CS3331 Numerical Methods

Quiz 1

Oct 3

Name: _____, ID: _____

1. Consider the equation $x^2 = 0.25$

   (a) What is the relative error of the answer $x_1 = 0.48, x_2 = -0.48$? (10pt)

   The exact solution is $x = \pm 0.5$. relative error $= \dfrac{|0.5 - 0.48|}{|0.5|} = 0.04$

   (b) What is the backward error of the above answer? (10pt) (In this problem, the input is 0.25.)

   The backward error is $|0.48^2 - 0.25|$.

   (c) How many significant digits does the above answer have? (10pt)

   Since the relative error $= 0.04 < 5 \times 10^{-2}$, there are 2 significant digits in the answer.

2. Suppose a sequence $x_0, x_1, x_2, \cdots$ satisfies the recursion $x_k = -4x_{k-1}^2 + 8x_{k-1} - 3$.

(a) Suppose this sequence converges to 1. What are the order of convergence and the asymptotic error constant? (10pt)

$$x_k - 1 = -4x_{k-1}^2 + 8x_{k-1} - 3 - 1 = -4(x_{k-1}^2 - 2x_{k-1} + 1) = -4(x_{k-1} - 1)^2.$$

$$\lim_{k \to \infty} \frac{|x_k - 1|}{|x_{k-1} - 1|^2} = 4.$$

The order of convergence is 2, and the asymptotic error constant is 4.

(b) What is the necessary and sufficient condition for this sequence converging to 1? (10pt)

$$|x_0 - 1| < \frac{1}{4}.$$