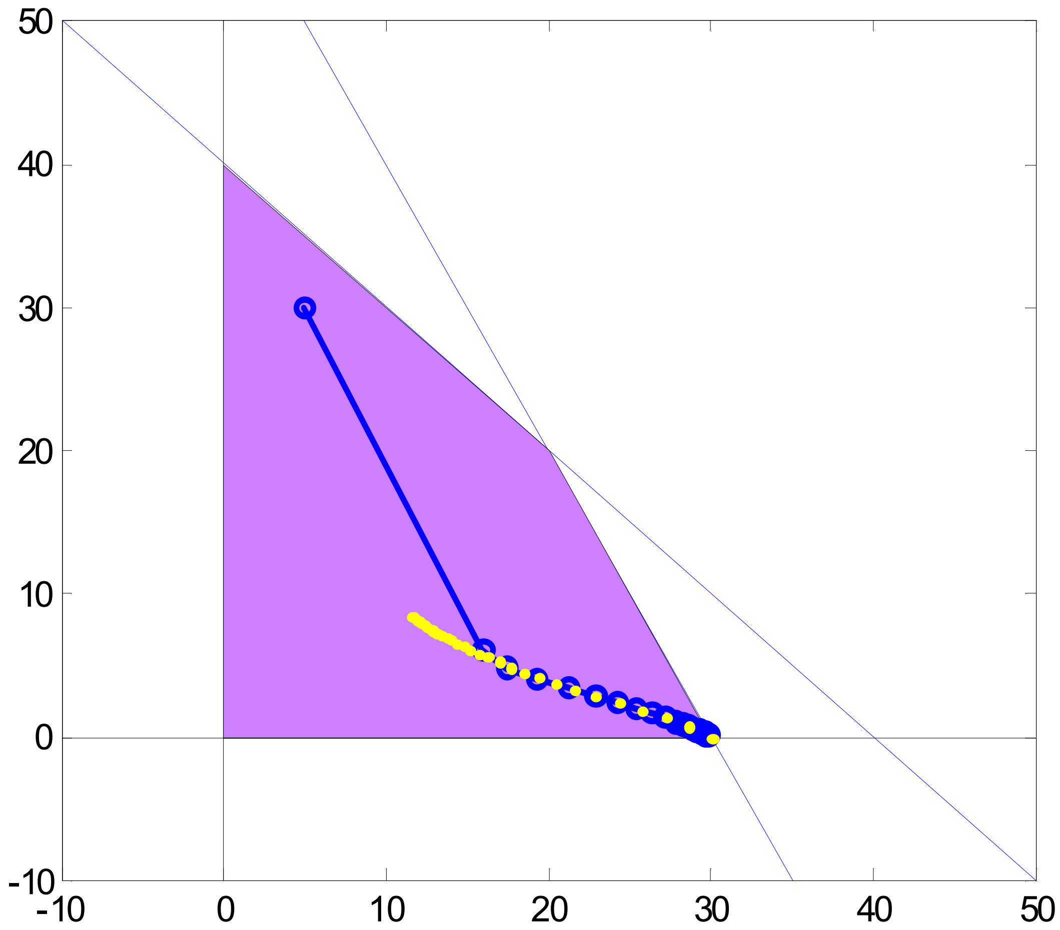
Path following algorithm

- Search direction is the solution of the following system,

$$\begin{pmatrix} 0 & A^T & I \\ A & 0 & 0 \\ S & 0 & X \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta \lambda \\ \Delta s \end{pmatrix} = \begin{pmatrix} -r_c \\ -r_b \\ -r_{XS} + \sigma_k \mu_k e \end{pmatrix}$$

- Duality measurement $\mu = \frac{1}{n} \sum_{i=1}^{n} x_i s_i = \frac{x^T s}{n}$
- Centering parameter σ_k.

Short-step path following

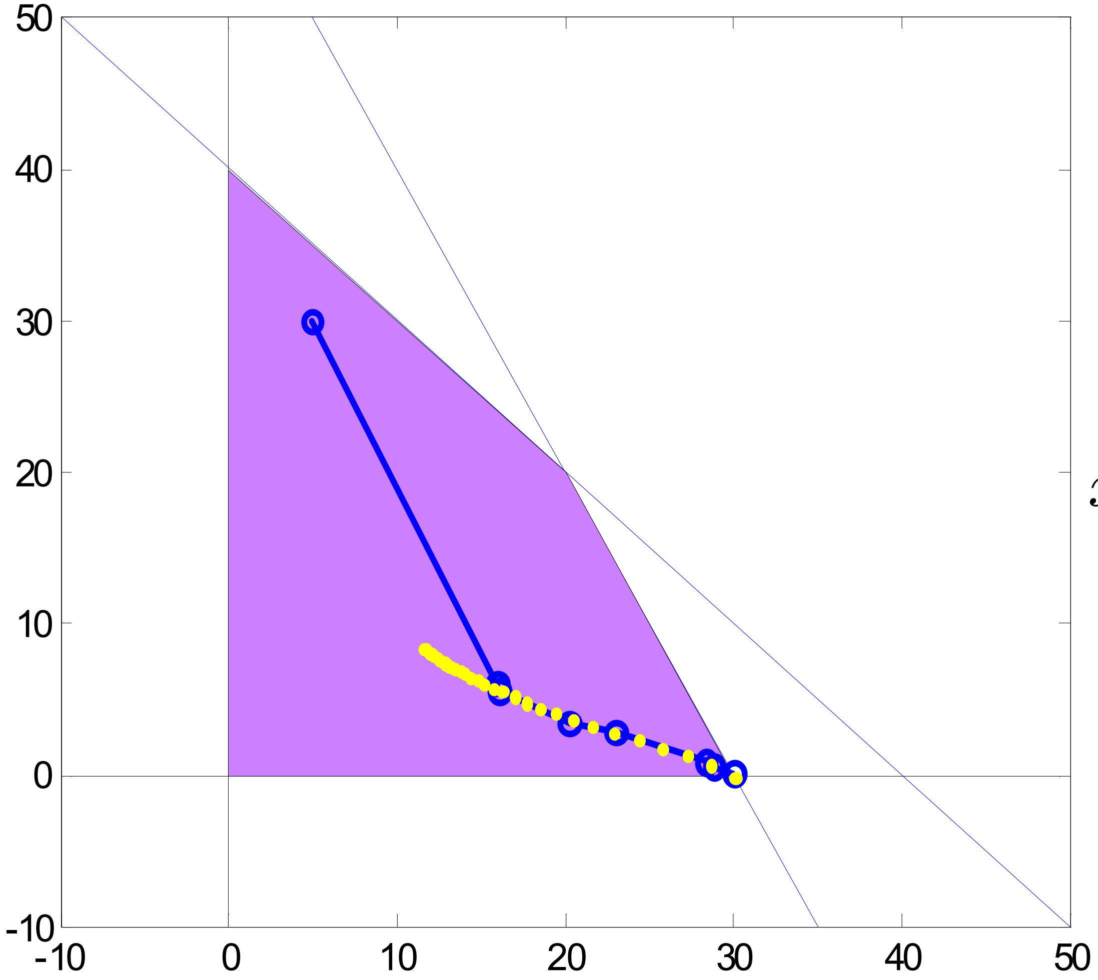


- No line search
- Fixed σ_k.
- $\mathcal{N}_2$.

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 5 \\ 30 \\ 5 \\ 10 \end{pmatrix}$$

$$s = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ .25 \end{pmatrix}$$

Long-step path-following



- Line search
- Varied σ_k.
- $\mathcal{N}_{-\infty}$.

$$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 5 \\ 30 \\ 5 \\ 10 \end{pmatrix}$$

$$s = \begin{pmatrix} s_1 \\ s_2 \\ s_3 \\ s_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \\ .25 \end{pmatrix}$$