CS3331 Numerical Methods

Quiz 2, Oct 17th

Name: _____, ID: _____

1. Suppose all five methods taught in class are able to converge to a simple root. What methods converge linearly? what methods converges superlinearly? and what methods converge quadratically? (20pt)

   - Linearly: Bisection, false position (or regula falsi).
   - Superlinearly: Secant method, Muller's method.
   - Quadratically: Newton's method.

2. Given a nonlinear function $f(x) = 4x^2 - 12x + 3$, and

$$\begin{aligned} f(0) &= 4 * 0^2 - 12 * 0 + 3 = 3 \\ f(1) &= 4 * 1^2 - 12 * 1 + 3 = -5 \\ f'(x) &= 8x - 12 \end{aligned}$$

(a) If the bisection method is used to solve $f(x) = 0$ with initial interval [0,1], what is the next interval? (10pt)

Mid point = (0+1)/2 =1/2

$$f(1/2) = 4 * (1/2)^2 - 12 * (1/2) + 3 = -2$$

Since $f(0)$ and $f(1/2)$ have different signs, the next interval is [0,1/2].

(b) If Newton's method is used to solve $f(x) = 0$ with the initial guess $x_0 = 0$. What is the next approximation x_1? (10pt)

$$\begin{aligned} x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ &= 0 - \frac{4 * 0^2 - 12 * 0 + 3}{8 * 0 - 12} \\ &= -\frac{3}{-12} = 1/4 \end{aligned}$$

(c) In secant method, the derivative is replaced by an approximation. If $x_0 = 1, x_1 = 0$, what is the derivative approximation? (10pt)

$$\frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{3 - (-5)}{0 - 1} = \frac{8}{-1} = -8$$