CS3331 Numerical Methods

Quiz 3, Oct 31th

Name: ________________________, ID: ________________________

1. Matrix $\mathbf{A} = \begin{pmatrix} 4 & -4 & 0 \\ -4 & 8 & 8 \\ 0 & 8 & 20 \end{pmatrix}$ is symmetric positive definite.

   (a) What is the 1-norm of $\mathbf{A}$? (12pt)

   First column sum $= |4| + |-4| + |0| = 8$
   Second column sum $= |4| + |8| + |8| = 20$
   Third column sum $= |0| + |8| + |20| = 28$
   1-norm of $\mathbf{A}$ is $\max(8, 20, 28) = 28$.

   (b) The LU decomposition with pivoting of $\mathbf{A}$ is $\mathbf{PA} = \mathbf{LU}$. The U-factor is $\mathbf{U} = \begin{pmatrix} 4 & -4 & 0 \\ 0 & 8 & 20 \\ 0 & 0 & -2\ \cancel{-8} \end{pmatrix}$, and the permutation matrix $\mathbf{P} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$. What is the lower triangular matrix $\mathbf{L}$? (18pt)

$$\mathbf{L} = \begin{pmatrix} 1 \ (2pt) & 0 & 0 \\ 0 \ (4pt) & 1 \ (2pt) & 0 \\ -1 \ (4pt) & .5 \ (4pt) & 1 \ (2pt) \end{pmatrix}$$

(c) The Cholesky decomposition of $\mathbf{A} = \begin{pmatrix} 4 & -4 & 0 \\ -4 & 8 & 8 \\ 0 & 8 & 20 \end{pmatrix}$ is $\mathbf{A} =$ ~~$\bar{\mathbf{L}}^T\bar{\mathbf{L}}$~~ $\bar{\mathbf{L}}\bar{\mathbf{L}}^T$. What is the lower triangular matrix $\bar{\mathbf{L}}$? (*NO pivoting*) (20pt)

$\bar{\mathbf{L}} =$

2 4pt	0	0
-2 4pt	2 4pt	0
0	4 4pt	2 4pt

$$\bar{\mathbf{L}} = \begin{pmatrix} \ell_{11} & 0 & 0 \\ \ell_{21} & \ell_{22} & 0 \\ 0 & \ell_{32} & \ell_{33} \end{pmatrix}$$

$$\bar{\mathbf{L}}\bar{\mathbf{L}}^T = \begin{pmatrix} \ell_{11}^2 & \ell_{11}\ell_{21} & 0 \\ \ell_{11}\ell_{21} & \ell_{21}^2 + \ell_{22}^2 & \ell_{22}\ell_{32} \\ 0 & \ell_{22}\ell_{32} & \ell_{32}^2 + \ell_{33}^2 \end{pmatrix} = \begin{pmatrix} 4 & -4 & 0 \\ -4 & 8 & 8 \\ 0 & 8 & 20 \end{pmatrix}$$

$$\begin{array}{lcll} \ell_{11}^2 & = & 4 & \Rightarrow \quad \ell_{11} = 2 \\ \ell_{11}\ell_{21} & = & -4 & \Rightarrow \quad \ell_{21} = -4/2 = -2 \\ \ell_{21}^2 + \ell_{22}^2 & = & 8 & \Rightarrow \quad \ell_{22} = \sqrt{8 - (-2)^2} = 2 \\ \ell_{22}\ell_{32} & = & 8 & \Rightarrow \quad \ell_{32} = 8/2 = 4 \\ \ell_{32}^2 + \ell_{33}^2 & = & 20 & \Rightarrow \quad \ell_{33} = \sqrt{20 - (4)^2} = 2 \end{array}$$